Interaction Notes

Note 74

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Electromagnetic Pulse Penetration through Small Apertures

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Abstract

The electromagnetic field penetration through small apertures is discussed for general aperture shapes. Approximate analytical expressions are obtained for the penetration fields. In particular the fields that penetrate a small aperture into the region between two parallel, perfectly conducting plates are determined and some numerical results are obtained.

INTRODUCTION

Electromagnetic Shielding has drawn much attention and a great deal is known about it.[1-3] However little is known about the effect on shielding effectiveness caused by the presence of small apertures[4]. It has been found that the field penetrating a small aperture may be much larger than the skin depth attenuated field[5,6]. For that reason a comprehensive study of the field penetration through small apertures is presented here. This work is an extension of earlier work[7-9].

The field penetration through a small aperture into a long cylindrical shell immersed in a uniform axial magnetic field has been studied both experimentally and theoretically by Bombardt[5,6]. The theoretical study followed the formulation developed by Kaden[10] and it entailed the use of the quasi-static approximation. Bombardt obtained good agreement between the theoretical and experimental results for a pulsed magnetic field and a circular aperture, indicating the validity of the quasi-static approximation. Bombardt did not determine the contribution to the penetration fields from an impressed electric field. This paper also employs the quasi-static analysis but a general impressed field (both electric and magnetic) is considered as well as arbitrary aperture shapes. Comparison of the data obtained with Bombardt experimental and theoretical data yields excellent agreement.

Particular advantages of using the quasi-static solution for determining the field penetration through apertures are that the penetration field is

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[1]Superscripts refer to the List of References at the end of this paper.

expressed in a form independent of the external field configuration and that the dependence of the penetration field on the aperture configuration is expressed simply in terms of the elements of a dyadic. This dyadic relates the impressed field components to the dipole moments of the equivalent magnetic sources for the field distribution within the aperture.

ANALYSIS

Aperture in a Plane

Consider an elliptical aperture oriented in a perfectly conducting-infinitesimally thin sheet as shown in figure 1. Further consider the field incident on the aperture from below, the $z < 0$ region. The x-axis and y-axis of a cartesian coordinate system is oriented along the major and minor axes, respectively, of the elliptical aperture. The z-axis is directed perpendicular to the sheet.

One may define equivalent magnetic charge and current distributions in the aperture to represent the appropriate field components for determining the field penetrating the aperture. This was first accomplished by Bethe[1]. With a knowledge of the equivalent source distributions the penetration field may be expressed in terms of the multipole expansion. If the aperture is small in comparison to the wavelength of the incident radiation, then the penetration field may be expressed in terms of the dipole moments to determine the field at distances large compared to the aperture dimensions.

At the outset harmonic time dependence is assumed, but suppressed. The temporal analysis is obtained by using Fourier superposition. Equivalent dipole moments of the aperture field distribution may be expressed in terms of a dyadic $\overleftrightarrow{\alpha}$. They are[12]

$$\vec{M}_o = \overleftrightarrow{\alpha}\cdot\vec{H}_o \qquad (1)$$

$$\vec{P}_o = \overleftrightarrow{\alpha}\cdot\vec{D}_o \qquad (2)$$

where $\vec{H}_o$ and $\vec{D}_o$ are the magnetic field strength and the electric flux density, respectively, that would exist at the position of the aperture if it were

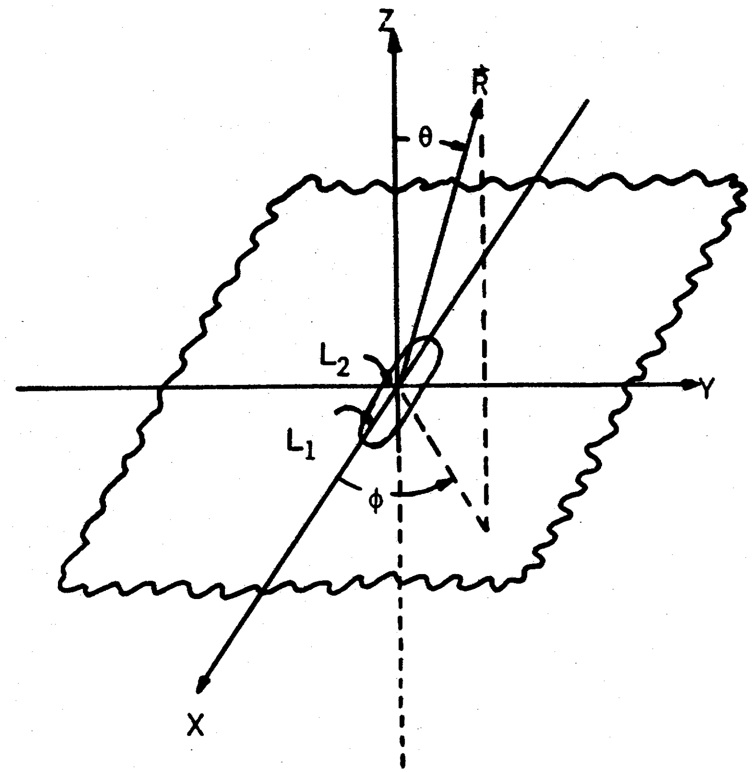


FIGURE 1: ELLIPTICAL APERTURE IN A PERFECTLY CONDUCTING SHEET.

completely shorted. The elements of the dyadic for an elliptical aperture are

$$\alpha_{11} = -\frac{2\pi}{3}\frac{\ell_1^3\, e^2}{K(e^2)-E(e^2)} \tag{3}$$

$$\alpha_{22} = -\frac{2\pi}{3}\frac{\ell_1^3 e^2(1-e^2)}{E(e^2)- (1-e^2)K(e^2)} \tag{4}$$

$$\alpha_{33} = -\frac{2\pi}{3}\frac{\ell_1^3\,(1-e^2)}{E(e^2)} \tag{5}$$

$$\alpha_{ij} = 0 \qquad i \neq j \tag{6}$$

Thus

$$\overleftrightarrow{\alpha} = \alpha_{11}\,\hat{x}\hat{x} + \alpha_{22}\,\hat{y}\hat{y} + \alpha_{33}\,\hat{z}\hat{z} \tag{7}$$

where $E(e^2)$ and $K(e^2)$ are elliptic integrals of the first and second kinds[14], respectively, e is the eccentricity of the elliptic aperture and ℓ_1 is the length of the semi major axis. If ℓ_2 is the length of the semi minor axis then

$$e = \sqrt{1 - (\ell_2/\ell_1)^2} \tag{8}$$

The field components about an electric dipole oriented along the polar axis of a spherical coordinate system are[13]

$$\left.\begin{aligned}
E_r(r,\theta) &= \frac{P_o\, e^{-jk_o r}}{2\pi\varepsilon_o r}\left(\frac{jk_o}{r} + \frac{1}{r^2}\right)\cos\theta \\
E_\theta(r,\theta) &= \frac{P_o\, e^{-jk_o r}}{4\pi\varepsilon_o r}\left(\frac{jk_o}{r} + \frac{1}{r^2} - k_o^2\right)\sin\theta \\
B_\phi(r,\theta) &= \frac{\eta_o k_o P_o}{4\pi r}\, e^{-jk_o r}\left(-k_o + j\frac{1}{r}\right)\sin\theta
\end{aligned}\right\} \tag{9}$$

and for the magnetic dipole oriented along the polar axis[3]

$$\left.\begin{aligned}
E_\phi(r,\theta) &= -\frac{\eta_o k_o M_o}{4\pi r}\, e^{-jk_o r}\,(-k_o + j\,\frac{1}{r})\,\sin\theta \\
B_r(r,\theta) &= \frac{\mu_o M_o\, e^{-jk_o r}}{2\pi r}\left(j\,\frac{k_o}{r} + \frac{1}{r^2}\right)\cos\theta \\
B_\theta(r,\theta) &= \frac{\mu_o M_o}{4\pi r}\, e^{-jk_o r}\left(j\,\frac{k_o}{r} + \frac{1}{r^2} - k_o^2\right)\sin\theta
\end{aligned}\right\} \qquad (10)$$

In the foregoing $\eta_o = \sqrt{\mu_o/\varepsilon_o} \simeq 120\ \pi$ ohms. If the magnetic dipole is aligned along the x-axis, then the field components are

$$E_y(x,y,z) = -\frac{\eta_o k_o M_{ox}}{4\pi r}\, e^{-jk_o r}\,(-k_o + j\,\frac{1}{r})\,\frac{z}{r} \qquad (11)$$

$$E_z(x,y,z) = -\frac{\eta_o k_o M_{ox}}{2\pi r}\, e^{-jk_o r}\,(-k_o + j\,\frac{1}{r})\,\frac{y}{r} \qquad (12)$$

$$\begin{aligned}
B_x(x,y,z) = {} & \frac{\mu_o M_{ox}}{2\pi r}\, e^{-jk_o r}\left(j\,\frac{k_o}{r} + \frac{1}{r^2}\right)\left(\frac{x}{r}\right)^2 \\
& - \frac{\mu_o M_{ox}}{4\pi r}\, e^{-jk_o r}\left(j\,\frac{k_o}{r} + \frac{1}{r^2} - k_o^2\right)\frac{r^2 - x^2}{r^2}
\end{aligned} \qquad (13)$$

$$\begin{aligned}
B_y(x,y,z) = {} & \frac{\mu_o M_{ox}}{2\pi r}\, e^{-jk_o r}\left(j\,\frac{k_o}{r} + \frac{1}{r^2}\right)\frac{xy}{r^2} \\
& + \frac{\mu_o M_{ox}}{4\pi r}\, e^{-jk_o r}\left(j\,\frac{k_o}{r} + \frac{1}{r^2} - k_o^2\right)\frac{xy}{r^2}
\end{aligned} \qquad (14)$$

$$B_z(x,y,z) = \frac{\mu_o M_{ox}}{2\pi r} e^{-jk_o r} \left(j \frac{k_o}{r} + \frac{1}{r^2} \right) \frac{xy}{r^2}$$

$$+ \frac{\mu_o M_{ox}}{4\pi r} e^{-jk_o r} \left(j \frac{k_o}{r} + \frac{1}{r^2} - k_o^2 \right) \frac{xz}{r^2} \quad (15)$$

If the dipole is oriented along the y axis then the field components are

$$E_x(x,y,z) = - \frac{\eta_o k_o M_{oy}}{4\pi r} e^{-jk_o r} (-k_o + j \frac{1}{r}) \frac{z}{r} \quad (16)$$

$$E_z(x,y,z) = - \frac{\eta_o k_o M_{oy}}{4\pi r} e^{-jk_o r} (- k_o + j \frac{1}{r}) \frac{x}{r} \quad (17)$$

$$B_x(x,y,z) = \frac{\mu_o M_{oy}}{2\pi r} e^{-jk_o r} \left(j \frac{k_o}{r} + \frac{1}{r^2} \right) \frac{xy}{r^2}$$

$$+ \frac{\mu_o M_{oy}}{4\pi r} e^{-jk_o r} \left(j \frac{k_o}{r} + \frac{1}{r^2} - k_o^2 \right) \frac{xy}{r^2} \quad (18)$$

$$B_y(x,y,z) = \frac{\mu_o M_{oy}}{2\pi r} e^{-jk_o r} \left(j \frac{k_o}{r} + \frac{1}{r^2} \right) \frac{y^2}{r^2}$$

$$- \frac{\mu_o M_{oy}}{4\pi r} e^{-jk_o r} \left(j \frac{k_o}{r} + \frac{1}{r^2} - k_o^2 \right) \frac{r^2-y^2}{r^2} \quad (19)$$

$$B_z(x,y,z) = \frac{\mu_o M_{oy}}{2\pi r} e^{-jk_o r} \left(j \frac{k_o}{r} + \frac{1}{r^2} \right) \frac{yz}{r^2}$$

$$+ \frac{\mu_o M_{oy}}{4\pi r} e^{-jk_o r} \left(j \frac{k_o}{r} + \frac{1}{r^2} - k_o^2 \right) \frac{yz}{r^2} \quad (20)$$

Expressing the electric dipole fields in cartesian components yields

$$E_x(x,y,z) = \frac{P_o}{2\pi\varepsilon_o r} e^{-jk_or}\left(j\frac{k_o}{r}+\frac{1}{r^2}\right)\frac{xz}{r^2}$$

$$+ \frac{P_o}{4\pi\varepsilon_o r} e^{-jk_or}\left(j\frac{k_o}{r}+\frac{1}{r^2}-k_o^2\right)\frac{xz}{r^2} \qquad (21)$$

$$E_y(x,y,z) = \frac{P_o}{2\pi\varepsilon_o r} e^{-jk_or}\left(j\frac{k_o}{r}+\frac{1}{r^2}\right)\frac{yz}{r^2}$$

$$+ \frac{P_o}{4\pi\varepsilon_o r} e^{-jk_or}\left(j\frac{k_o}{r}+\frac{1}{r^2}-k_o^2\right)\frac{yz}{r^2} \qquad (22)$$

$$E_z(x,y,z) = \frac{P_o}{2\pi\varepsilon_o r} e^{-jk_or}\left(j\frac{k_o}{r}+\frac{1}{r^2}\right)\frac{z^2}{r^2}$$

$$- \frac{P_o}{4\pi\varepsilon_o r} e^{-jk_or}\left(j\frac{k_o}{r}+\frac{1}{r^2}-k_o^2\right)\frac{r^2-z^2}{r^2} \qquad (23)$$

$$B_x(x,y,z) = -\frac{\eta_o k_o P_o}{4\pi r} e^{-jk_or}\left(-k_o + j\frac{1}{r}\right)\frac{y}{r} \qquad (24)$$

$$B_y(x,y,z) = \frac{\eta_o k_o P_o}{4\pi r} e^{-jk_or}\left(-k_o + j\frac{1}{r}\right)\frac{x}{r} \qquad (25)$$

The time histories of the foregoing field components may be obtained by Fourier superposition. The results are

$$\tilde{E}_x(x,y,z,t) = -\frac{\eta_o}{4\pi}\frac{z}{r^2} L_1[\tilde{M}_{oy}] + \frac{1}{4\pi\varepsilon_o}\frac{xz}{r^3} L_3[\tilde{P}_o] \qquad (26)$$

$$\tilde{E}_y(x,y,z,t) = -\frac{\eta_o}{4\pi}\frac{z}{r^2} L_1 [\tilde{M}_{ox}] + \frac{1}{4\pi\varepsilon_o}\frac{yz}{r^3} L_3 [\tilde{P}_o] \tag{27}$$

$$\tilde{E}_z(x,y,z,t) = -\frac{\eta_o}{4\pi}\left\{\frac{y}{r^2} L_1 [\tilde{M}_{ox}] + \frac{x}{r^2} L_1 [\tilde{M}_{oy}]\right\}$$

$$+ \frac{1}{4\pi\varepsilon_o}\left\{\frac{z^2}{r^3} L_3 [\tilde{P}_o] - \frac{1}{r} L_2 [\tilde{P}_o]\right\} \tag{28}$$

$$\tilde{B}_x(x,y,z,t) = \frac{\mu_o}{4\pi}\left\{\frac{x^2}{r^3} L_3 [\tilde{M}_{ox}] - \frac{1}{r} L_2 [\tilde{M}_{ox}]\right\}$$

$$+ \frac{\mu_o}{4\pi}\frac{xy}{r^3} L_3 [\tilde{M}_{oy}] - \frac{\eta_o}{4\pi}\frac{y}{r^2} L_1 [\tilde{P}_o] \tag{29}$$

$$\tilde{B}_y(x,y,z,t) = \frac{\mu_o}{4\pi}\left\{\frac{y^2}{r^3} L_3 [\tilde{M}_{oy}] - \frac{1}{r} L_2 [\tilde{M}_{oy}]\right\}$$

$$+ \frac{\mu_o}{4\pi}\frac{xy}{r^3} L_3 [\tilde{M}_{ox}] + \frac{\eta_o}{4\pi}\frac{x}{r^2} L_1 [\tilde{P}_o] \tag{30}$$

$$\tilde{B}_z(x,y,z,t) = \frac{\mu_o}{4\pi}\frac{z}{r^3}\left\{x L_3 [\tilde{M}_{ox}] + y L_3 [\tilde{M}_{oy}]\right\} \tag{31}$$

where the foregoing operators are used

$$L_1 [f(t)] = \left\{\left(\frac{1}{c^2}\frac{\partial^2}{\partial\tau^2} + \frac{1}{rc}\frac{\partial}{\partial\tau}\right) f(\tau)\right\}_{\tau=t-r/c} \tag{32}$$

$$L_2 [f(t)] = L_1 [f(t)] + \frac{1}{r^2} f(t - r/c) \tag{33}$$

$$L_3\ [f(t)] = L_2\ [f(t)] + 2\left\{\left(\frac{1}{rc}\ \frac{\partial}{\partial\tau} + \frac{1}{r^2}\right) f(\tau)\right\}_{\tau=t-r/c} \tag{34}$$

with $c = 3 \times 10^8$ m/sec.

By using the appropriate coordinate transformation, the foregoing expressions may be used to determine the field components from an arbitrarily located aperture.

Dipole Moments and the Incident Field

For plane wave incidence on a perfectly conducting plane, it is well known that the normal component of the electric field at surface of the plane is

$$\hat{n}\cdot\vec{E}_o = 2\ \hat{n}\cdot\vec{E}^{inc} \tag{35}$$

where $\vec{E}^{inc}$ is the incident electric field vector and $\hat{n}$ is a unit vector normal to the plane. The tangential component of the magnetic field is

$$\hat{n} \times \vec{H}_o = 2\ \hat{n} \times \vec{H}^{inc} \tag{36}$$

Using the foregoing in (1) and (2) yields

$$\tilde{P}_o(\tau) = +\ 2\ \ell_1^3\ \alpha_{33}^1\ D_z^{inc}\ (\tau) \tag{37}$$

$$\tilde{M}_{ox}(\tau) = -\ 2\ \ell_1^3\ \alpha_{11}^1\ H_x^{inc}\ (\tau) \tag{38}$$

$$\tilde{M}_{oy}(\tau) = -\ 2\ \ell_1^3\ \alpha_{22}^1\ H_y^{inc}\ (\tau) \tag{39}$$

where

$$\left.\begin{aligned} \alpha^1_{11} &= -\ \alpha_{11}/\ell_1^3 \\ \alpha^1_{22} &= -\ \alpha_{22}/\ell_1^3 \\ \alpha^1_{33} &= +\ \alpha_{33}/\ell_1^3 \end{aligned}\right\} \qquad (40)$$

for an elliptical aperture.

For E-plane incidence and the propagation vector $\vec{k} = (-k_o \sin\theta_o \cos\phi_o, -k_o \sin\theta_o \sin\phi_o, -k_o \cos\theta_o)$

$$\left.\begin{aligned} E_z^{inc} &= 0 \\ H_x^{inc}(t) &= -\frac{1}{\eta_o} E^{inc}(t) \cos\theta_o \cos\phi_o \\ H_y^{inc}(t) &= -\frac{1}{\eta_o} E^{inc}(t) \cos\theta_o \sin\phi_o \end{aligned}\right\} \qquad (41)$$

And for H-plane incidence

$$\left.\begin{aligned} E_z^{inc}(t) &= E^{inc}(t) \sin\theta_o \\ H_x^{inc}(t) &= \frac{1}{\eta_o} E^{inc}(t) \cos\phi_o \\ H_y^{inc}(t) &= \frac{1}{\eta_o} E^{inc}(t) \sin\phi_o \end{aligned}\right\} \qquad (42)$$

where $E^{inc}(t)$ is the total electric field incident on the aperture.

The non-dimensional dyadic elements for an elliptical aperture, as defined in (40), are given in Table 1. For apertures that are not elliptical an approximation to the dyadic elements must be used. An equivalent aperture eccentricity may be defined for arbitrarily shaped apertures as shown in Figure 2. The dyadic elements such as those for the elliptical aperture

TABLE 1: Nondimensional Dyadic Elements

for Elliptical Apertures

e^2	α'_{11}	α'_{22}	α'_{33}
0	2.666 667	2.666 667	1.333 333
0.1	2.564 030	2.369 213	1.231 387
0.2	2.455 493	2.077 049	1.125 236
0.3	2.339 876	1.796 515	1.014 331
0.4	2.215 577	1.510 016	0.897 988
0.5	2.080 122	1.236 050	0.775 332
0.6	1.929 904	0.969 252	0.645 209
0.7	1.758 534	0.710 467	0.506 027
0.8	1.628 746	0.460 909	0.355 437
0.9	1.279 395	0.222 554	0.189 577
1.0	0	0	0

may be determined using the equivalent eccentricity in (3) - (5). The equivalent semi major axis length should be chosen so that the area of the arbitrarily formed aperture is equal the area of the ellipse with the equivalent eccentricity. For example, the rectangular aperture shown in Figure 2 has as the equivalent semi major axis length

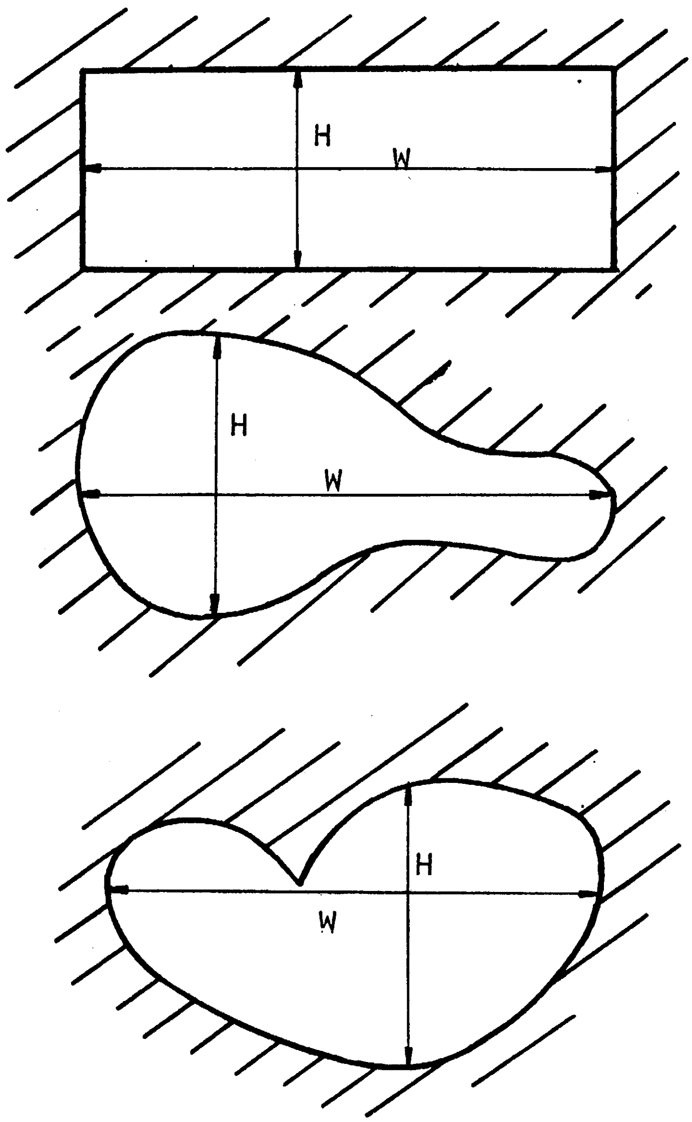
$$\ell_1 = \frac{w}{2\sqrt{\pi}} \tag{43}$$

For a perfectly square aperture the equivalent eccentricity is zero and equivalent semi major axis length is the same as (43) where w is the length of one side of the aperture. Then using $e = 0$ and ℓ_1 as shown in (43) in (3) - (5) the dyadic elements for the aperture may be approximated. However the foregoing approximations are not expected to be accurate if the shape of the aperture differs radically from an ellipse.

If the aperture is located in a finite size object then (35) and (36) are no longer valid. But E_o may be expressed in terms of the surface current charge distribution that would exist at the aperture if it were shorted and

$$\vec{H}_o = \hat{n} \times \vec{K}$$

expresses the magnetic field in terms of the surface current distribution. This brings up an important question of aperture penetration for finite bodies with a resonant surface current. In this situation the scattered magnetic field may be orders of magnitude greater than the incident magnetic field and the use of (36) would grossly underestimate the field penetration.



$$\text{Eccentricity} = [1-(H/W)^2]^{\frac{1}{2}}$$

FIGURE 2: SOME APERTURE SHAPES WITH EQUIVALENT ECCENTRICITIES.

Two Parallel Plates

The problem of field penetration into the region between two parallel plates is of considerable interest since it applies to the degradation of shield integrity caused by the presence of small apertures. The preceding analysis may be extended to two parallel plates, one having an aperture and the other continuous, by using image theory. The image of the electric dipole moment is colinear with the dipole vector; however the image of the magnetic dipole is antiparallel with the magnetic dipole vector. Taking this in consideration a doubly infinite array of images is formed as shown in figure 3. The field components at a particular point in space may be obtained by an algebraic addition of all the contributions from the aperture dipoles and the image dipoles. This was done in computing the data that are presented in figures 4-11.

In figure 4 the x-component of the electric field along the z axis between the plates is shown. As would be expected the field component gets quite large very near the aperture. Here

$$a_2 = \begin{cases} -\alpha'_{22} \cos\theta_o \sin\phi_o & \text{E - Polarization} \\ \alpha'_{22} \sin\phi_o & \text{H - Polarization} \end{cases} \qquad (44)$$

Figure 5 exhibits the z component of the electric field along the z axis. Here

$$a_3 = \begin{cases} 0 & \text{E - polarization} \\ \alpha'_{33} \sin\theta_o & \text{H - polarization} \end{cases} \qquad (45)$$

And figure 6 exhibits the only non zero component of the magnetic field along the z axis. The penetration field components at the surface of the plate

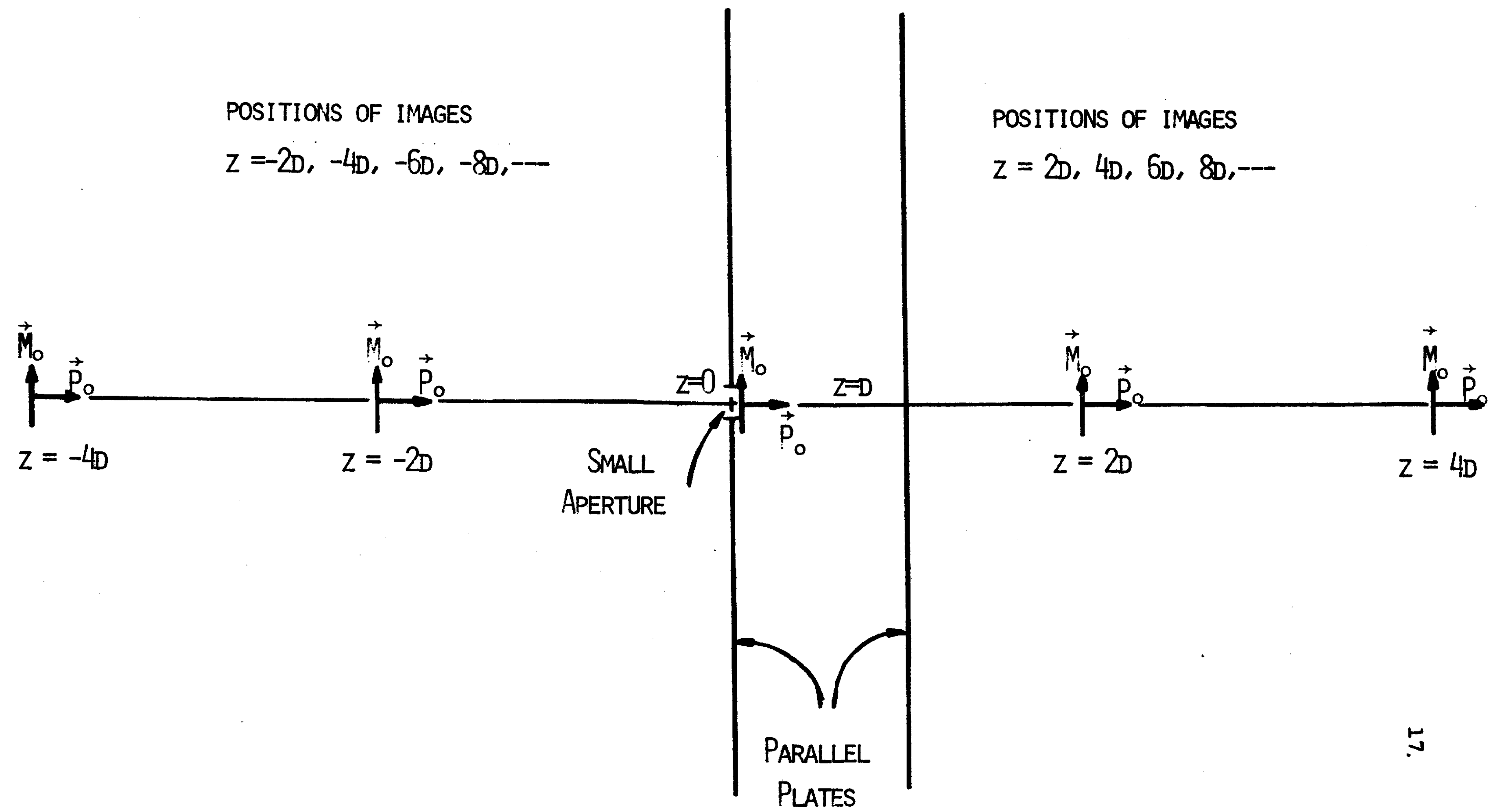


FIGURE 3: PARALLEL PLATES WITH SMALL APERTURE AND EQUIVALENT DIPOLE MOMENTS. POSITIONS AND ORIENTATION OF THE IMAGE DIPOLE MOMENTS ARE SHOWN FOR DETERMINING THE FIELD BETWEEN THE PLATES.

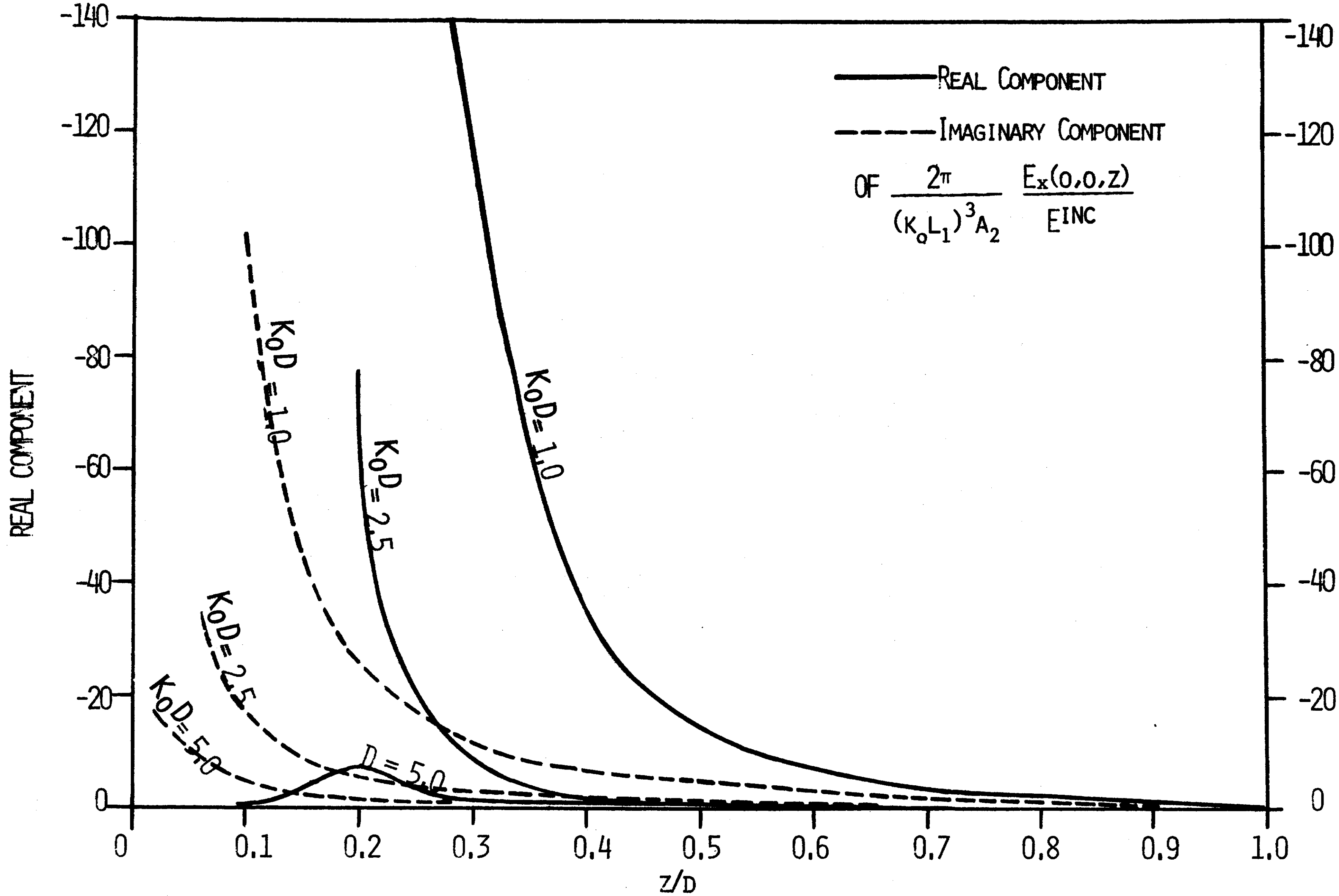


FIGURE 4: X-COMPONENT OF THE ELECTRIC FIELD BETWEEN THE PLATES ALONG A LINE PASSING THROUGH THE CENTER OF THE APERTURE AND PERPENDICULAR TO THE PLATES.

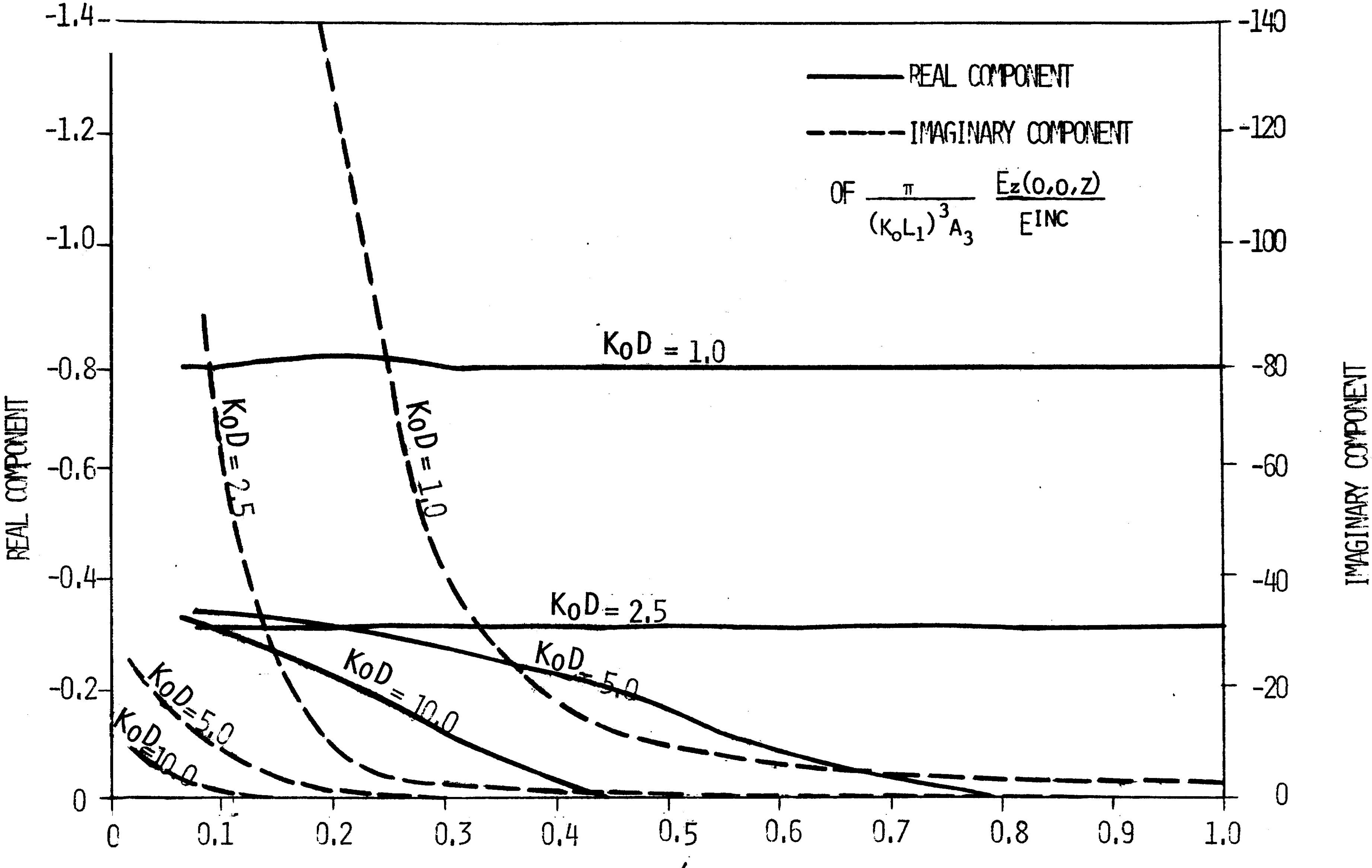


FIGURE 5: Z-COMPONENT OF THE ELECTRIC FIELD BETWEEN THE PLATES ALONG A LINE PASSING THROUGH THE CENTER OF THE APERTURE AND PERPENDICULAR TO THE PLATES.

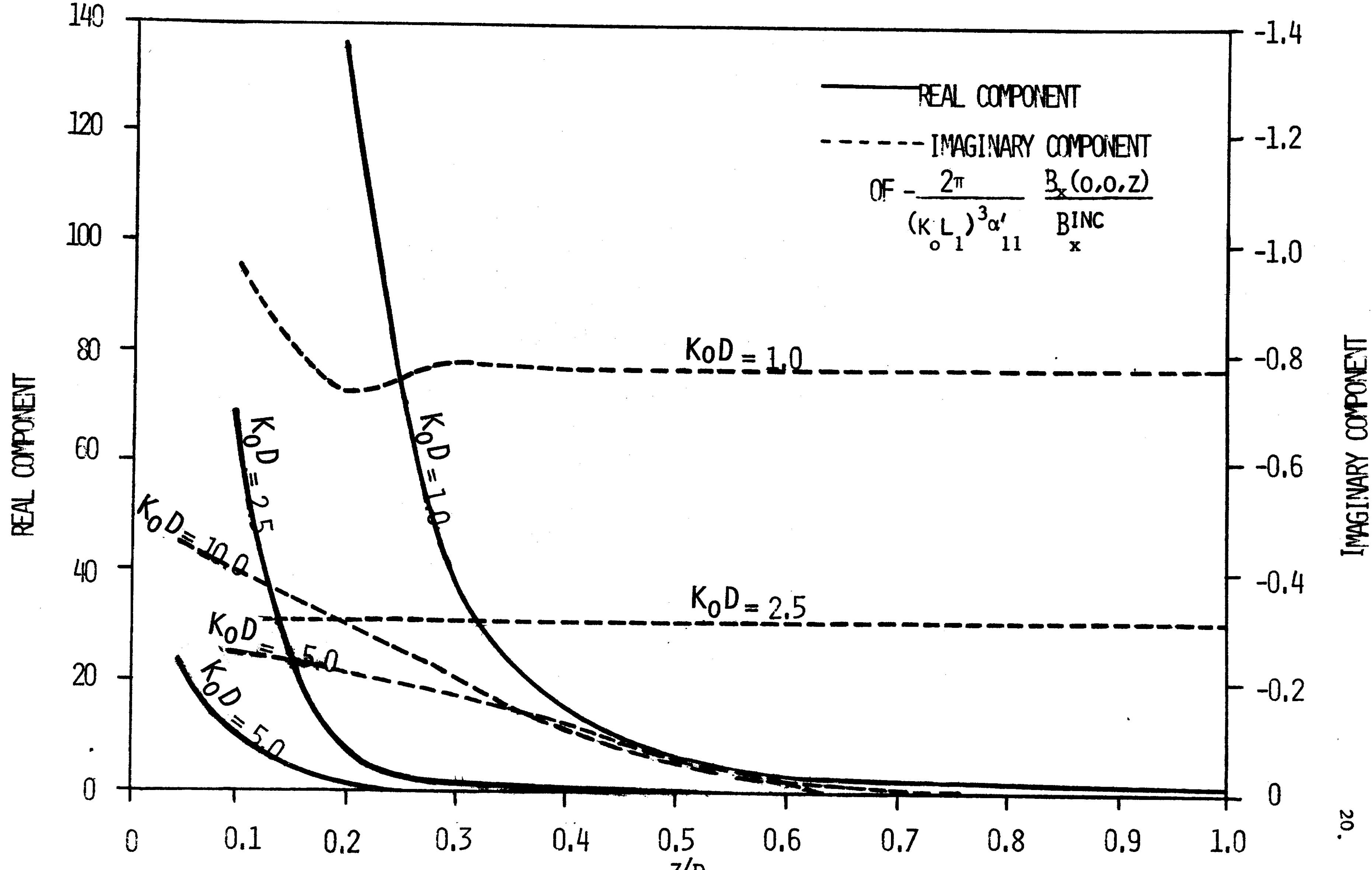


FIGURE 6: X-COMPONENT OF THE MAGNETIC FIELD BETWEEN THE PLATES ALONG A LINE PASSING THROUGH THE CENTER OF THE APERTURE AND PERPENDICULAR TO THE PLATES.

are shown in figures 7-11. The dominant feature of the foregoing curves is that the contribution from the images is in general quite small for the parameters studied.

The expressions used to compute the penetration field exhibits some peculiar behavior for certain plate separations and frequencies. In particular, when

$$k_o d = m\pi \qquad m = 1, 2, 3, ---$$

some components of the field are unbounded. Evidently the foregoing represents resonance conditions. The appendix discusses this result in detail plus a modification of the formulation to treat the resonant case.

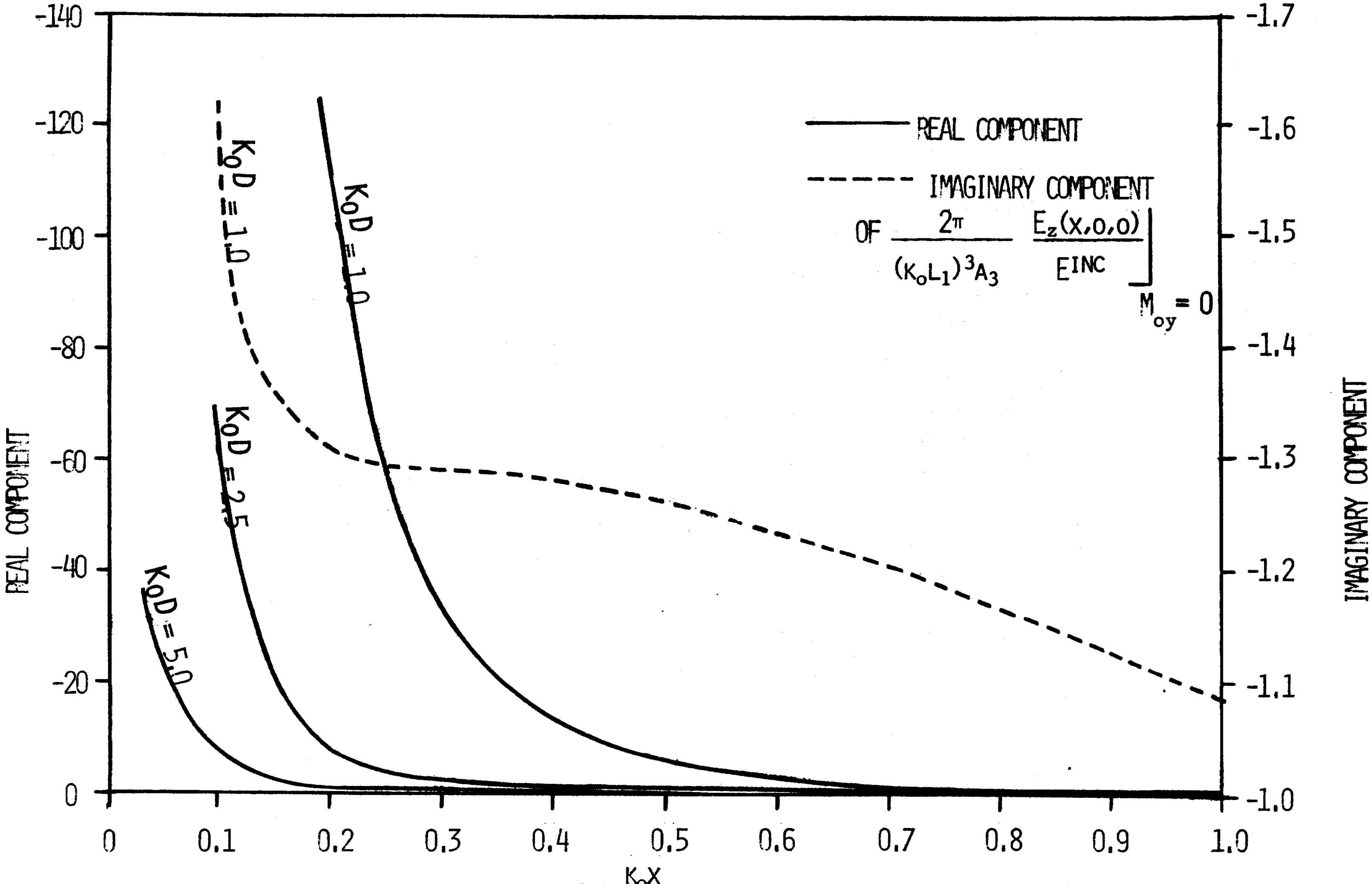


FIGURE 7: CONTRIBUTION OF THE ELECTRIC DIPOLE MOMENT TO THE NORMAL COMPONENT OF THE ELECTRIC FIELD AT THE SURFACE OF THE PLATE WITH THE APERTURE.

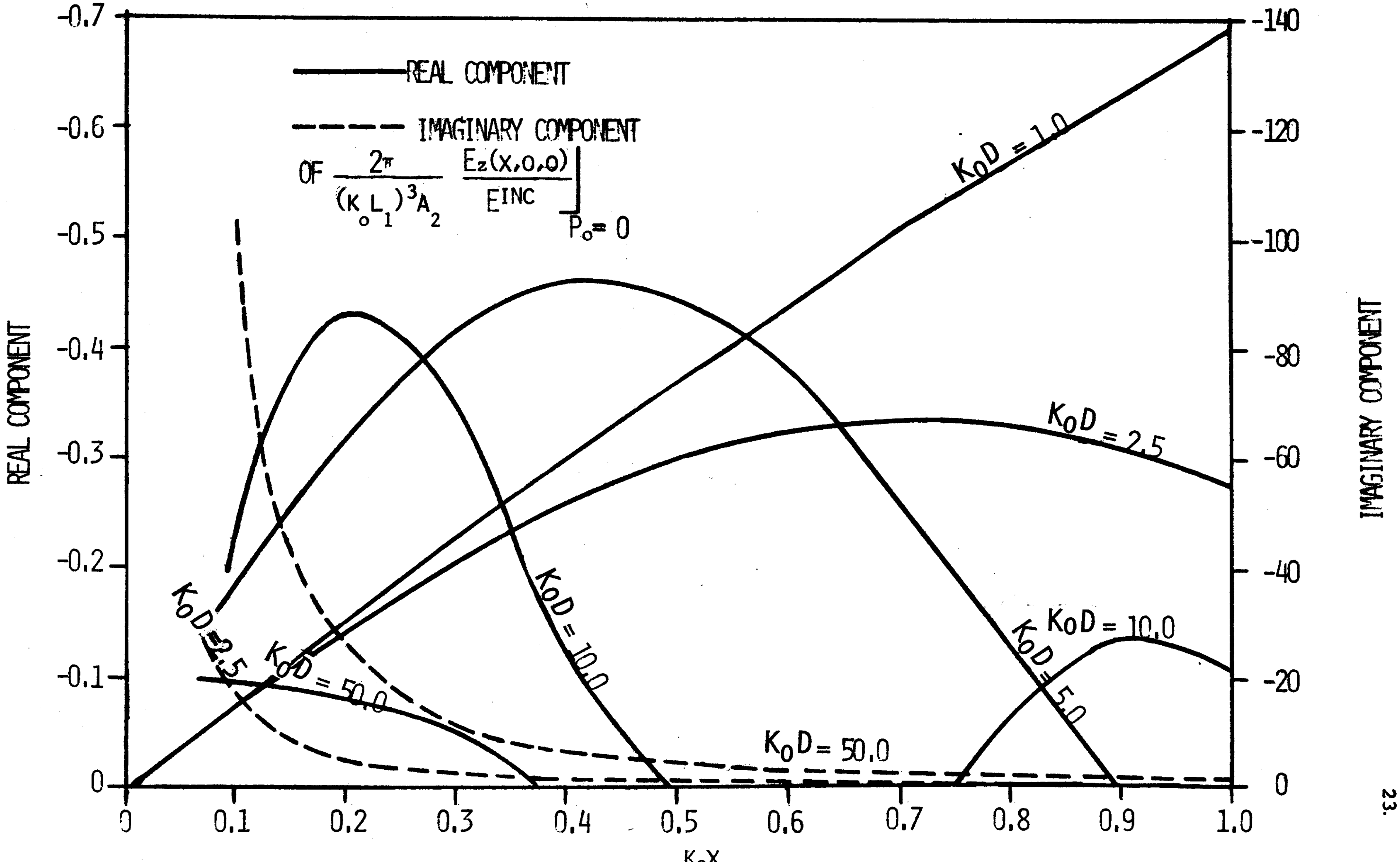


FIGURE 8: CONTRIBUTION OF THE MAGNETIC DIPOLE MOMENT TO THE NORMAL COMPONENT OF THE ELECTRIC FIELD AT THE SURFACE OF THE PLATE WITH THE APERTURE.

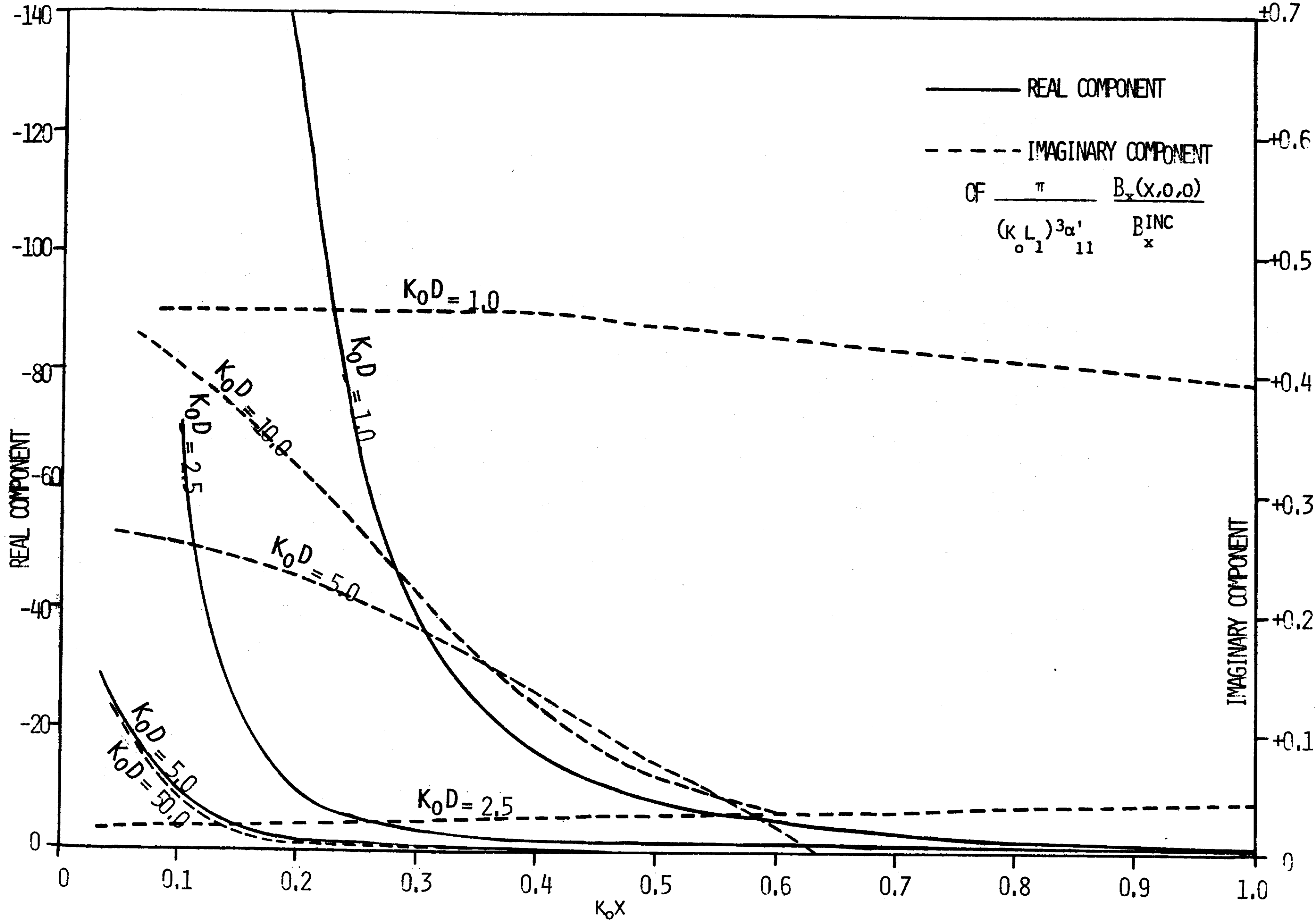


FIGURE 9: MAGNETIC FIELD AT THE SURFACE OF THE PLATE WITH THE APERTURE.

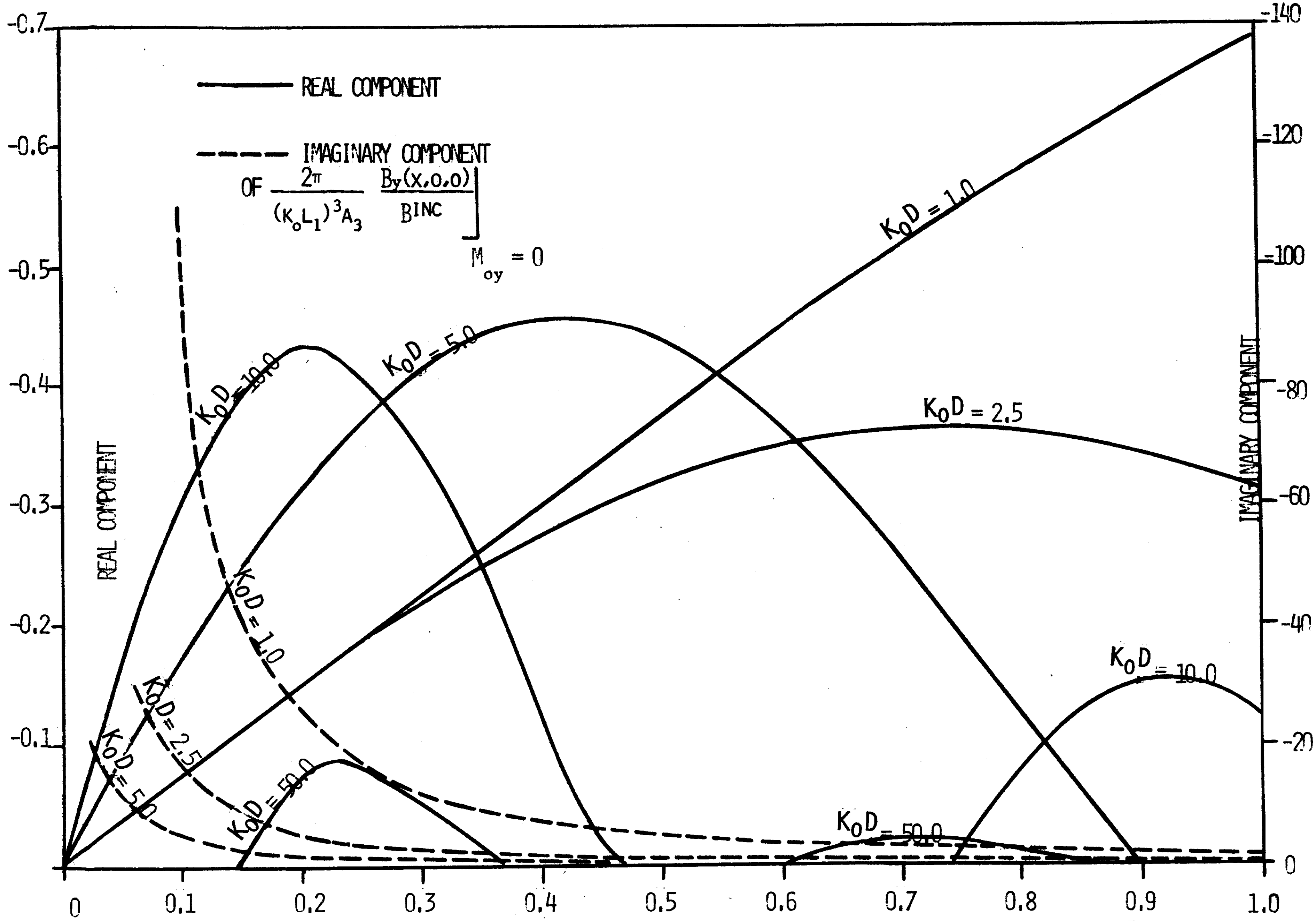


FIGURE 10: CONTRIBUTION OF THE ELECTRIC DIPOLE MOMENT TO THE MAGNETIC FIELD AT THE SURFACE OF THE PLATE WITH THE APERTURE.

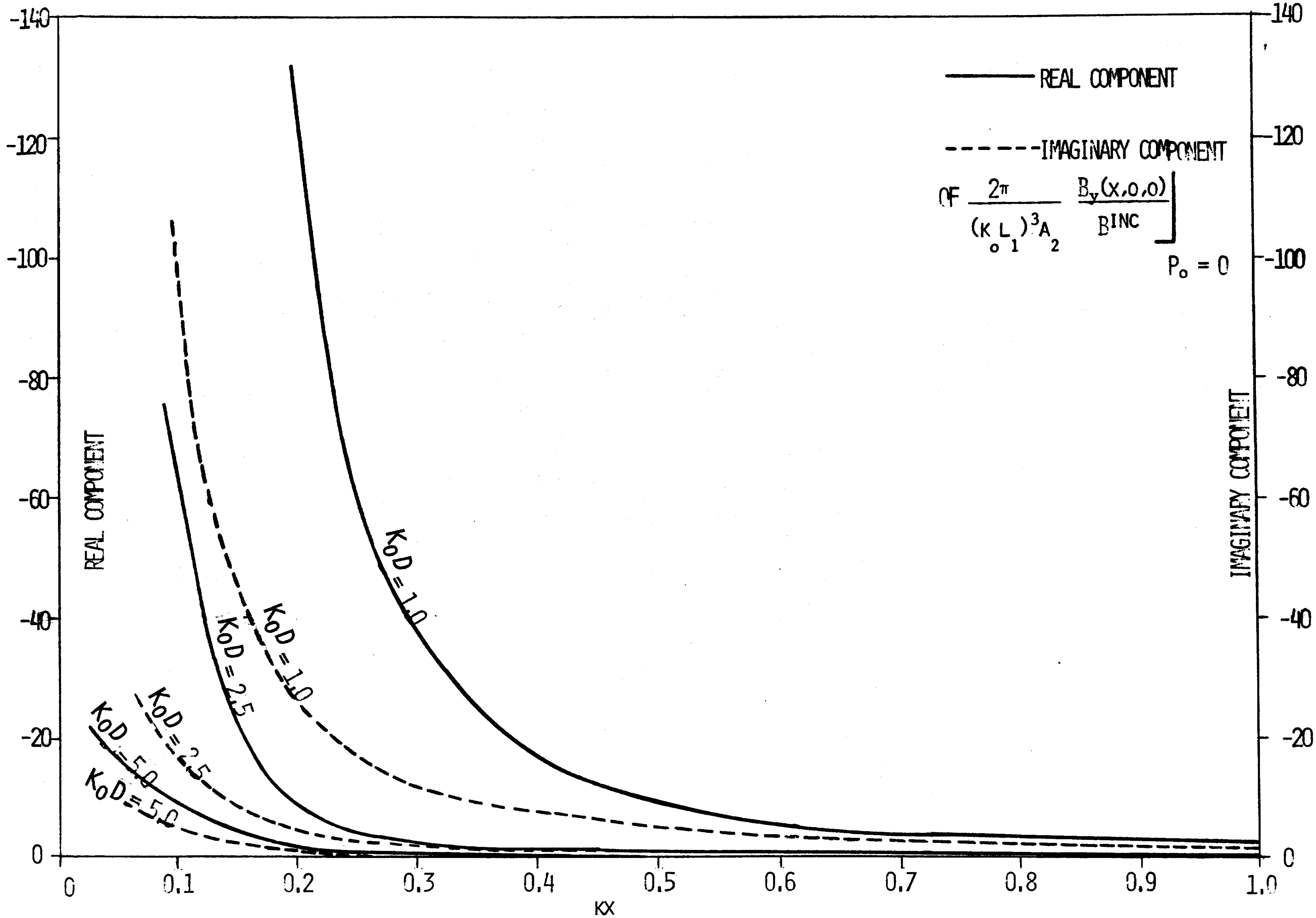


FIGURE 11: CONTRIBUTION OF THE MAGNETIC DIPOLE MOMENT TO THE MAGNETIC FIELD AT THE SURFACE OF THE PLATE WITH THE APERTURE.

Penetration Current and Charge about the Aperture

For many practical problems the currents and charges on the inside surface about an aperture are needed. An approximation to the behavior of these quantities may be obtained by examining the dipole field. It should be remembered however that this field should be combined with the fields of the image dipoles. As seen from the previous section the image dipole contribution is often negligible.

Considering an elliptical aperture the dipole field components at the surface of the conducting plate are

$$\left[\frac{B_x(x,0,0)}{B^{inc}}\right] e^{j|k_o x|} = \frac{a_1 (k_o \ell_1)^3}{2\pi (k_o x)^2}\left(1 - j\,\frac{1}{|k_o x|}\right) \tag{46}$$

$$\left[\frac{B_y(x,0,0)}{B^{inc}}\right] e^{j|k_o x|} = -\frac{a_2 (k_o \ell_1)^3}{2\pi |k_o x|}\left(1 - j\,\frac{1}{|k_o x|} - \frac{1}{|k_o x|^2}\right)$$

$$+\frac{a_3 (k_o \ell_1)^3}{2\pi |k_o x|}\left(1 - j\,\frac{1}{|k_o x|} - \frac{1}{|k_o x|^2}\right) \tag{47}$$

$$\left[\frac{E_z(x,0,0)}{E^{inc}}\right] e^{j|k_o x|} = -\frac{a_2 (k_o \ell_1)^3}{2\pi |k_o x|}\left(1 - j\,\frac{1}{|k_o x|}\right)$$

$$+\frac{a_3 (k_o \ell_1)^3}{2\pi |k_o x|}\left(1 - j\,\frac{1}{|k_o x|} - \frac{1}{|k_o x|^2}\right) \tag{48}$$

$$\text{Where} \quad a_1 = \begin{cases} -\alpha'_{11} \cos\theta_o \cos\phi_o & \text{E Polarization} \\ \alpha'_{11} \cos\phi_o & \text{H Polarization} \end{cases} \tag{49}$$

It must be remembered however that the foregoing approximations are strictly valid only at distances from the aperture that are large as compared to the aperture dimensions, i.e. $r >> \ell_1$. The coefficients in the higher order multipole expansion are of the order $(k_o a)^{2n+1}$ for the $n^{\underline{th}}$ multipole and the radial dependence goes as $(k_o r)^{-(2n+1)}$.

To determine the time behavior of the corresponding field components a particular time dependence of the incident field is considered. A convenient time dependence also used by Bombardt is

$$\tilde{E}^{inc}(t) = \beta t\, e^{-\alpha t} \tag{50}$$

where

$$\alpha = \frac{1}{T}$$

$$\beta = \frac{e}{T} \tilde{E}^{inc}(T)$$

Here T is the rise time of the pulse and $\tilde{E}^{inc}(T)$ is the peak value of the electric field.

The surface charge on the inner surface of the plate containing an aperture may be obtained by using (50) in (28). Along the x-axis it is

$$\tilde{D}_z(x,0,0,t) \simeq -\frac{a_3}{2\pi}\left(\frac{\ell_1}{|x|}\right)^3 E_o\, E_z^{inc}(t - |x|/c) \tag{51}$$

provided

$$\frac{\ell_1}{cT} << 1 \tag{52}$$

and

$$\frac{\ell_1}{cT} << \frac{|x|}{{}_{1}} \frac{(t-|x|/c)^{\alpha}}{2} \tag{53}$$

The x-component of the magnetic field under the foregoing restrictions is

$$\tilde{H}_x(x,0,0,t) \simeq -\frac{a_1}{2\pi}\left(\frac{\ell_1}{|x|}\right)^3 \frac{E^{inc}(t-|x|/c)}{\eta_o} \tag{54}$$

and the corresponding y-component,

$$\tilde{H}_y(x,0,0,t) \simeq -\frac{a_2}{2\pi}\left(\frac{\ell_1}{|x|}\right)^3 \frac{E^{inc}(t-|x|/c)}{\eta_o} \tag{55}$$

It is noted that (52) and (53) are equivalent to the small aperture approximation, i.e. $k_o\ell_1 << 1$ for significant frequency content of the pulse.

Comparison with Bombardt's Results

In studying the quasi-static field transmission through circular apertures Bombardt[5,6] considered a small aperture in a long cylindrical shell immersed in a uniform axial magnetic field. The geometry of the study is shown in figure 12. In this problem the penetration field may be expressed in terms of only a magnetic dipole moment directed antiparallel to the impressed magnetic field. However the effect of the cylinder walls must be considered. From the preceeding results it is found that the penetration fields are generally only slightly perturbed by the walls of the shield. Hence the penetration inside the cylindrical shell and near the center of the shell should be nearly that occuring for the aperture in a plate. This is the approximation used by Bombardt.

The time dependence for the magnetic field is considered to be that given in (50) for the electric field. Following the development of the foregoing section the penetration field may be easily obtained. The time history of the z-component of the magnetic field on the axis of the cylinder is found to be

$$\frac{\tilde{H}_z(o,y,z_o t)}{H_o(t)} = - \frac{3\,\alpha'_{22}\,\ell_1^3 z_o}{4\pi}\left(\frac{y}{r^5}\right) \tag{56}$$

where $H_o(t)$ is the impressed magnetic field directed along the y axis. This equation agrees exactly with equation (9) of Bombardt's first paper[5] provided the aperture is circular, i.e. $\alpha'_{22} = 8/3$. In Bombardt's second paper on the subject a higher order expansion is used to determine the magnetic field. Figure 13 exhibits experimental data and theoretical data obtained by Bombardt compared with (56). Excellent agreement is obtained.

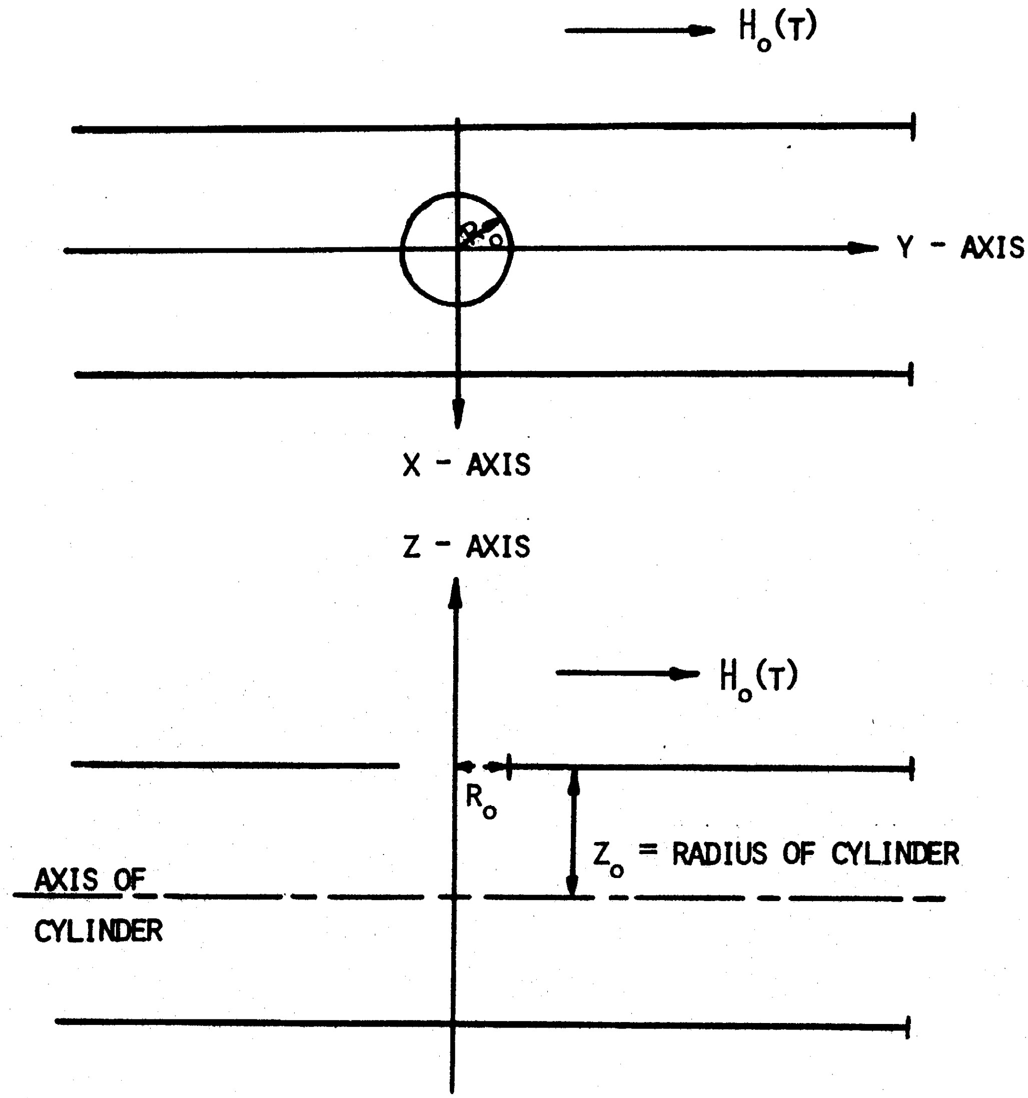


FIGURE 12: SMALL APERTURE IN A LONG CYLINDRICAL SHELL IMMERSED IN A UNIFORM AXIAL MAGNETIC FIELD, $H_o(T)$.

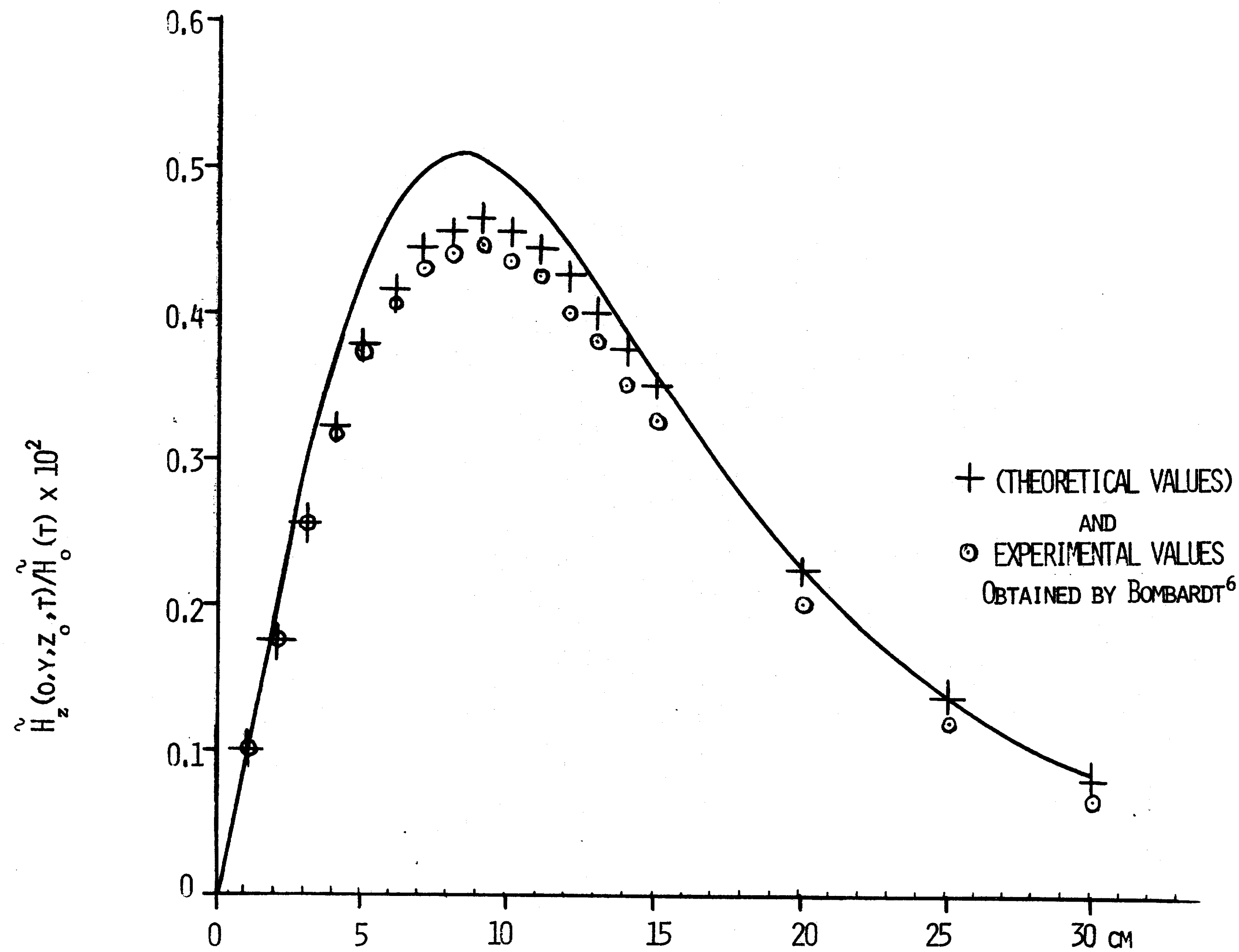


FIGURE 13: PENETRATION FIELD FOR A CIRCULAR APERTURE IN A LONG CYLINDRICAL SHELL IMMERSED IN A UNIFORM AXIAL MAGNETIC FIELD, $H_o(T)$. $Z = 16.5$ CM AND $L_1 = 5$ CM

CONCLUSION

A general theory of aperture penetration is presented for arbitrarily shaped apertures. Results are presented for circular and elliptical apertures and a comparison with experimental data is presented. The penetration field is expressed in terms of the dipole moments of the equivalent sources for the aperture field distribution. Therefore the formulation applies only for apertures that are much smaller than the operating wavelength.

It is shown that pulses penetrate small apertures with essentially no distortion and that the fields of the apertures are mostly local. Furthermore, the field configuration is approximately that about perpendicular static electric and magnetic dipoles except for cases very near resonant conditions. It is because of this behavior that pulses penetrate small apertures with essentially no distortion in time history.

Future studies should include determining the field penetration for cavities near resonance and with a finite Q. These topics are only briefly discussed in this paper.

Although the formulation presented in this paper strictly applies only for a small aperture in a parallel plate shield the results have a much broader application. They are:

1. The field penetrating a small aperture (dimensions $\lesssim \lambda/10$) in an arbitrarily formed electromagnetic shield is approximately that of static-crossed electric and magnetic dipoles, except for frequencies very near resonance (within $\sim$10%) of the interior cavity of the shield. If the interior cavity formed by the shield is a low Q cavity ($Q \lesssim 10$) then the aforementioned restriction vanishes.

2. The foregoing magnetic dipole moment of the equivalent source distribution of the aperture lies in the plane of the aperture and the corresponding electric dipole moment is perpendicular to the aperture.

3. The cartesian components of the magnetic dipole moment are directly proportional to the corresponding components of the surface current density that would exist at the position of the aperture if it were electrically shorted.

4. The magnitude of the electric dipole moment is directly proportional to the surface charge density that would exist at the position of the aperture if it were electrically shorted.

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APPENDIX

When the aperture is illuminated at the resonant frequency of the cavity formed by the parallel plates, multiple reflections within the cavity may significantly perturb the aperture field. The contribution to the normal component of the electric field at the aperture arising from the multiple reflections is

$$E_z(0,0,0) = - j \frac{P_o' \; k_o^3}{\pi \; \varepsilon_o} \sum_{n=1}^{\infty} \frac{e^{-j2nk_od}}{(2nk_od)} \left(-1 + j \frac{1}{2nk_od}\right) \tag{A1}$$

where P_o' is the aperture electric dipole moment corrected for multiple reflections. This corrected dipole moment is

$$P_o' = \ell_1^3 \, \alpha_{33}' \, [2 \, E_z^{inc} + E_z \; (0,0,0)] \tag{A2}$$

Using (A1) in (A2) yeilds

$$P_o' = 2 \, \ell_1^3 \, \alpha_{33}' \, E_z^{inc} / \left\{ 1 + \frac{j(k_o\ell_1)^3 \, \alpha_{33}'}{\pi} \sum_{n=1}^{\infty} \frac{e^{-j2nk_od}}{(2nk_od)^2} \left[-1 + j \frac{1}{2nk_od}\right] \right\} \tag{A3}$$

In general, the correction is small so that

$$P_o' \simeq P_o = 2 \, \ell_1^3 \; \alpha_{33}' \; E_z^{inc}$$

However, when the magnetic dipole moment is considered a significant correction may be obtained.

The contribution to the x-component of the magnetic field at the aperture produced by the multiple reflections is

$$H_x(0,0,0) = \frac{M'_{ox}\ k_o^3}{2\ \pi} \sum_{n=1}^{\infty} \frac{e^{-j2nk_od}}{2nk_od} \left[-1 + j\ \frac{1}{2nk_od} + \frac{1}{(2nk_od)^2}\right] \quad \text{(A4)}$$

where M'_{ox} is the x-component of the aperture magnetic dipole moment corrected for multiple reflections. This corrected dipole moment is

$$M'_{ox} = -\ell_1^3\ \alpha'_{11}\ [2\ H_x^{inc} + H_x(0,0,0)] \quad \text{(A5)}$$

As before using (A4) in (A5) yields

$$M'_{ox} = -\ 2\ \ell_1^3\ \alpha'_{11}\ H_x^{inc} / \left\{ 1 + \frac{(k_o\ell_1)^3\ \alpha'_{11}}{2\pi} \times \sum_{n=1}^{\infty} \frac{e^{-j2nk_od}}{2nk_od} \left[-1 + j\ \frac{1}{2nk_od} + \frac{1}{(2nk_od)^2}\right] \right\} \quad \text{(A6)}$$

Similarly the y-component of the corrected magnetic dipole moment is obtained.

$$M'_{oy} = -\ 2\ \ell_1^3\ \alpha'_{22}\ H_y^{inc} / \left\{ 1 + \frac{(k_o\ell_1)^3\ \alpha'_{22}}{2\pi} \times \sum_{n=1}^{\infty} \frac{e^{-j2nk_od}}{2nk_od} \left[-1 + j\ \frac{1}{2nk_od} + \frac{1}{(2nk_od)^2}\right] \right\} \quad \text{(A7)}$$

Note that when resonance occurs, i.e. $k_od = m\pi$, where m is an integer, the sums in (A6) and (A7) diverge thus yielding

$$M'_o \xrightarrow[k_o d \to m\pi]{} 0 \qquad \text{(A8)}$$

Thus at resonance a significant perturbation of the aperture occurs because of multiple reflections. Evidently to determine the penetration field at resonance requires a limit be evaluated.

Since the corrected electric dipole moment does not approach zero as a resonant condition is obtained it must be concluded that the electric dipole moment of the aperture can not excite a resonant mode of the parallel plate cavity. However, for the general cavity, it must be expected that the electric dipole may excite a resonant mode. Thus near the excitable resonances the internal reflections (image dipoles) significantly effect the aperture dipole moments.

The foregoing discussion and conclusions apply only to high Q cavities. Many cavities formed by electromagnetic shields are relatively low Q, i.e. $Q \sim 10$. For these cavities the importance of multiple reflection considerably diminishes. However, this is a topic requiring further investigation.