SC-R-71 3250

TRANSMISSION LINE COUPLED TO A CYLINDER IN AN INCIDENT FIELD

Ronold W. P. King
Gordon McKay Laboratory
Harvard University
Cambridge, Massachusetts 02138

Charles W. Harrison, Jr.
Sandia Laboratories
Albuquerque, New Mexico 87115

Date Completed: May 1971

ABSTRACT

An approximate formula is derived for the currents in the loads terminating a two-wire line located parallel to and near a circular conducting cylinder of finite length. The line and the cylinder are illuminated by an incident plane wave. The analysis takes account of the coupling between the line and the cylinder and hence of the effects of possible axial resonances in the latter. The configuration simulates exposed conductors used to interconnect the electronic apparatus in a rocket. The analysis is directed toward determining possible hazards due to induced currents.

INTRODUCTION

When an arbitrary electromagnetic field is incident upon a rocket with exposed conductors that interconnect electronic apparatus in the manner shown schematically in Fig. 1, currents are induced that may generate unwanted and possibly dangerous voltages in the equipment. This investigation seeks to determine specifically the currents in the loads terminating the ends of a two-wire line placed close to a conducting cylinder as shown in Fig. 2 when illuminated by a plane normally incident electromagnetic wave that has its electric vector parallel to the axes of the cylinder and the two-wire line.

The cylinder with radius a_1 extends from $z = -h_1$ to $z = h_1$; it is in the same plane as the two conductors of the transmission line which extend from $z = -h_2'$ to $z = h_2$. Their common radius is a_2 and their distances from the axis of the cylinder are $d_2 = d - b/2$ and $d_3 = d + b/2$ where b is the spacing of the line and d the distance from the axis of the cylinder to the center of the line. The following inequalities are satisfied:

$$\beta a_2 < \beta a_1 << 1 \quad ; \quad \beta a_2 < \beta b << 1 \quad ; \quad \beta d << 1 \tag{1a}$$

$$a_1 << h_1 \quad , \quad a_2 << (h_2 + h_2')/2 \quad ; \quad h_2 + h_2' < 2h_1 \tag{1b}$$

The load terminating the line at $z = -h_2'$ is Z_0, that at $z = h_2$ is Z_s.

EXCITATION FUNCTIONS AND VECTOR POTENTIALS

The incident electric vector is given by

$$E_z^{inc}(x) = E_z^{inc}(0)\ e^{-j\beta x} \tag{2}$$

with the origin of coordinates at the center of the cylinder. The following excitation functions can be defined for the three conductors:

$$U_1 = E_z^{inc}(0)/\beta \quad , \quad U_2 = E_z^{inc}(-d+b/2)/\beta = E_z^{inc}(0)\ e^{j\beta(d-b/2)}/\beta_0$$

$$U_3 = E_z^{inc}(-d-b/2)/\beta = E_z^{inc}(0)\ e^{j\beta(d+b/2)}/\beta_0 \qquad (3)$$

If it is assumed that the distances between the conductors are all large compared with their radii, the vector potentials on their surfaces due to the currents in all of them are well approximated by:

$$A_{1z}(z) = \frac{\mu_0}{4\pi}\{\int_{-h_1}^{h_1} I_1(z')\ K_{11}(z,z')\ dz' + \int_{-h_2'}^{h_2} [I_2(z')\ K_{12}(z,z') + I_3(z')\ K_{13}(z,z')]dz'\} \qquad (4a)$$

$$A_{2z}(z) = \frac{\mu_0}{4\pi}\{\int_{-h_1}^{h_1} I_1(z')\ K_{21}(z,z')\ dz' + \int_{-h_2'}^{h_2} [I_2(z')\ K_{22}(z,z') + I_3(z')\ K_{23}(z,z')]dz'\} \qquad (4b)$$

$$A_{3z}(z) = \frac{\mu_0}{4\pi}\{\int_{-h_1}^{h_1} I_1(z')\ K_{31}(z,z')\ dz' + \int_{-h_2'}^{h_2} [I_2(z')\ K_{32}(z,z') + I_3(z')\ K_{33}(z,z')]dz'\} \qquad (4c)$$

where

$$K_{jk}(z,z') = \frac{e^{-j\beta R_{jk}}}{R_{jk}} \quad , \quad R_{jk} = \sqrt{(z_j - z_k')^2 + C_{jk}^2} \qquad (5)$$

with

$$C_{11} = a_1 \quad , \quad C_{22} = C_{33} = a_2 \quad ; \quad C_{23} = C_{32} = b \qquad (6a)$$

$$C_{12} = C_{21} = d - b/2 \quad ; \quad C_{13} = C_{31} = d + b/2 \qquad (6b)$$

Note that these distances are measured from a point z on the surface of one conductor where the vector potential is calculated to the point z' which locates the element of current I dz' on the axis of another conductor. The actual current is on the surface and involves distances that are functions of the angle θ. However, the average distance to the center is an excellent approximation when the conductors are electrically thin as required by (1a) and when they are not too close together compared with their radii. Actually this latter condition is not satisfied when the transmission line is very close to the cylinder with $d - b/2$ only very slightly larger than a_1 and with $a_2 << a_1$.

The vector potentials (4a-c) are good approximations for electrically thin conductors that are separated by distances between axes that are large compared with the radius of any of them. Since the cylinder 1 may have a radius a_1 that is not small compared with its distances from conductors 2 and 3, the transverse distribution of current in it may depart significantly from the rotationally symmetrical near the adjacent transmission line. It follows that the implied approximation in (4a-c) that the average distance to the currents in the cylinder is to the axis in calculating the vector potentials is not necessarily a good one. Insofar as the total current $I_1(z)$ in the cylinder is concerned, the effect of the adjacent transmission line in modifying the transverse distribution is insignificant. On the other hand, in determining the vector potential on the conductors of the transmission line due to the current in the cylinder, the proximity of localized image currents in the cylinder cannot be neglected when the line is very close to its surface. For this purpose the cylinder may be represented by the total axial current $I_1(z')$ and by the localized image currents $I_4(z')$ and $I_5(z')$ in the images of the two conductors of the line in the cylinder as in Fig. 3.

These images are located at distances d_{24} and d_{35} from conductors 2 and 3 which are given by the relations [1]

$$d_{24} = \frac{a_1^2 - a_2^2 - D_4^2}{D_4} \doteq \frac{a_1^2 - D_4^2}{D_4} \tag{7a}$$

$$d_{35} = \frac{a_1^2 - a_2^2 - D_5^2}{D_5} \doteq \frac{a_1^2 - D_5^2}{D_5} \tag{7b}$$

where D_4 and D_5 are the distances of the images of conductors 2 and 3 from the axis of cylinder 1. The approximate expressions on the right assume that $a_2^2 << a_1^2$. The currents in the image conductors are equal and opposite to the currents in the conductors themselves. Thus,

$$I_4(z') = -I_2(z') \quad , \quad I_5(z') = -I_3(z') \tag{8}$$

The vector potentials on conductors 2 and 3 may now be approximated as follows:

$$A_{2z} \doteq \frac{\mu_0}{4\pi} \{\int_{-h_1}^{h_1} I_1(z') \frac{e^{-j\beta R_{21}}}{R_{21}} dz' + \int_{-h_2'}^{h_2} I_2(z') [\frac{e^{-j\beta R_{22}}}{R_{22}} - \frac{e^{-j\beta R_{24}}}{R_{24}}] dz'$$

$$+ \int_{-h_2'}^{h_2} I_3(z') [\frac{e^{-j\beta R_{23}}}{R_{23}} - \frac{e^{-j\beta R_{25}}}{R_{25}}] dz'\} \tag{9a}$$

$$A_{3z} \doteq \frac{\mu_0}{4\pi} \{\int_{-h_1}^{h_1} I_1(z') \frac{e^{-j\beta R_{31}}}{R_{31}} dz' + \int_{-h_2'}^{h_2} I_2(z') [\frac{e^{-j\beta R_{32}}}{R_{32}} - \frac{e^{-j\beta R_{34}}}{R_{34}}] dz'$$

$$+ \int_{-h_2'}^{h_2} I_3(z') [\frac{e^{-j\beta R_{33}}}{R_{33}} - \frac{e^{-j\beta R_{35}}}{R_{35}}] dz'\} \tag{9b}$$

where

$$R_{24} = \sqrt{(z - z')^2 + d_{24}^2} \quad , \quad R_{25} = \sqrt{(z - z')^2 + d_{25}^2}$$
$$R_{34} = \sqrt{(z - z')^2 + d_{34}^2} \quad , \quad R_{35} = \sqrt{(z - z')^2 + d_{35}^2} \tag{10}$$

Note that since $d_{24} = d - b/2 - D_4$ and $d_{35} = d + b/2 - D_5$, it follows that

$$D_4 = \frac{a_1^2}{d - b/2} \quad , \quad d_{24} = \frac{(d - b/2)^2 - a_1^2}{d - b/2} \quad , \quad d_{34} = d_{24} + b \tag{11a}$$

$$D_5 = \frac{a_1^2}{d + b/2} \quad , \quad d_{35} = \frac{(d + b/2)^2 - a_1^2}{d + b/2} \quad , \quad d_{25} = d_{35} - b \tag{11b}$$

The distance between the image conductors is

$$b_i = D_4 - D_5 = a_1^2 \left[\frac{b}{d^2 - b^2/4}\right] \tag{12}$$

The several distances introduced in (10)-(12) are shown in Fig. 3.

APPROXIMATE EVALUATION OF THE DIFFERENCE INTEGRALS

As a consequence of the conditions (1a) all of the conductors are separated by distances that are electrically small. This permits the approximate evaluation of the difference integrals in (9a,b) following the pattern used in transmission-line theory [2]. That is, with $R_a = [(z - z')^2 + a^2]^{1/2}$, $R_b = [(z - z')^2 + b^2]^{1/2}$,

$$\int_{-h}^{h} I(z') \left[\frac{e^{-j\beta R_a}}{R_a} - \frac{e^{-j\beta R_b}}{R_b}\right] dz' \doteq I(z) \cdot [2 \ln(b/a)] \tag{13}$$

provided $a < b \ll h$, $\beta a < \beta b \ll 1$. With (13) and the notation

$$p_{24} = (\zeta_0/2\pi)\ln(d_{24}/a_2) \quad , \quad p_{35} = (\zeta_0/2\pi)\ln(d_{35}/a_2)$$
$$p_{25} = (\zeta_0/2\pi)\ln(d_{25}/b) \quad , \quad p_{34} = (\zeta_0/2\pi)\ln(d_{34}/b) \tag{14}$$

the vector potentials in (9a,b) become:

$$A_{2z}(z) \doteq \frac{\mu_0}{4\pi}\int_{-h_1}^{h_1} I_1(z')\,\frac{e^{-j\beta R_{21}}}{R_{21}}\,dz' + \frac{\mu_0}{\zeta_0}\,[I_2(z)\,p_{24} + I_3(z)\,p_{25}] \tag{15}$$

$$A_{3z}(z) \doteq \frac{\mu_0}{4\pi}\int_{-h_1}^{h_1} I_1(z')\,\frac{e^{-j\beta R_{31}}}{R_{31}}\,dz' + \frac{\mu_0}{\zeta_0}\,[I_2(z)\,p_{34} + I_3(z)\,p_{35}] \tag{16}$$

where $\zeta_0 = \sqrt{\mu_0/\epsilon_0} \doteq 120\pi$ ohms.

SIMULTANEOUS EQUATIONS FOR THE CURRENTS IN THE LINE

The boundary condition on the total electric field on the surface of each conductor is

$$E_z(z) = E_z^{inc}(z) - (j\omega/\beta^2)(\partial^2/\partial z^2 + \beta^2)\,A_z(z) = 0 \tag{17}$$

The solutions of this equation with the definitions (3) are:

$$4\pi\mu_0^{-1}\,A_{1z}(z) = \frac{-j4\pi}{\zeta_0}\,[C_1 \cos\beta z + U_1] \tag{18}$$

$$4\pi\mu_0^{-1}\,A_{2z}(z) = \frac{-j4\pi}{\zeta_0}\,[C_2 \cos\beta z + D_2 \sin\beta z + U_2] \tag{19}$$

$$4\pi\mu_0^{-1}\,A_{3z}(z) = \frac{-j4\pi}{\zeta_0}\,[C_3 \cos\beta z + D_3 \sin\beta z + U_3] \tag{20}$$

Note that the coefficient D_1 in (18) vanishes because the current satisfies the symmetry condition $I_1(-z) = I_1(z)$ and has no discontinuity in slope at $z = 0$.

When (19) and (20) are combined respectively with (15) and (16) to eliminate $A_{2z}(z)$ and $A_{3z}(z)$, the following equations are obtained:

$$I_2(z)\, p_{24} + I_3(z)\, p_{25} = -\frac{\zeta_0}{4\pi} \int_{-h_1}^{h_1} I_1(z') \frac{e^{-j\beta R_{21}}}{R_{21}}\, dz' - j[C_2 \cos \beta z + D_2 \sin \beta z + U_2] \tag{21}$$

$$I_2(z)\, p_{34} + I_3(z)\, p_{35} = -\frac{\zeta_0}{4\pi} \int_{-h_1}^{h_1} I_1(z') \frac{e^{-j\beta R_{31}}}{R_{31}}\, dz' - j[C_3 \cos \beta z + D_3 \sin \beta z + U_3] \tag{22}$$

Since the currents in the closely-spaced transmission line have only a relatively small localized effect on the current in the large cylinder, it is a satisfactory approximation to neglect the contributions to the vector potential on the surface of the cylinder by the currents in the line. Thus, with (4a) and (18)

$$4\pi\mu_0^{-1} A_{1z}(z) \doteq \int_{-h_1}^{h_1} I_1(z') \frac{e^{-j\beta R_{11}}}{R_{11}}\, dz' = \frac{-j4\pi}{\zeta_0} [C_1 \cos \beta z + U_1] \tag{23}$$

The approximate solution of this integral equation is [3]:

$$I_1(z) \doteq \frac{j4\pi}{\zeta_0} U_1 K_1 (\cos \beta z - \cos \beta h_1) \tag{24a}$$

where

$$K_1 = [\Psi_{dU} \cos \beta h_1 - \Psi_U(h_1)]^{-1} \tag{24b}$$

The parameters Ψ_{dU} and $\Psi_U(h)$ are defined in the Appendix. With (24a) it follows that [4]:

$$\int_{-h_1}^{h_1} I_1(z') \frac{e^{-j\beta R_{21}}}{R_{21}} dz' = \frac{j4\pi}{\zeta_0} U_1 K_1 \int_{-h_1}^{h_1} (\cos \beta z' - \cos \beta h_1) \frac{e^{-j\beta R_{21}}}{R_{21}} dz'$$

$$\doteq \frac{j4\pi}{\zeta_0} U_1 K_1 \Psi_{21}(z) \tag{25a}$$

$$\int_{-h_1}^{h_1} I_1(z') \frac{e^{-j\beta R_{31}}}{R_{31}} dz' \doteq \frac{j4\pi}{\zeta_0} U_1 K_1 \Psi_{31}(z) \tag{25b}$$

where

$$\Psi_{k1}(z) = C_f(h_1,z) - E_f(h_1,z) \cos \beta h_1 \tag{26}$$

with

$$f = d - b/2 \quad \text{for} \quad k = 2 \quad , \quad f = d + b/2 \quad \text{for} \quad k = 3 \tag{27}$$

The integral functions $C_f(h,x)$ and $E_f(h,x)$ are defined in the Appendix.

With (25a,b), the simultaneous equations (21) and (22) become:

$$I_2(z)\, p_{24} + I_3(z)\, p_{25} = -j[U_1 K_1 \Psi_{21}(z) + C_2 \cos \beta z + D_2 \sin \beta z + U_2] \tag{28}$$

and

$$I_2(z)\,p_{34} + I_3(z)\,p_{35} = -j[U_1K_1\Psi_{31}(z) + C_3 \cos \beta z + D_3 \sin \beta z + U_3] \qquad (29)$$

These are the equations for the currents in the two conductors of the transmission line in terms of the incident electric field. The effects of coupling to the cylinder and each other are included. The boundary conditions to be used in the evaluation of the four constants C_2, D_2, C_3, and D_3 are:

$$I_2(h_2) + I_3(h_2) = 0 \quad , \quad I_2(-h_2') + I_3(-h_2') = 0 \qquad (30)$$

$$V(h_2) = I_2(h_2)\,Z_s \quad , \quad V(-h_2') = -I_2(-h_2')\,Z_0 \qquad (31)$$

where Z_s and Z_0 are the impedances terminating the transmission line at $z = h_2$ and $z = -h_2'$ and $V_s = V(h_2)$ and $V_0 = V(-h_2')$ are the voltages across these impedances as shown in Fig. 3.

THE SCALAR POTENTIAL DIFFERENCE

The scalar potential on each conductor is obtained from the vector potential with the help of the Lorentz condition. Thus with (19) and (20),

$$\phi_2(z) = \frac{j\omega}{\beta^2}\frac{\partial A_{2z}(z)}{\partial z} = -C_2 \sin \beta z + D_2 \cos \beta z \qquad (32)$$

$$\phi_3(z) = \frac{j\omega}{\beta^2}\frac{\partial A_{3z}(z)}{\partial z} = -C_3 \sin \beta z + D_3 \cos \beta z \qquad (33)$$

It follows that the potential difference between the two conductors of the line is

$$V(z) = \phi_2(z) - \phi_3(z) = -(C_2 - C_3) \sin \beta z + (D_2 - D_3) \cos \beta z \qquad (34)$$

With (31) it follows that

$$V(h_2) = -(C_2 - C_3) \sin \beta h_2 + (D_2 - D_3) \cos \beta h_2 = I_2(h_2) Z_s \quad (35)$$

$$V(-h_2') = (C_2 - C_3) \sin \beta h_2' + (D_2 - D_3) \cos \beta h_2' = -I_2(-h_2') Z_0 \quad (36)$$

These equations are easily solved for $(C_2 - C_3)$ and $(D_2 - D_3)$ in terms of $I_2(h_2)$ and $I_2(-h_2')$. The results are:

$$(C_2 - C_3) = -[I_2(h_2) Z_s \cos \beta h_2' + I_2(-h_2') Z_0 \cos \beta h_2]/\sin \beta s \quad (37)$$

$$(D_2 - D_3) = [I_2(h_2) Z_s \sin \beta h_2' - I_2(-h_2') Z_0 \sin \beta h_2]/\sin \beta s \quad (38)$$

where

$$s = h_2 + h_2' \quad (39)$$

is the length of the transmission line.

EVALUATION OF THE CURRENTS IN THE LOADS

In order to make use of (37) and (38) in (28) and (29) it is convenient to subtract (29) from (28) and then successively set $z = h_2$ and $z = -h_2'$. Use can be made of (30) by setting $I_3(h_2) = -I_2(h_2)$, $I_3(-h_2') = -I_2(-h_2')$. Thus,

$$I_2(h_2) Z_{ca} = -j[(C_2 - C_3) \cos \beta h_2 + (D_2 - D_3) \sin \beta h_2 + F] \quad (40a)$$

$$I_2(-h_2') Z_{ca} = -j[(C_2 - C_3) \cos \beta h_2' - (D_2 - D_3) \sin \beta h_2' + F'] \quad (40b)$$

where

$$Z_{ca} = \frac{\zeta_0}{2\pi} (p_{24} - p_{25} - p_{34} + p_{35}) = \frac{\zeta_0}{2\pi} \left[2 \ln \frac{b}{a_2} + \ln \frac{d_{24}d_{35}}{d_{25}d_{34}}\right] \quad (41)$$

is the antisymmetric characteristic impedance and

$$F = U_1K_1[\Psi_{21}(h_2) - \Psi_{31}(h_2)] + U_2 - U_3$$

$$\doteq 2U_1K_1(\cos \beta h_2 - \cos \beta h_1)\ \ln[(d + b/2)/(d - b/2)] + U_2 - U_3 \qquad (42a)$$

$$F' = U_1K_1[\Psi_{21}(-h_2') - \Psi_{31}(-h_2')] + U_2 - U_3$$

$$\doteq 2U_1K_1(\cos \beta h_2' - \cos \beta h_1)\ \ln[(d + b/2)/(d - b/2)] + U_2 - U_3 \qquad (42b)$$

The approximate final forms in (42a,b) follow from the relation

$$\Psi_{21}(z) - \Psi_{31}(z) \doteq 2[\cos \beta z - \cos \beta h_1]\ \ln[(d + b/2)/(d - b/2)] \qquad (43)$$

which is derived in the Appendix. Note that from (3)

$$U_1 = E^{inc}(0)/\beta \quad \text{and} \quad (U_2 - U_3) = [-j2E^{inc}(0)/\beta]\ e^{j\beta d} \sin(\beta b/2) \qquad (44)$$

The substitution of (37) and (38) in (40a) and (40b) leads to the following simultaneous equations for $I_2(h_2)$ and $I_2(-h_2')$:

$$I_2(h_2)[Z_s \cos \beta s + jZ_{ca} \sin \beta s] + I_2(-h_2')\ Z_0 = F \sin \beta s \qquad (45a)$$

$$I_2(h_2)\ Z_s + I_2(-h_2')[Z_0 \cos \beta s + jZ_{ca} \sin \beta s] = F' \sin \beta s \qquad (45b)$$

These are easily solved with the results:

$$I_s = I_2(h_2) = [F(Z_{ca} \sin \beta s - jZ_0 \cos \beta s) + jF'Z_0]/\Delta \qquad (46a)$$

$$I_0 = I_2(-h_2') = [F'(Z_{ca} \sin \beta s - jZ_s \cos \beta s) + jFZ_s]/\Delta \qquad (46b)$$

$$\Delta = Z_{ca}(Z_0 + Z_s) \cos \beta s + j(Z_{ca}^2 + Z_0Z_s) \sin \beta s \qquad (46c)$$

These are the desired currents, viz., I_s in Z_s and I_0 in Z_0. Note that if the line extends from $z = h_2'$ to $z = h_2$ or from $z = -h_2'$ to $z = -h_2$ instead of from $z = -h_2'$ to $z = h_2$ as assumed, the corresponding changes in sign must be made in the several formulas.

CURRENTS AT ALL POINTS ALONG THE TRANSMISSION LINE

Although the specific purpose of this investigation is to evaluate the currents in the loads as given by (46a,b,c), it is of interest to complete the analysis and indicate how the currents at all points along the two conductors of the line can be determined. Note that if (46a,b,c) are used in (37) and (38), explicit formulas are obtained for the difference constants $(C_2 - C_3)$ and $(D_2 - D_3)$. In order to solve (28) and (29) for $I_2(z)$ and $I_3(z)$ it is necessary to obtain expressions for $(C_2 + C_3)$ and $(D_2 + D_3)$ so that all four constants C_2, C_3, D_2, and D_3 can be determined.

Let (28) and (29) be added and z set equal to h_2 and $-h_2'$ successively. The results are:

$$I_2(h_2)\, Z_{cs} = -j[(C_2 + C_3) \cos \beta h_2 + (D_2 + D_3) \sin \beta h_2 + G] \tag{47a}$$

$$I_2(-h_2')\, Z_{cs} = -j[(C_2 + C_3) \cos \beta h_2' - (D_2 + D_3) \sin \beta h_2' + G'] \tag{47b}$$

where

$$Z_{cs} = \frac{\zeta_0}{2\pi} (p_{24} - p_{25} + p_{34} - p_{35}) = \frac{\zeta_0}{2\pi} \ln(d_{24}d_{34}/d_{25}d_{35}) \tag{48}$$

and

$$G = U_1K_1[\Psi_{21}(h_2) + \Psi_{31}(h_2)] + U_2 + U_3 \tag{49a}$$

$$G' = U_1K_1[\Psi_{21}(-h_2') + \Psi_{31}(-h_2')] + U_2 + U_3 \tag{49b}$$

The equations (47a,b) can be rearranged as follows:

$$(C_2 + C_3) \cos \beta h_2 + (D_2 + D_3) \sin \beta h_2 = jI_2(h_2)\, Z_{cs} - G \tag{50a}$$

$$(C_2 + C_3) \cos \beta h_2' - (D_2 + D_3) \sin \beta h_2' = jI_2(-h_2') Z_{cs} - G' \tag{50b}$$

These can be solved for $(C_2 + C_3)$ and $(D_2 + D_3)$ to give

$$(C_2 + C_3) = \{[jI_2(h_2) Z_{cs} - G] \sin \beta h_2' + [jI_2(-h_2') Z_{cs} - G'] \sin \beta h_2\}/\sin \beta s \tag{51a}$$

$$(D_2 + D_3) = \{[jI_2(h_2) Z_{cs} - G] \cos \beta h_2' - [jI_2(-h_2') Z_{cs} - G'] \cos \beta h_2\}/\sin \beta s \tag{51b}$$

where $I_2(h_2)$ and $I_2(-h_2')$ are in (46a,b), G and G' in (49a,b) with (68) in the Appendix.

Since $(C_2 - C_3)$ and $(D_2 - D_3)$ are given in (37) and (38) with (46a,b), and $(C_2 + C_3)$ and $(D_2 + D_3)$ are available in (51a,b), explicit formulas for C_2, C_3, D_2, D_3 are readily obtained. If these are substituted in (28) and (29), all quantities on the right are known so that $I_2(z)$ and $I_3(z)$ can be determined. The equal and opposite transmission-line part of the currents is $I^a(z) = [I_2(z) + I_3(z)]/2$.

SPECIAL CASE A: TERMINATED LINE NOT VERY CLOSE TO CYLINDER

When the transmission line is sufficiently far from the cylinder so that the inequalities $d \gg a_1$ and $d \gg b$ are satisfied while $\beta d \ll 1$ is still valid, it follows from (11a,b) and (12) that $b_i \doteq 0$ and $d_{24} \doteq d_{25} \doteq d - b/2$, $d_{34} \doteq d_{35} \doteq d + b/2$. With these values (41) gives

$$Z_{ca} \doteq Z_c = (\zeta_0/\pi)\ln(b/a_2) \tag{52}$$

which is the characteristic impedance of the isolated two-wire line.

SPECIAL CASE B: TERMINATED LINE VERY CLOSE TO CYLINDER

When the transmission line is so near to the cylinder and so closely spaced that the following inequalities are satisfied:

$$(d_c/2) = d - a_1 \ll a_1 \quad , \quad b \ll a_1 \quad , \quad b \ll d \tag{53}$$

it follows with (11a,b) that

$$d_{24} = \frac{[a_1 + (d_c - b)/2]^2 - a_1^2}{a_1 + (d_c - b)/2} \doteq d_c - b \quad ; \quad d_{34} \doteq d_c \tag{54a}$$

$$d_{35} = \frac{[a_1 + (d_c + b)/2]^2 - a_1^2}{a_1 + (d_c + b)/2} \doteq d_c + b \quad ; \quad d_{25} \doteq d_c \tag{54b}$$

Since conductor 2 and its image, conductor 4, may be very close together, the approximate expression for p_{24} in (14) is not adequate. The accurate formula that includes the proximity effect is [5]:

$$p_{24} = (\zeta_0/2\pi)\ln(d_{24e}/a_2) \quad ; \quad d_{24e} = (d_{24}/2)[1 + \sqrt{1 - (2a_2/d_{24})^2}] \tag{55}$$

With (54a,b), (41) gives

$$Z_{ca} = (\zeta_0/2\pi)\{2 \ln(b/a_2) + \ln[(d_c + b)(d_c - b + \sqrt{(d_c - b)^2 + 4a_2^2})/2d_c^2]\} \tag{56a}$$

where $d_c/2$ is the distance from the surface of the cylinder to the center of the transmission line. Note that when the line conductor 2 is in contact with the cylinder, $d_c = b + 2a_2$ so that with $a_2 \ll b$,

$$Z_{ca} \doteq (\zeta_0/2\pi)\ln[2(1 + \sqrt{2})b/a_2] = (\zeta_0/2\pi)[\ln(2b/a_2) + 0.881] \tag{56b}$$

Similarly in (43),

$$2 \ln\left[\frac{d + b/2}{d - b/2}\right] \doteq 2 \ln\left[\frac{1 + (d_c + b)/2a_1}{1 + (d_c - b)/2a_1}\right] \doteq \frac{d_c + b}{a_1} - \frac{d_c - b}{a_1} = \frac{2b}{a_1} \tag{57}$$

where use has been made of (53) and the series expansion of the logarithms.

SPECIAL CASE C: SINGLE CONDUCTOR CONNECTED TO CYLINDER BY TERMINATING IMPEDANCES

If conductor 2 of the transmission line is brought in contact with the cylinder, it may be removed and the terminating impedances Z_0 and Z_s connected directly to the cylinder as indicated on the right in Fig. 1 where Z_0 and Z_s are the input impedances of coaxial lines into the rocket. In this case $d_c = b$ and from (56a) the characteristic impedance of the line formed by conductor 3 and the cylinder is

$$Z_{ca} = (\zeta_0/2\pi)\ell n(2b/a_2) \qquad (58)$$

The formulas (46a,b) give the currents in Z_s and Z_0 terminating the single conductor to the cylinder. Since Z_{ca} in (58) differs by only 52.8 ohms from Z_{ca} in (56b), it is clear that the currents in Z_0 and Z_s are of the same order of magnitude when Z_0 and Z_s terminate a two-wire line (with spacing b) very close to or in contact with the surface of the cylinder as when Z_0 and Z_s are connected between the ends of a single conductor with its axis at a distance b from the surface of the cylinder. This is true as long as the conditions in (53) are satisfied. Essentially the only difference arises from the somewhat different values of Z_{ca} obtained from (56a) with $d_c = b$ or $d_c = b + 2a_2$.

CONCLUSION

General formulas have been derived for the currents in the conductors of a two-wire line that is parallel to the axis of and quite near to a conducting cylinder when both line and cylinder are illuminated by an incident plane electromagnetic wave with the electric vector parallel to the common axis. In particular, explicit formulas are given for the currents in the loads terminating both ends of the line when this is close to or in contact with the cylinder. Formulas for the currents in the loads connecting the ends of a single wire to the surface of the cylinder are also given.

APPENDIX

The functions Ψ_{dU} and $\Psi_U(h_1)$ which appear in (24b) are defined as follows:

$$\Psi_{dU} = (1 - \cos\beta h_1)^{-1} \int_{-h_1}^{h_1} (\cos\beta z' - \cos\beta h_1)[K(0,z') - K(h_1,z')]\, dz' \tag{59}$$

$$\Psi_U(h_1) = \int_{-h_1}^{h_1} (\cos\beta z' - \cos\beta h_1)\, K(h_1,z')\, dz' \tag{60}$$

with

$$K(z,z') = \frac{e^{-j\beta R}}{R} \tag{61}$$

$$R = [(z - z')^2 + a_1^2]^{1/2} \tag{62}$$

The integral functions in (26) are defined as follows:

$$C_f(h,x) = \int_{-h}^{h} \cos\beta x' \frac{e^{-j\beta R}}{R}\, dx' \tag{63}$$

$$E_f(h,x) = \int_{-h}^{h} \frac{e^{-j\beta R}}{R}\, dx' \tag{64}$$

where

$$R = [(x - x')^2 + f^2]^{1/2} \tag{65}$$

They are readily evaluated by computer.

The combination $\Psi_{21}(x) - \Psi_{31}(x)$ which occurs in (42a,b) with $x = h_2$ and $x = -h_2'$ is readily evaluated from (63) and (64). Thus,

$$\Psi_{21}(x) - \Psi_{31}(x) = \int_{-h_1}^{h_1} (\cos \beta x' - \cos \beta h_1) \times \left(\frac{e^{-j\beta R_{21}}}{R_{21}} - \frac{e^{-j\beta R_{31}}}{R_{31}}\right) dx' \tag{66}$$

This has the same form as (13) and satisfies the same conditions so that with $R_{21} = [(x - x')^2 + (d - b/2)^2]^{1/2}$ and $R_{31} = [(x - x')^2 + (d + b/2)^2]^{1/2}$ and (1a), it follows that

$$\Psi_{21}(x) - \Psi_{31}(x) \doteq (\cos \beta x - \cos \beta h_1)\cdot 2 \ \ln[(d + b/2)/(d - b/2)] \tag{67}$$

The combination $\Psi_{21}(x) + \Psi_{31}(x)$ which occurs in (49a,b) cannot be simplified in this manner. It is given by

$$\Psi_{21}(x) + \Psi_{31}(x) = C_{(d+b/2)}(h_1,x) + C_{(d-b/2)}(h_1,x) - [E_{(d+b/2)}(h_1,x) + E_{(d-b/2)}(h_1,x)] \cos \beta h_1 \tag{68}$$

where the C and E functions are defined in (63) and (64). They are readily evaluated by computer.

REFERENCES

[1] R. W. P. King, Transmission-Line Theory, pp. 60-61, Dover Publications, Inc., 1965.

[2] Ibid., pp. 60-61.

[3] R. W. P. King and C. W. Harrison, Jr., Antennas and Waves: A Modern Approach, p. 520, Eq.(8.3.19b), M.I.T. Press, 1969.

[4] Ibid., p. 156, Eq.(3.4.32).

[5] R. W. P. King, Transmission-Line Theory, p. 29, Eq.(40a), Dover Publications, Inc., 1965.

LEGENDS FOR FIGURES

Fig. 1. Rocket with external single and two-wire lines.

Fig. 2. Two-wire transmission line coupled to cylinder.

Fig. 3. Two-wire line close to cylinder with approximate image.

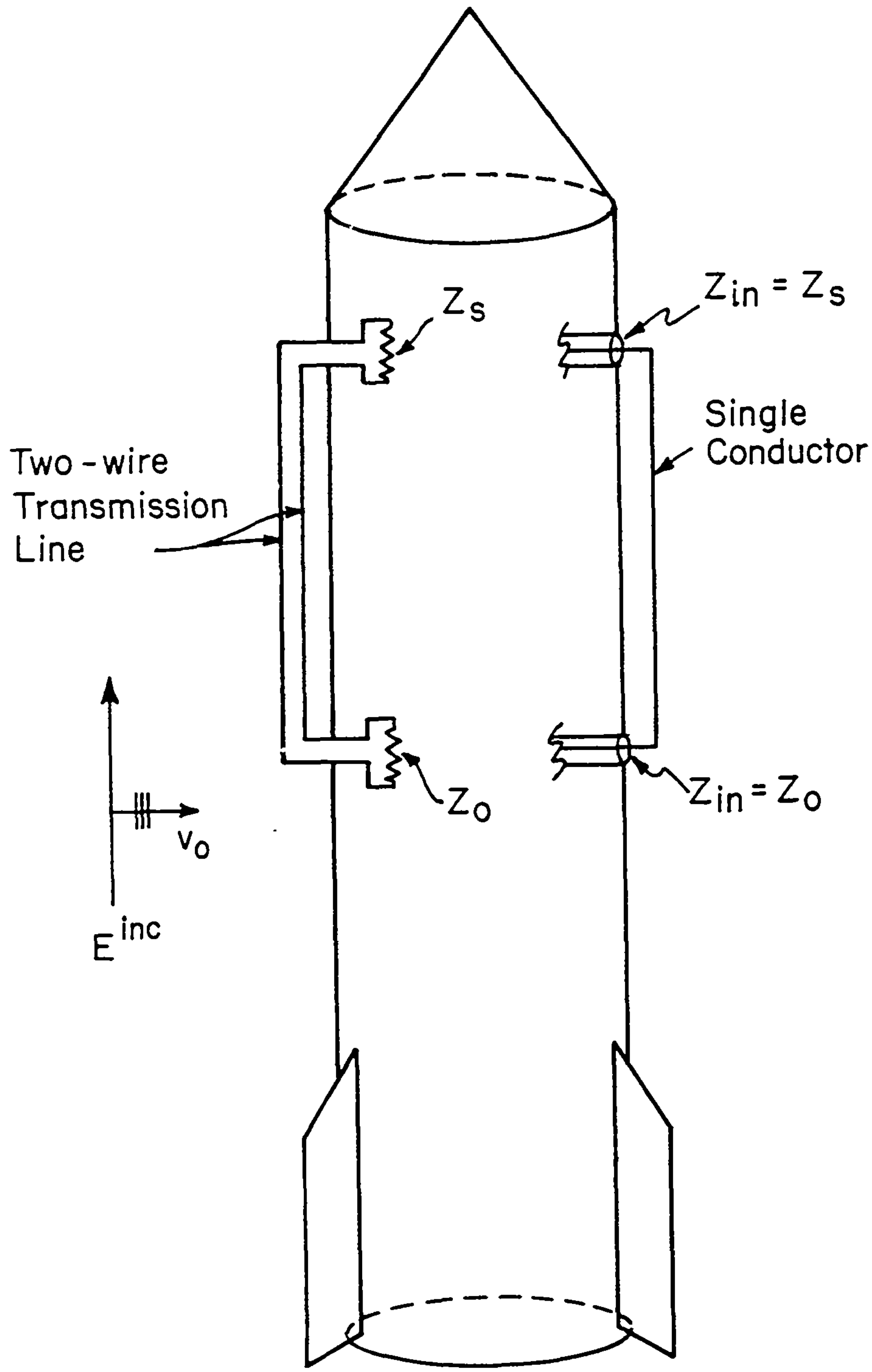

FIG. 1 ROCKET WITH EXTERNAL SINGLE AND TWO-WIRE LINES.

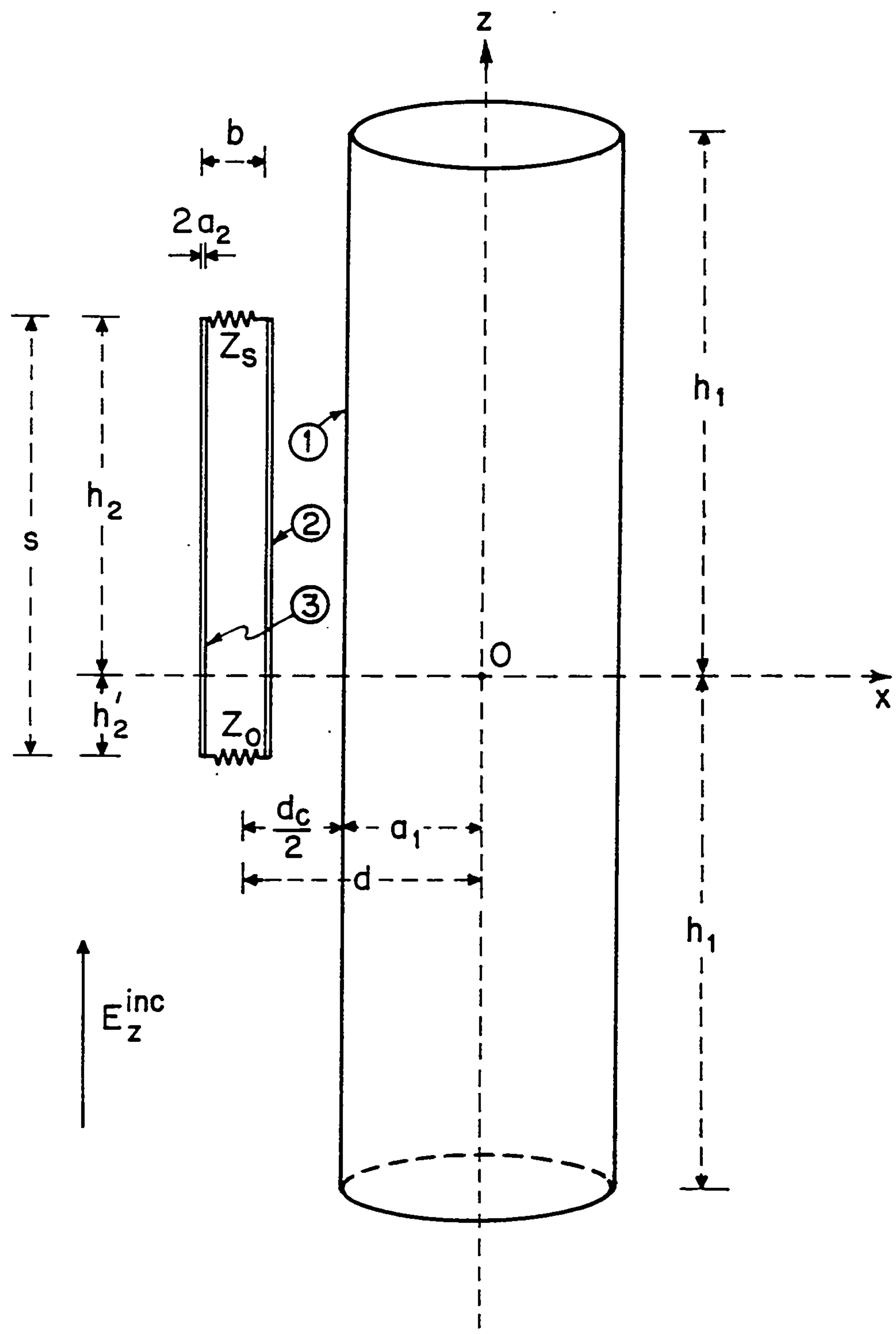

FIG. 2 TWO-WIRE TRANSMISSION LINE COUPLED TO CYLINDER

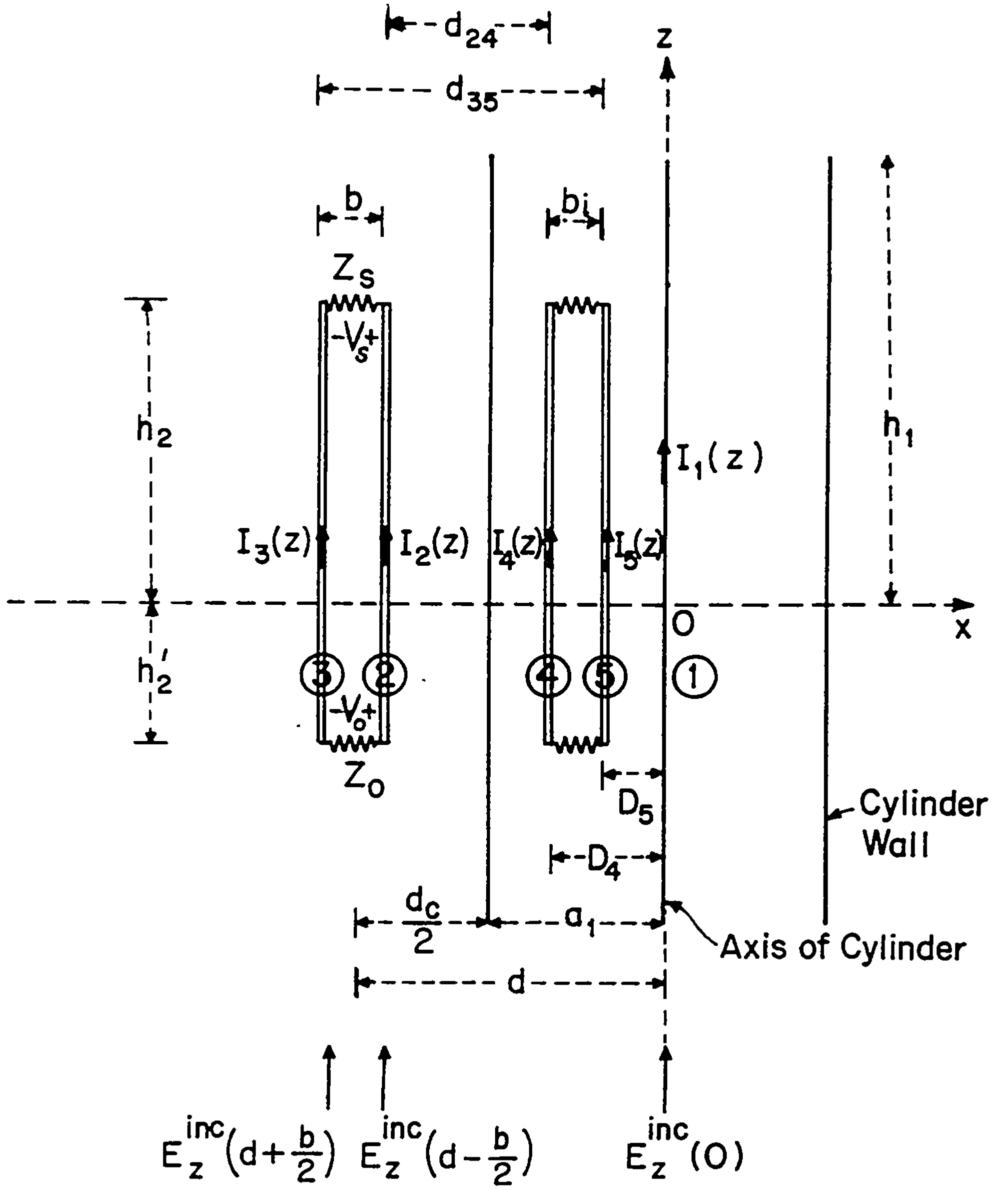

FIG. 3 TWO-WIRE LINE CLOSE TO CYLINDER WITH APPROXIMATE IMAGE.