SC-R-71 3209

BOUNDS ON THE LOAD CURRENTS OF EXPOSED ONE- AND TWO-CONDUCTOR TRANSMISSION LINES ELECTROMAGNETICALLY COUPLED TO A ROCKET

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April 1971

ACKNOWLEDGMENT

The writer is indebted to Professor Ronold W. P. King, Sandia Laboratories Consultant, for reviewing the manuscript.

SUMMARY

Three circuits are analyzed: One consists of an isolated two-wire transmission line with terminating impedances; another of a single conductor with terminating impedances grounded to an infinite, perfectly conducting plane; and, finally, a terminated two-wire transmission line in the vicinity of an infinite perfectly conducting plane. In all cases, a plane monochromatic electromagnetic wave is incident on the wires with the electric vector parallel to their axes. The wires are oriented with respect to the incident field and the ground plane, if present, for maximum response. The objective is to derive formulas for the currents in the load impedances of the three circuit configurations described above. The writer then presents a heuristic argument to the effect that solutions of these problems bracket the response of exposed unshielded one- and two-wire transmission lines arranged parallel to the axis of a rocket and close to its surface. The established upper and lower bounds for the load currents are sufficiently close together to be of considerable practical value in the study of the electromagnetic compatibility of rockets.

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BOUNDS ON THE LOAD CURRENTS OF EXPOSED ONE- AND TWO-CONDUCTOR TRANSMISSION LINES ELECTROMAGNETICALLY COUPLED TO A ROCKET

Prolegomena

In Reference 1, an analysis is given for a circuit consisting of an exposed unshielded longitudinal wire, two terminating impedances, and the rocket for the case of transverse electromagnetic field excitation. An entirely different problem arises when the polarization of the incident field is such that the electric vector is parallel to the axes of the conductors and lies in their common plane. An accurate solution of this problem appears difficult to achieve, but bounds of practical utility on the values of the load currents are relatively easy to obtain. In Reference 2, another circuit consisting of an exposed terminated two-wire transmission line electromagnetically coupled to a rocket is treated theoretically. The axes of the wires and rocket are assumed to be parallel and so oriented with respect to the incident electromagnetic field that maximum response is obtained; i.e., maximum currents in the load impedances. Omitted from discussion are two limiting cases. Evidently, when the rocket is not present, the complete circuit consists of an isolated two-wire transmission line with impedance terminations. Also, when the rocket becomes large in terms of the wavelength of the incident field illumination, it may be replaced by an infinite, perfectly conducting plane. The circuit then consists of a terminated two-wire transmission line parallel to and some distance removed from the plane. It is desirable to obtain precisely the currents in the line terminations under these special circumstances. Formulas are derived in this paper to permit these calculations.

The circuits of general interest are shown in Figure 1. To bracket the response of these composite circuits requires analysis of three circuits that might be termed "building blocks." The first of these is an isolated terminated two-wire transmission line, shown in Figure 2.

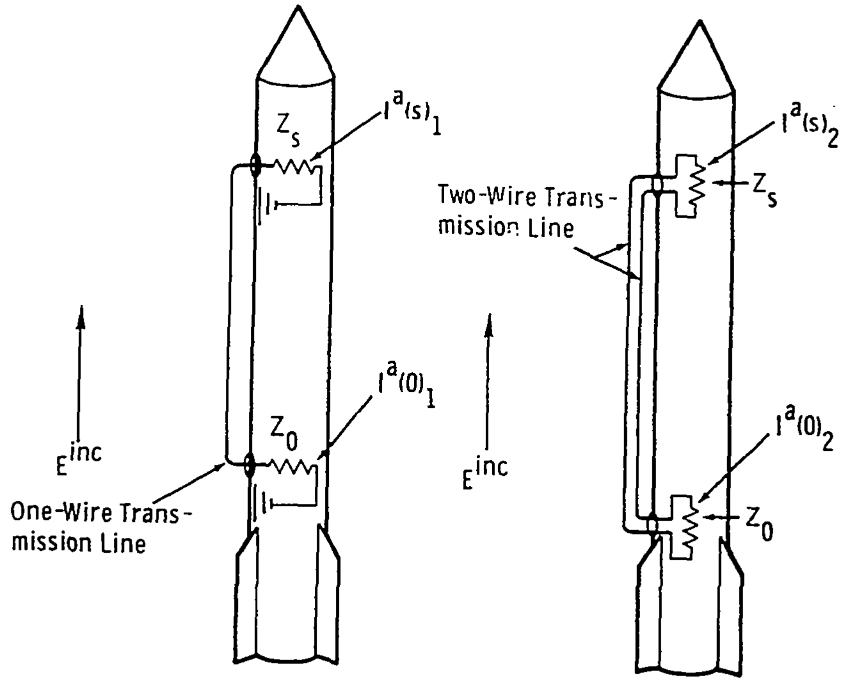

Figure 1. Rockets with External Cables

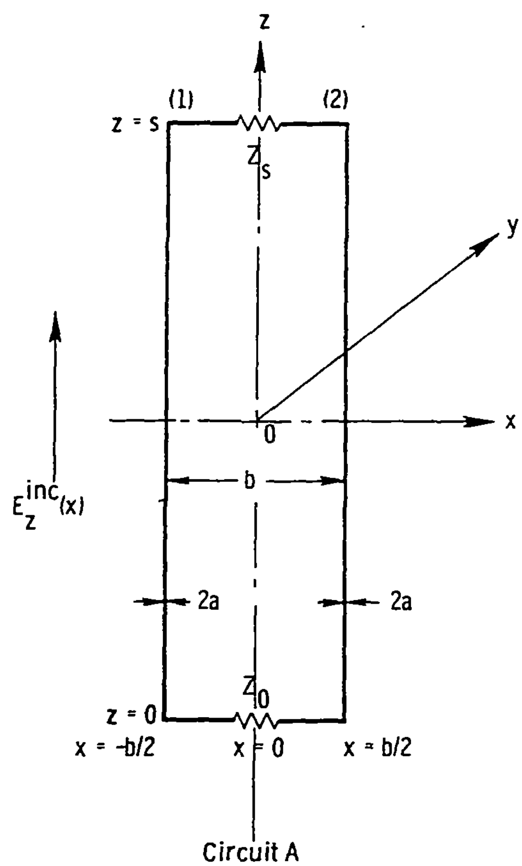

Figure 2.

Isolated Impedance Terminated Transmission Line

A single-wire line with terminations grounded to an infinite, perfectly conducting plane is illustrated by Figure 3, and a terminated two-wire transmission line in proximity to a perfectly conducting plane is portrayed by Figure 4.

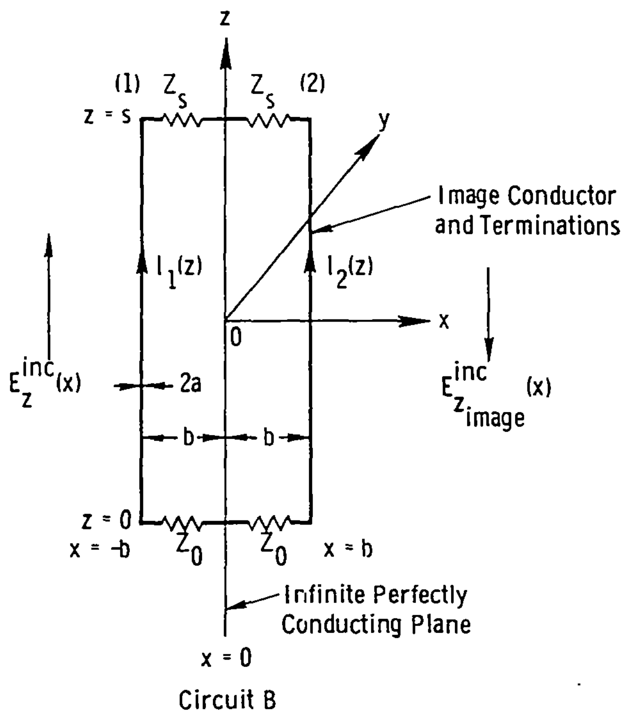

Figure 3.

Single Conductor with Terminating Impedances Grounded to a Perfectly Conducting Plane

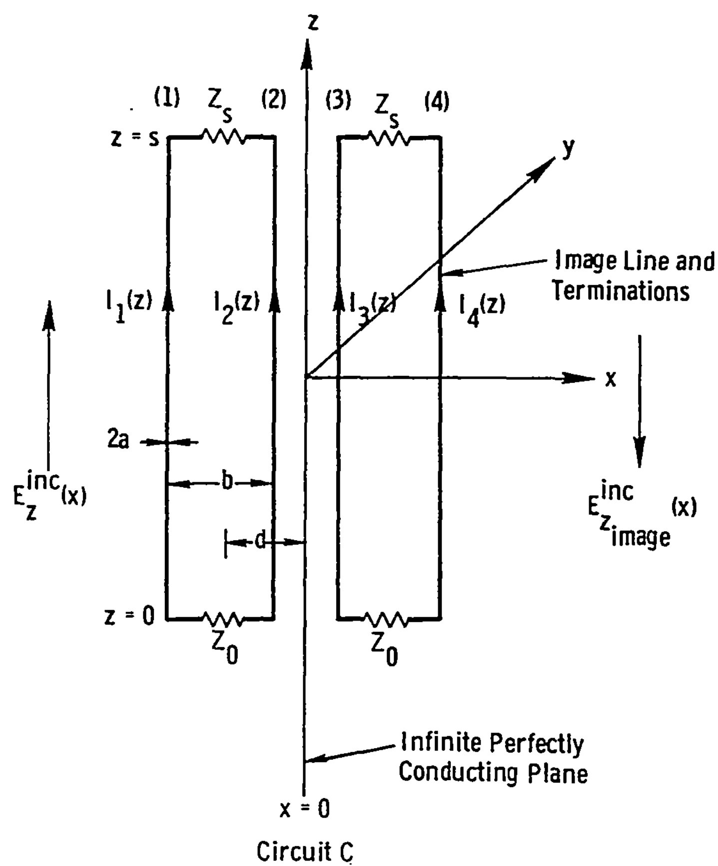

Figure 4.

Two-Wire Line with Terminating Impedances Oriented Edgewise with Respect to an Infinite Perfectly Conducting Ground Plane

Isolated Two-Conductor Terminated Transmission Line (Circuit A)

Figure 2 illustrates an isolated two-wire transmission line. The wires are of radius a and are spaced a distance b apart. The origin of a Cartesian coordinate system is at the midpoint of the line. The line lies in the xz plane, with conductors parallel to the z-axis. The terminating impedances Z_o and Z_s are located at z = 0 and z = s, respectively.

The incident plane wave propagates in the positive x direction with electric field parallel to the z-axis. The analytical representation is

$$E_z^{inc}(x) = E_z^{inc}(0)\ e^{-j\beta x} \ , \tag{1}$$

where the reference for phase is at x = 0. $\beta = 2\pi/\lambda$ is the radian wave number and λ is the free-space wavelength.

It is convenient to define an excitation function U for each of the conductors. For wires 1 and 2, this function may be written

$$E_z^{inc}\left(-\frac{b}{2}\right) = E_z^{inc}(0)\ e^{j\frac{\beta b}{2}} = -\beta U_1 \ , \tag{2}$$

and

$$E_z^{inc}\left(\frac{b}{2}\right) = E_z^{inc}(0)\ e^{-j\frac{\beta b}{2}} = -\beta U_2 \ , \tag{3}$$

respectively.

By analogy with previous work,[2] the simultaneous integral equations for the <u>total currents</u> $I_1(z)$ and $I_2(z)$ in the line conductors are:

$$A_1(z) = \frac{\mu_o}{4\pi}\left[\int_0^s I_1(z')\frac{e^{-j\beta R_{11}}}{R_{11}}\,dz' + \int_0^s I_2(z')\frac{e^{-j\beta R_{12}}}{R_{12}}\,dz'\right]$$

$$= -\frac{j}{v_p}\left\{C_1 \cos \beta_z + D_1 \sin \beta_z + U_1\right\}, \tag{4}$$

and

$$A_2(z) = \frac{\mu_o}{4\pi}\left[\int_0^s I_1(z')\frac{e^{-j\beta R_{21}}}{R_{21}}\,dz' + \int_0^s I_2(z')\frac{e^{-j\beta R_{22}}}{R_{22}}\,dz'\right]$$

$$= -\frac{j}{v_p}\left\{C_2 \cos \beta_z + D_2 \sin \beta_z + U_2\right\}, \tag{5}$$

where

$$\left.\begin{aligned} R_{11} = R_{22} &= \sqrt{(z - z')^2 + a^2} \\ R_{12} = R_{21} &= \sqrt{(z - z')^2 + b^2} \end{aligned}\right\}, \tag{6}$$

$\mu_o = 4\pi \times 10^{-7}$ H/m is the permeability of free space, v_p is the velocity of light, $v_p = 3 \times 10^8$ m/sec, and the constants of integration are C and D.

It is convenient to separate Equations (4) and (5) into symmetrical and anti-symmetrical components. For the vector potentials $A_1(z)$ and $A_2(z)$, this is accomplished by writing

$$A_1(z) = \frac{1}{2}\left\{A_1(z) + A_2(z)\right\} + \frac{1}{2}\left\{A_1(z) - A_2(z)\right\}, \tag{7}$$

$$A_2(z) = \frac{1}{2}\left\{A_1(z) + A_2(z)\right\} - \frac{1}{2}\left\{A_1(z) - A_2(z)\right\}, \tag{8}$$

and then setting

$$A_1(z) = A^s(z) + A^a(z) \ , \tag{9}$$

$$A_2(z) = A^s(z) - A^a(z) \ , \tag{10}$$

where

$$A^s(z) = \frac{1}{2}\left\{A_1(z) + A_2(z)\right\} \ , \tag{11}$$

$$A^a(z) = \frac{1}{2}\left\{A_1(z) - A_2(z)\right\} \ . \tag{12}$$

In a similar manner,

$$I^s(z) = \frac{1}{2}\left\{I_1(z) + I_2(z)\right\} \ , \tag{13}$$

$$I^a(z) = \frac{1}{2}\left\{I_1(z) - I_2(z)\right\} \ , \tag{14}$$

$$U^s = \frac{1}{2}\left\{U_1 + U_2\right\} = - \frac{E_z^{inc}(0)}{\beta} \cos\left(\frac{\beta b}{2}\right) \ , \tag{15}$$

$$U^a = \frac{1}{2}\left\{U_1 - U_2\right\} = - \frac{jE_z^{inc}(0)}{\beta} \sin\left(\frac{\beta b}{2}\right) \ . \tag{16}$$

In writing Equations (15) and (16), use has been made of Equations (2) and (3).

Employing the above definitions for the symmetrical and antisymmetrical components of vector potentials, currents and excitation functions (4) and (5) become

$$4\pi\mu_o^{-1} A^s(z) = \int_0^s I^s(z')\left[\frac{e^{-j\beta R_{11}}}{R_{11}} + \frac{e^{-j\beta R_{12}}}{R_{12}}\right] dz'$$

$$= -j\,\frac{4\pi}{\zeta_o}\left[C^s \cos\beta z + D^s \sin\beta z + U^s\right] , \tag{17}$$

$$4\pi\mu_o^{-1} A^a(z) = \int_0^s I^a(z')\left[\frac{e^{-j\beta R_{11}}}{R_{11}} - \frac{e^{-j\beta R_{12}}}{R_{12}}\right] dz'$$

$$= -j\,\frac{4\pi}{\zeta_o}\left[C^a \cos\beta z + D^a \sin\beta z + U^a\right] . \tag{18}$$

Here,

$$\left.\begin{aligned} C^s &= \tfrac{1}{2}(C_1 + C_2) , & C^a &= \tfrac{1}{2}(C_1 - C_2) \\ D^s &= \tfrac{1}{2}(D_1 + D_2) , & D^a &= \tfrac{1}{2}(D_1 - D_2) \end{aligned}\right\} , \tag{19}$$

and $\zeta_o = 120\pi$ ohms is the characteristic impedance of free space.

The reader should observe that $I^s(z)$ is a codirectional or antenna current. This current is excited on the wires by E_z^{inc} because $U^s \neq 0$. This current causes no voltage drop across the load impedances Z_o and Z_s if they are lumped and centrally located. The current $I^a(z)$ is a bidirectional or transmission line current. The voltages across Z_o and Z_s are due solely to the antisymmetrical component of current excited by the incident field in the structure. Accordingly, interest centers in solving Equation (18) for $I^a(0)$ and $I^a(s)$ subject to the boundary conditions.

It can be shown that the left side of Equation (18) is well approximated by[3]

$$\int_0^s I^a(z')\left[\frac{e^{-j\beta R_{11}}}{R_{11}} - \frac{e^{-j\beta R_{12}}}{R_{12}}\right]dz' \simeq 2I^a(z)\ \ell n\left(\frac{b}{a}\right), \tag{20}$$

when transmission line theory applies, i.e., $\beta b << 1$. Accordingly,

$$I^a(z) = -j\ \frac{2}{Z_{cA}}\left[C^a \cos\beta z + D^a \sin\beta z + U^a\right], \tag{21}$$

where

$$Z_{cA} = \frac{\zeta_o}{\pi}\ \ell n\left(\frac{b}{a}\right), \tag{22}$$

is the characteristic impedance of a lossless two-conductor transmission line.

For the antisymmetric current $I^a(z)$, ordinary transmission line equations apply so that the potential difference across the wires at any position z can be obtained directly from Equation (21). It is given by

$$V^a(z) = \frac{j}{\omega c}\ \frac{2I^a(z)}{\partial z} = 2\left[-C^a \sin\beta z + D^a \cos\beta z\right], \tag{23}$$

where use has been made of the relations $\beta = \omega\sqrt{\ell^e c}$ and $Z_{cA} = \sqrt{\frac{\ell^e}{c}}$ for a dissipationless line. Evidently,

$$V^a(s) = I^a(s)Z_s = 2\left[-C^a \sin\beta s + D^a \cos\beta s\right], \tag{24}$$

$$V^a(0) = -I^a(0)Z_o = 2D^a, \tag{25}$$

where, from Equation (21),

$$I^a(0) = -j\ \frac{2}{Z_{cA}}\left[C^a + U^a\right], \tag{26}$$

and

$$I^a(s) = -j \frac{2}{Z_{cA}} \left[C^a \cos \beta s + D^a \sin \beta s + U^a \right] . \tag{27}$$

Equations (24) through (27) may be solved simultaneously for C^a and D^a. The currents in the terminating impedances Z_o and Z_s are then given by Equations (26) and (27), respectively. Using Equation (16) it is readily shown that

$$I_A^a(0) = -j2E_z^{inc}(0)\beta^{-1}D_A^{-1} \sin\left(\frac{\beta b}{2}\right)\left[Z_{cA} \sin \beta s + jZ_s(1 - \cos \beta s)\right] , \tag{28}$$

$$I_A^a(s) = -j2E_z^{inc}(0)\beta^{-1}D_A^{-1} \sin\left(\frac{\beta b}{2}\right)\left[Z_{cA} \sin \beta s + jZ_o(1 - \cos \beta s)\right] , \tag{29}$$

where

$$D_A = Z_{cA}(Z_o + Z_s) \cos \beta s + j\left(Z_{cA}^2 + Z_s Z_o\right) . \tag{30}$$

These are the final expressions for the load currents in an isolated two-wire transmission line displayed by Figure 2 and delineated as Circuit A.

Single-Conductor Transmission Line With Terminations Grounded to an Infinite Perfectly Conducting Plane (Circuit B)

In this section, the objective is to derive formulas for the load currents in a circuit consisting of a single conductor with terminating impedances grounded to a perfectly conducting plane, as illustrated by Figure 3. In carrying out the analysis, it is convenient to substitute an image of the conductor and its terminations for the ground plane. The circuits are then electrically equivalent. However, it is necessary to remember that in obtaining the excitation functions for the wires both the direct and image fields must be included.

The impedance termination at $z = 0$ is $2Z_o$, and at $z = s$ it is $2Z_s$ (see Figure 3). For convenience, the distance between the actual conductor and its image is designated 2b; the radius of the wire is designated a. The reference for phase is taken at the midpoint of the line-image configuration.

Now

$$\left.\begin{aligned} E_z^{inc}(x) &= E_z^{inc}(0)\, e^{-j\beta x} \\ E_{z_{image}}^{inc}(x) &= -E_z^{inc}(0)\, e^{j\beta x} \end{aligned}\right\} , \tag{31}$$

so that

$$-\beta U_1 = E_z^{inc}(-b) + E_{z_{image}}^{inc}(-b)$$

$$= E_z^{inc}(0)\left[e^{\beta b} - e^{-j\beta b}\right] = j2E_z^{inc}(0) \sin(\beta b) \; . \tag{32}$$

By symmetry, it is evident that

$$U_2 = -U_1 \; . \tag{33}$$

Accordingly,

$$U^s = \frac{1}{2}(U_1 + U_2) = 0 \; , \tag{34}$$

$$U^a = \frac{1}{2}(U_1 - U_2) = U_1 \; , \tag{35}$$

where use has been made of Equations (15) and (16).

Since $U^s = 0$, it is clear, from Equation (17), that an antenna current cannot exist in this circuit, since there is no driving mechanism for this mode.

Inasmuch as the circuits shown in Figures 2 and 3 differ in no geometrical respects, the final formulas for the load currents $I_B^a(0)$ and $I_B^a(s)$ in the loads of Circuit B may be written down by analogy with Equations (28) through (30). Thus,

$$I_B^a(0) = -j4E_z^{inc}(0)\beta^{-1}D_B^{-1} \sin \beta b\left[Z_{cB} \sin \beta s + j2Z_s(1 - \cos \beta s)\right] , \tag{36}$$

$$I_B^a(s) = -j4E_z^{inc}(0)\beta^{-1}D_B^{-1} \sin \beta b\left[Z_{cB} \sin \beta s + j2Z_o(1 - \cos \beta s)\right] , \tag{37}$$

where

$$D_B = 2Z_{cB}(Z_o + Z_s) \cos \beta s + j\left(Z_{cB}^2 + 4Z_sZ_o\right)\sin \beta s , \tag{38}$$

and

$$Z_{cB} = \frac{\zeta_o}{\pi} \ln \left(\frac{2b}{a}\right) . \tag{39}$$

Terminated Two-Wire Transmission Line Oriented Edgewise With Respect to a Perfectly Conducting Ground Plane (Circuit C)

Figure 4 shows a two-wire transmission with terminating impedances oriented with respect to the incident field and infinite perfectly conducting ground plane to obtain maximum load currents. The ground plane may be dispensed with by introducing an image transmission line and incident field, as was done for Circuit B. The ground plane represents the exterior surface of a missile of infinite length and radius (refer to Figure 1).

The simultaneous integral equations for Circuit C are

$$4\pi\mu_o^{-1}A_1(z) = \int_0^s I_1(z')\,\frac{e^{-j\beta R_{11}}}{R_{11}}\,dz' + \int_0^s I_2(z')\,\frac{e^{-j\beta R_{12}}}{R_{12}}\,dz'$$

$$- \int_0^s I_2(z')\,\frac{e^{-j\beta R_{13}}}{R_{13}}\,dz' - \int_0^s I_1(z')\,\frac{e^{-j\beta R_{14}}}{R_{14}}\,dz'$$

$$= -j\,\frac{4\pi}{\zeta_o}\left\{C_1 \cos\beta z + D_1 \sin\beta z + U_1\right\}\ , \tag{40}$$

$$4\pi\mu_o^{-1}A_2(z) = \int_0^s I_1(z')\,\frac{e^{-j\beta R_{21}}}{R_{21}}\,dz' + \int_0^s I_2(z')\,\frac{e^{-j\beta R_{22}}}{R_{22}}\,dz'$$

$$- \int_0^s I_2(z')\,\frac{e^{-j\beta R_{23}}}{R_{23}}\,dz' - \int_0^s I_1(z')\,\frac{e^{-j\beta R_{24}}}{R_{24}}\,dz'$$

$$= -j\,\frac{4\pi}{\zeta_o}\left\{C_2 \cos\beta z + D_2 \sin\beta z + U_2\right\}\ , \tag{41}$$

where

$$\left.\begin{aligned}
R_{11} &= R_{22} = \sqrt{(z - z')^2 + a^2} \\
R_{12} &= R_{21} = \sqrt{(z - z')^2 + b^2} \\
R_{13} &= R_{24} = \sqrt{(z - z')^2 + 4d^2} \\
R_{23} &= \sqrt{(z - z')^2 + (2d - b)^2} \\
R_{14} &= \sqrt{(z - z')^2 + (2d + b)^2}
\end{aligned}\right\} . \tag{42}$$

In writing Equations (40) and (41) use has been made of the symmetry conditions

$$\left.\begin{aligned} I_4(z) &= -I_1(z) \\ I_3(z) &= -I_2(z) \end{aligned}\right\} . \tag{43}$$

Although total currents appear in the integral equations, these currents become transmission line currents at z = 0 and z = s. Consider the first integral in Equation (40). It may be written in the form

$$\int_0^s I_1(z') \frac{e^{-j\beta R_{11}}}{R_{11}} dz' = \int_0^s I_1(z') \frac{e^{-j\beta R_g}}{R_g} dz'$$

$$+ \int_0^s I_1(z') \left\{ \frac{e^{-j\beta R_{11}}}{R_{11}} - \frac{e^{-j\beta R_g}}{R_g} \right\} dz' , \tag{44}$$

where

$$R_g = \sqrt{(z - z')^2 + g^2} . \tag{45}$$

But Equation (44) is in the same form as (20). Hence,

$$\int_0^s I_1(z') \left\{ \frac{e^{-j\beta R_{11}}}{R_{11}} - \frac{e^{-j\beta R_g}}{R_g} \right\} dz' \simeq I_1(z)\Psi_a , \tag{46}$$

provided that $2k_o(b + d) << 1$. Ψ_a will be defined presently. In Equation (45) g is the equivalent radius of the transmission line and its image. It can be determined easily but is of no importance in the present analysis.[4-6]

Every integral in Equations (40) and (41) is treated in the manner described above. The result is

$$J_g(z) + I_1(z)\Psi_a + I_2(z)\Psi_b - I_2(z)\Psi_{2d} - I_1(z)\Psi_{2d+b}$$

$$= -j\,\frac{4\pi}{\zeta_o}\left\{C_1 \cos\beta z + D_1 \sin\beta z + U_1\right\} , \tag{47}$$

$$J_g(z) + I_1(z)\Psi_b + I_2(z)\Psi_a - I_2(z)\Psi_{2d-b} - I_1(z)\Psi_{2d}$$

$$= -j\,\frac{4\pi}{\zeta_o}\left\{C_2 \cos\beta z + D_2 \sin\beta z + U_2\right\} , \tag{48}$$

where

$$J_g(z) = \int_0^s \sum_{n=1}^{4} I_n(z')\,\frac{e^{-j\beta R_g}}{R_g}\,dz' , \tag{49}$$

$$\left.\begin{aligned}
\Psi_a &= 2\ \ell n\left(\frac{g}{a}\right)\\
\Psi_b &= 2\ \ell n\left(\frac{g}{b}\right)\\
\Psi_{2d} &= 2\ \ell n\left(\frac{g}{2d}\right)\\
\Psi_{2d+b} &= 2\ \ell n\left(\frac{g}{2d+b}\right)\\
\Psi_{2d-b} &= 2\ \ell n\left(\frac{g}{2d-b}\right)
\end{aligned}\right\} . \tag{50}$$

Subtracting Equation (48) from (47) yields

$$I_1(z)\alpha_1 = I_2(z)\alpha_2 - j\,\frac{4\pi}{\zeta_o}\left\{(C_1 - C_2)\cos\beta z + (D_1 - D_2)\sin\beta z + (U_1 - U_2)\right\} . \tag{51}$$

In Equation (51),

$$\left.\begin{aligned} \alpha_1 &= \Psi_a - \Psi_b - \Psi_{2d+b} + \Psi_{2d} \\ \alpha_2 &= \Psi_a - \Psi_b - \Psi_{2d-b} + \Psi_{2d} \end{aligned}\right\} . \tag{52}$$

The next step is to add $I_1(z)\alpha_2$ to both sides of Equation (51). The result is

$$I_1(z) = I_T(z)\left(\frac{\alpha_2}{\alpha_1 + \alpha_2}\right) - j\,\frac{8\pi}{\zeta_o(\alpha_1 + \alpha_2)}\left\{C^a \cos\beta z + D^a \sin\beta z + U^a\right\} . \tag{53}$$

C^a and D^a are defined in Equation (19). By Equation (16)

$$U^a = \frac{1}{2}(U_1 - U_2) .$$

By one of Kirchhoff's laws,

$$\left.\begin{aligned} I_T(0) &= I_1(0) + I_2(0) = 0 \\ I_T(0) &= I_1(s) + I_2(s) = 0 \end{aligned}\right\} . \tag{54}$$

It follows that

$$I_1^a(0) = -j\,\frac{2}{Z_{cC}}\left[C^a + U^a\right] , \tag{55}$$

$$I_1^a(s) = -j\,\frac{2}{Z_{cC}}\left[C^a \cos\beta s + D^a \sin\beta s + U^a\right] , \tag{56}$$

where

$$Z_{cC} = \frac{\zeta_o}{4\pi}(\alpha_1 + \alpha_2) = \frac{\zeta_o}{2\pi}\,\ell n\left[\frac{b^2(4d^2 - b^2)}{4a^2d^2}\right] . \tag{57}$$

Observe that Equations (55) and (56) are in the same form as Equations (26) and (27), respectively. Also, $I_1(0) = I_1^a(0)$ and $I_1(s) = I_1^a(s)$.

The potential differences (on voltage drops) across Z_o and Z_s are

$$\phi_1(0) - \phi_2(0) = -I_1^a(0)Z_o \ , \tag{58}$$

$$\phi_1(s) - \phi_2(s) = I_1^a(s)Z_s \ , \tag{59}$$

respectively. Now,

$$\phi_1\begin{pmatrix}0\\s\end{pmatrix} = j\,\frac{\omega}{\beta^2}\left[\frac{\partial A_1(z)}{\partial z}\right]_{z=\left|\begin{smallmatrix}0\\s\end{smallmatrix}\right|} \ , \tag{60}$$

and

$$\phi_2\begin{pmatrix}0\\s\end{pmatrix} = j\,\frac{\omega}{\beta^2}\left[\frac{\partial A_2(z)}{\partial z}\right]_{z=\left|\begin{smallmatrix}0\\s\end{smallmatrix}\right|} \ , \tag{61}$$

where $A_1(z)$ and $A_2(z)$ are given by Equations (40) and (41), and $\omega = 2\pi f$ is the radian frequency.

It follows that

$$-I_1^a(0)Z_o = 2D^a \ , \tag{62}$$

and

$$I_1^a(s)Z_s = 2\left[-C^a \sin\beta s + D^a \cos\beta s\right] \ . \tag{63}$$

These equations are the same as Equations (25) and (24). Evidently, to complete the problem requires only the determination of $U^a = (U_1 - U_2)/2$. This calculation is modeled after Equations (31) and (32). Thus,

$$U_1 = -j \frac{2E_z^{inc}(0)}{\beta} \sin \beta\left(d + \frac{b}{2}\right) , \tag{64}$$

$$U_2 = -j \frac{2E_z^{inc}(0)}{\beta} \sin \beta\left(d - \frac{b}{2}\right) , \tag{65}$$

so that

$$U^a = -j \frac{2E_z^{inc}(0)}{\beta} \cos \beta d \sin \left(\frac{\beta b}{2}\right) . \tag{66}$$

The final solution to the problem may now be written down. It is

$$I_C^a(0) = -j4E_z^{inc}(0)\beta^{-1}D_C^{-1} \cos \beta d \sin \frac{\beta b}{2}\left[Z_{cC} \sin \beta s + jZ_s(1 - \cos \beta s)\right] , \tag{67}$$

$$I_C^a(s) = -j4E_z^{inc}(0)\beta^{-1}D_C^{-1} \cos \beta d \sin \frac{\beta b}{2}\left[Z_{cC} \sin \beta s + jZ_o(1 - \cos \beta s)\right] , \tag{68}$$

where

$$D_C = Z_{cC}(Z_o + Z_s) \cos \beta s + j\left(Z_{cC}^2 + Z_s Z_o\right) , \tag{69}$$

and

$$Z_{cC} = \frac{\zeta_o}{2\pi} \ln \left[\frac{b^2(4d^2 - b^2)}{4a^2 d^2}\right] .$$

The above formulas for $I_C^a(0)$ and $I_C^a(s)$ apply only to a two-wire transmission line oriented edgewise with respect to a perfectly conducting ground plane (see Figure 4).

Bounds on the Current in the Load Impedances of Exposed One- and Two-Conductor Transmission Lines Arranged Longitudinally on the Surface of a Rocket

The objective of this section is to justify in a heuristic manner the following inequalities:

$$I_A^a\begin{bmatrix}0\\s\end{bmatrix} \leq I^a\begin{bmatrix}0\\s\end{bmatrix}_1 \leq I_B^a\begin{bmatrix}0\\s\end{bmatrix}, \tag{70}$$

$$I_A^a\begin{bmatrix}0\\s\end{bmatrix} \leq I^a\begin{bmatrix}0\\s\end{bmatrix}_2 \leq I_C^a\begin{bmatrix}0\\s\end{bmatrix}, \tag{71}$$

where $I^a\begin{bmatrix}0\\s\end{bmatrix}_1$ and $I^a\begin{bmatrix}0\\s\end{bmatrix}_2$ apply to the circuits pictured in Figure 1. The subscripts indicate the number of wires involves. It is assumed that $\beta a < \beta b \ll 1$, $\beta d > \beta\frac{b}{2}$, and $\beta d \ll 1$. Moreover, a, b, s, and $E_z^{inc}(0)$ must have the same values throughout a given calculation whether either Equation (70) or (71) is in use.

The discussion in this section is limited to establishing Equation (70). Credence may be given Equation (71) by use of a parallel argument.

Low-silhouette telemetry antennas in the form of a shunt-driven inverted L[7,8] are frequently used on rockets having velocities under Mach 6. These antennas are sometimes only 1 inch in height and operate in the 225- to 260-MHz range. They are mounted on a large aluminum ground plane and matched to a 50-ohm line; i.e., for an SWR not exceeding 1:1.1 at the desired operating frequency. Following this, the antennas are placed in position for service on a rocket that may not exceed 12 inches in diameter. The remarkable thing is that the SWR on the feed line remains under 1:1.1 so that the power supplied by the generator is constant irrespective of the ground plane size and its contour. This situation is altered only when the dimensions of the ground plane become small in terms of the wavelength.

The single conductor with terminating impedances grounded to a perfectly conducting plane may be regarded as a low-silhouette antenna. Let the structure be driven by an impedanceless generator V^e in series with Z_o, as shown in Figure 5.

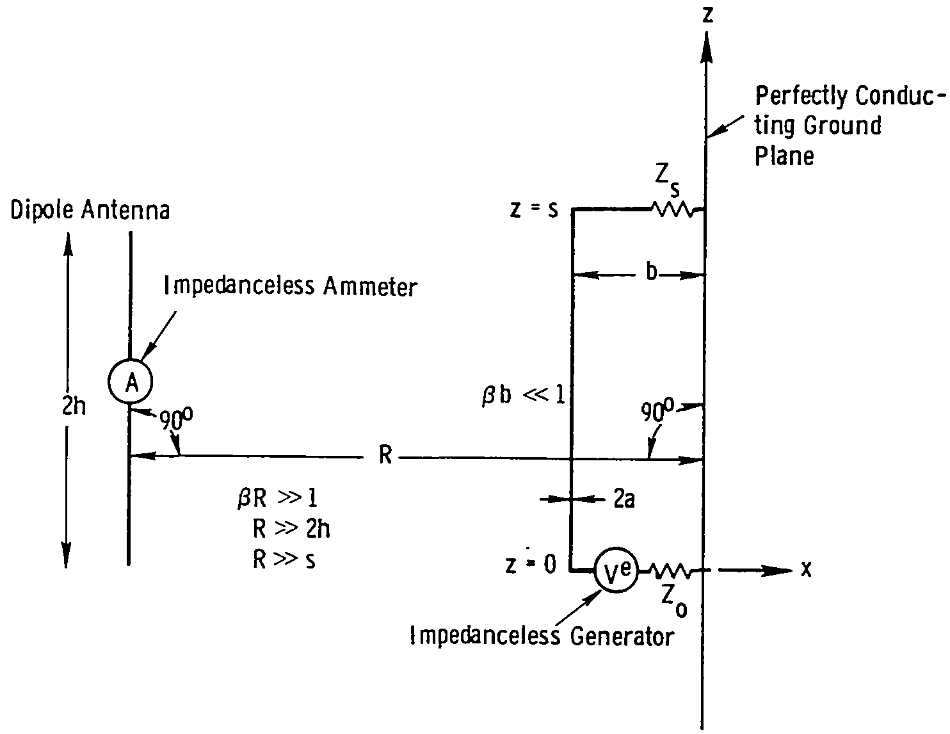

Figure 5. Single Conductor with Terminating Impedances Grounded to a Perfectly Conducting Plane as a Transmitting Antenna

In the far zone a dipole with an impedanceless ammeter in series with the conductor at its center is placed. Note the orientation of the dipole with respect to the transmitting antenna. With only the voltage, V^e, operating, a traveling wave exists on the ground plane as long as it is infinite, and radiation is confined to 2π steradians. Now, progressively reduce the size of the ground plane. Resonances occur, and a radial field (tangential to Z_o and Z_s) is developed. In addition, radiation takes place in 4π steradians. The ammeter reading is oscillatory because of resonances, but the peak readings become less and less because radiation increases on the back side of the ground plane, and the size of the reflector is being reduced. In other words, the field strength in the direction of the dipole is decreasing. (An antenna always becomes less directive as the reflector dimensions are made smaller.) It is to be remembered that constant power is being supplied by the generator during this process. The current $I^a(0)_1$ is always available. It is the reading of the ammeter in the dipole, for by the reciprocity theorem one may interchange ammeter and generator with no change in the ammeter reading. Note that when V^e is placed in the dipole, $E_z^{inc}(0)$ at the ground plane is fixed.

When the ammeter is in series with Z_o for fixed $E_z^{inc}(0)$, its reading will fluctuate as the ground plane size is reduced, but the maximum and minimum readings will be within the limits prescribed by Equation (70).

Conclusions

What the author set out to do is to set bounds on the load currents in the impedances Z_o and Z_s for the circuits shown in Figure 1. This is accomplished. In addition, accurate expressions are developed for these currents when the rocket is of infinite size. The problem is also solved for the case of the terminated two-wire transmission line when the rocket is completely decoupled from the line.

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