SC-R-713178

EXCITATION OF AN EXTERNAL, TERMINATED, LONGITUDINAL CONDUCTOR ON A ROCKET BY A TRANSVERSE ELECTROMAGNETIC FIELD

Ronold W. P. King
Gordon McKay Laboratory
Harvard University
Cambridge, Massachusetts 02138

Charles W. Harrison, Jr.
Sandia Laboratories
Albuquerque, New Mexico 87115

Date Completed: January 1971

ABSTRACT

A single conductor of arbitrary length is placed close to the surface of a conducting cylinder parallel to its axis. Impedances are connected between the ends of the cable and the cylinder. An incident monochromatic plane electromagnetic wave propagates in the longitudinal direction and is thus polarized in a transverse plane with respect to the axes of the external conductor and the cylinder. The component of the electric field tangent to the terminating impedances excites the circuit. The objective is to derive formulas for the currents in these impedances. The transverse dimensions of the conductor satisfy the inequalities: $a_1 \leq a_2 \leq \infty$, where a_1 is the radius of the external conductor, and a_2 is the radius of the cylinder. It is assumed that the distance d between adjacent points on the conductor and the rocket is small compared with the wavelength.

INTRODUCTION

Exposed unshielded conductors are used for interconnecting the electronic equipment in rockets. Such a conductor is usually arranged parallel to the axis of the rocket and sometimes recessed into its surface. It is not necessarily centered axially along the rocket; its length may range from a very small fraction to more than three-fourths of the length of the rocket. Its ends are connected directly to the cylinder through arbitrary impedances.

An incident electromagnetic wave is assumed to propagate axially toward the nose cone or toward the propulsion nozzles of the rocket. The field is thus cross-polarized with respect to the rocket and the external wire so that no axial currents are induced in either. However, if the radius of the rocket is large in terms of the wavelength of the incident field, significant transverse currents may be excited on its surface. The excitation of the circuit consisting of the conductor, the two terminating impedances and the rocket is due entirely to the component of the incident electric field that is directed parallel to the terminations. It follows that any extension of the rocket beyond the ends of the parallel conductor is immaterial. Accordingly, for the assumed polarization and direction of travel of the incident field, the currents in the terminating impedances are independent of the length of the rocket.

THEORETICAL CONSIDERATIONS

Fig. 1 shows a conductor of radius a_1 adjacent to a section of a cylinder of radius a_2. The terminating impedances Z_0 and Z_s are at $z = 0$ and $z = s$, respectively. Thus, the circuit to be analyzed consists of a closely-spaced two-conductor transmission line with unequal conductors excited by an electric field directed parallel to the terminations. In practice $a_1 << a_2$, but for completeness the general case with $a_1 \leq a_2 \leq \infty$, $a_1 << \lambda$, is investigated including the special cases $a_1 = a_2$ and $a_2 \to \infty$.

The voltages induced in the electrically short lengths d at z = 0 and z = s that contain the terminating impedances Z_0 and Z_s are

$$V_0^e = E^{inc} d$$
$$V_s^e = E^{inc} d e^{-jk_0 s} \quad (1)$$

where $E^{inc} = E^{inc}_{max} \cos \Psi$ is the component of the incident field traveling from Z_0 toward Z_s that is parallel to the terminations,

$$d = b - a_1 - a_2 << \lambda , \quad (2)$$

and k_0 is the free-space wave number. The angle Ψ is between the plane containing the axes of the conductor and the rocket and the direction of the incident electric vector $\vec{E}^{inc}_{max}$.

The currents along the transmission line generated respectively by V_0^e and V_s^e are [1]

$$I_z'(w) = V_0^e D^{-1} (Z_s \sinh \gamma w + Z_c \cosh \gamma w) \quad (3)$$

and

$$I_w''(z) = V_s^e D^{-1} (Z_0 \sinh \gamma z + Z_c \cosh \gamma z) \quad (4)$$

where

$$D = (Z_c^2 + Z_0 Z_s) \sinh \gamma s + Z_c (Z_0 + Z_s) \cosh \gamma s , \quad (5)$$

$\gamma = \alpha + j\beta$ is the propagation constant defined in the Appendix, and $w = s - z$. Note that $I_z'(w)$ is the current in the positive z direction referred to $w = 0$ or $z = s$; $I''_w(z)$ is the current in the positive w or negative z direction referred to $z = 0$. The characteristic impedance of the line is

$$Z_c = \sqrt{\frac{z^i + j\omega \ell^e}{g + j\omega c}} \tag{6a}$$

where z^i, ℓ^e, g and c are defined in the Appendix. When $z^i \doteq 0$ and $g \doteq 0$,

$$Z_c = \frac{\zeta_0}{2\pi} [\cosh^{-1}(b_1/2a_1) + \cosh^{-1}(b_2/2a_2)] \tag{6b}$$

where ζ_0 is the characteristic impedance of free space; $b_1/2$ and $b_2/2$ are the distances from the centers of the conductor 1 and the rocket 2 to the neutral plane of zero potential between them as shown in Fig. 1. These distances are given by

$$b_1/2 = \frac{b^2 + a_1^2 - a_2^2}{2b} \tag{7}$$

$$b_2/2 = \frac{b^2 + a_2^2 - a_1^2}{2b} \tag{8}$$

so that the distance between the axis of the conductor and the axis of the rocket is $b = b_1/2 + b_2/2$.

The current in impedance Z_0 is $I(0)$, that in Z_s is $I(s)$. They are given by:

$$I(0) = I'_z(s) - I''_w(0)$$

$$= E^{inc}\, d\, D^{-1} [Z_s \sinh \gamma s + Z_c \cosh \gamma s - Z_c e^{-jk_0 s}] \tag{9}$$

and

$$I(s) = I'_z(0) - I''_w(s)$$

$$= - E^{inc} e^{-jk_0 s} d D^{-1} [Z_0 \sinh \gamma s + Z_c \cosh \gamma s - Z_c e^{jk_0 s}] \quad (10)$$

In (9) and (10), Z_c is given by (6a).

If the two-conductor line is dissipationless, $\alpha = 0$, $\gamma = j\beta = jk_0$, and Z_c is given by (6b). For this case,

$$I(0) = jE^{inc} d D^{-1} (Z_s + Z_c) \sin k_0 s \quad (11)$$

$$I(s) = -jE^{inc} e^{-jk_0 s} d D^{-1} (Z_0 - Z_c) \sin k_0 s \quad (12)$$

where now

$$D = j(Z_c^2 + Z_0 Z_s) \sin k_0 s + Z_c(Z_0 + Z_s) \cos k_0 s \quad (13)$$

Note that if $\alpha = 0$, both $I(0)$ and $I(s)$ vanish when $k_0 s = n\pi$, $n = 1, 2, 3, \ldots$.

If $\gamma = \alpha + j\beta \doteq \alpha + jk_0$ with $\alpha s << 1$, i.e., the missile and conductor have only small losses, it follows that

$$\sinh \gamma s \doteq \alpha s \cos k_0 s + j \sin k_0 s$$
$$\cosh \gamma s \doteq \cos k_0 s + j\alpha s \sin k_0 s \quad (14)$$

When (14) is substituted into (9), (10) and (5), the results are:

$$I(0) = E^{inc} d D^{-1} [j(Z_s + Z_c + \alpha s Z_c) \sin k_0 s + Z_s \alpha s \cos k_0 s] \quad (15)$$

$$I(s) = -E^{inc} e^{-jk_0 s} d\, D^{-1} [j(Z_0 - Z_c + \alpha s Z_c) \sin k_0 s + Z_0 \alpha s \cos k_0 s] \qquad (16)$$

with

$$D = j[Z_c^2 + Z_0 Z_s + \alpha s Z_c (Z_0 + Z_s) \sin k_0 s + [Z_c(Z_0 + Z_s) + \alpha s (Z_c^2 + Z_0 Z_s)] \cos k_0 s \qquad (17)$$

In (15)-(17), Z_c is well approximated by (6b).

Clearly the general solutions for the currents I(0) and I(s) when the rocket and the parallel conductor have only small losses are given by (15)-(17) with (2) and (6b), provided $k_0 a_1 < k_0 b_1/2 < 1$ so that transmission-line theory applies. No restriction need be imposed on $k_0 a_2$ and $k_0 b_2/2$ since with $d << \lambda$ and as a result of the proximity effect the currents on the rocket 2 are localized near conductor 1 regardless of the size of the rocket.

SPECIAL CASES

Attention is now directed toward determining Z_{cA} and d_A when $a_1 = a_2$ (defined as case A with identifying subscript A) and Z_{cB} and d_B when $a_2 \to \infty$, $b_2 \to \infty$ (defined as case B with subscript B). Note that when $a_2 \to \infty$ and $b_2 \to \infty$, the surface of the rocket itself becomes the neutral plane of zero potential.

Case A:

When $a_1 = a_2$, $b_1/2 = b_2/2$ and (6b) yields

$$Z_{cA} = \frac{\zeta_0}{\pi} \cosh^{-1}(b_1/2a_1) \qquad (18)$$

It follows from (2) that

$$d_A = b - 2a_1 = b_1 - 2a_1 \qquad (19)$$

Case B:

The limit when $a_2 \to \infty$ and $b_2 \to \infty$ must be examined carefully. The substitution of $b = b_1/2 + b_2/2$ into (7) yields:

$$\frac{b_1^2}{2} + \frac{b_1 b_2}{2} = \left(\frac{b_1}{2} + \frac{b_2}{2}\right)^2 + a_1^2 - a_2^2 \tag{20}$$

from which it follows that

$$a_2 = \sqrt{\frac{b_2^2}{4} + a_1^2 - \frac{b_1^2}{4}} \tag{21}$$

Hence,

$$\frac{a_2}{b_2} = \sqrt{\frac{1}{4} + \frac{1}{b_2^2}\left(a_1 - \frac{b_1}{4}\right)} \tag{22}$$

For a_1 and b_1 fixed,

$$\lim_{b_2 \to \infty} \left(\frac{a_2}{b_2}\right) = \frac{1}{2} \tag{23}$$

or

$$\lim_{b_2 \to \infty} \left(\frac{b_2}{2a_2}\right) = 1 \tag{24}$$

When this limit is substituted in (6b),

$$Z_{cB} = \frac{\zeta_0}{2\pi} \cosh^{-1}(b_1/2a_1) \tag{25}$$

so that $Z_{cA} = 2Z_{cB}$.

From (21),

$$a_2 = \frac{b_2}{2}\sqrt{1 + \frac{4}{b_2^2}\left(a_1 - \frac{b_1}{4}\right)} = \frac{b_2}{2}\,[1 + O(1/b_2)^2] \qquad (26)$$

for $b_2 \to \infty$. Then

$$a_2 - \frac{b_2}{2} \to 0 \qquad (27)$$

and, from (2),

$$d_B = b - a_1 - a_2 = \frac{b_1}{2} + \frac{b_2}{2} - a_1 - a_2 \to \frac{b_1}{2} - a_1 \qquad (28)$$

When $a_1 = a_2$, Z_{cA} and d_A are used in (15) and (16); when $a_2 \to \infty$, use is made of Z_{cB} and d_B.

CONCLUSIONS

A general theory has been formulated for determining the r.f. pickup of a terminated external conductor placed axially parallel to the skin of a rocket when an incident electromagnetic field is polarized in the transverse plane.

APPENDIX

When losses are taken into account in the rocket and the parallel conductor, the propagation constant γ is given by

$$\gamma = \sqrt{zy} = \sqrt{(z^i + j\omega\ell^e)(g + j\omega c)} \tag{29}$$

where ω is the radian frequency $2\pi f$. Let

$$H = \cosh^{-1}(b_1/2a_1) + \cosh^{-1}(b_2/2a_2) \tag{30}$$

Then [2],

$$\ell^e = \frac{\mu}{2\pi} H \quad ; \quad c = 2\pi\varepsilon H^{-1} \quad ; \quad g = 2\pi\sigma_d H^{-1} \tag{31}$$

In (31), ordinarily $\mu = \mu_0 = 4\pi \times 10^{-7}$ H/m, ε is the absolute dielectric constant and σ_d is the conductivity of the dielectric surrounding the transmission line. When this is air, $\varepsilon = \varepsilon_0 = 8.85 \times 10^{-12}$ farad/m and $\sigma_d = 0$.

Accurate formulas for z^i are not available and are difficult to derive since the proximity effect is involved. An approximate formula for $z^i = z_1^i + z_2^i$ is [3]

$$z_1^i = \frac{(1 + j)}{2\pi a_1} \sqrt{\frac{\mu_1\omega}{2\sigma_1[1 - (2a_1/b_1)^2]}} \tag{32}$$

$$z_2^i = \frac{(1 + j)}{2\pi a_2} \sqrt{\frac{\mu_2\omega}{2\sigma_2[1 - (2a_2/b_2)^2]}} \tag{33}$$

In these expressions σ_1 and σ_2 are the conductivities of the parallel conductor and the rocket, respectively. The corresponding permeabilities are μ_1 and μ_2. In practice $a_2 >> a_1$ so that $|z_1^i| >> |z_2^i|$. Thus, z_2^i can be neglected as a first approximation.

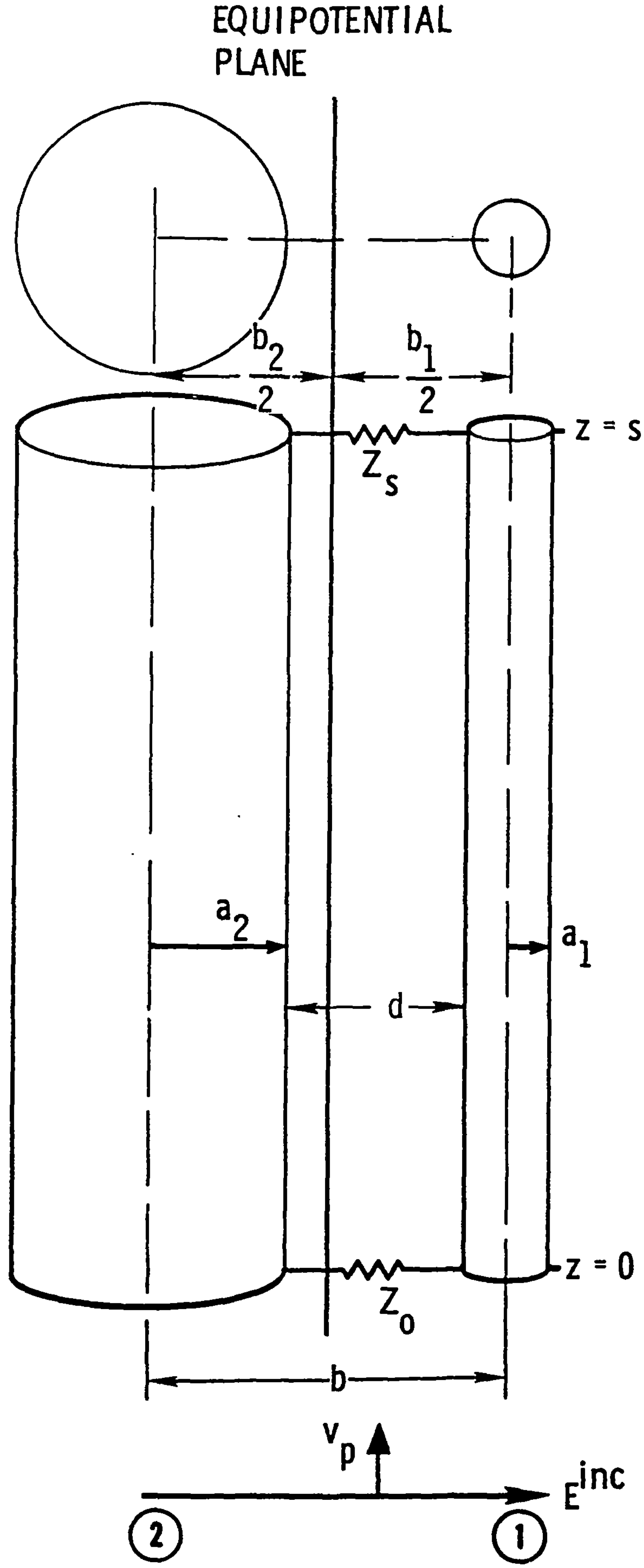

Fig. 1. Two conductor transmission line. Note that the impedances Z_0 and Z_s may be physically located just inside the cylinder of radius a_2 without altering the problem.

REFERENCES

[1] R. W. P. King, *Transmission-Line Theory*, McGraw-Hill Book Co., Inc., p. 83, 1955.

[2] *Ibid*., p. 28.

[3] *Ibid*., p. 30, where references are given to more accurate formulas.