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Design of Shielded Cables Using Saturable Ferromagnetic Materials

DAVID E. MEREWETHER, MEMBER, IEEE

Abstract—Previously, a numerical solution was evolved that may be used to analyze the effect of material saturation on the low frequency shielding characteristics of a thin ferromagnetic cable shield. Numerical solutions are difficult to employ in design problems and so a simplified theory has also been developed. This supplemental theory, described here, yields simple equations useful in the design of ferromagnetic cable shields to provide protection against some anticipated intense low frequency environment.

Introduction

DESIGN PROBLEMS are simplified if it is possible to relate, even approximately, the desired characteristics to the independent variables. The simplified theory to be described here will yield simple equations useful in the design of ferromagnetic cable shields; additionally, it will provide a better understanding of the physical processes involved in the ferromagnetic shielding problem. However, this theory will not supplant the numerical solution [1] previously described; only the numerical solution can predict the shielding provided by a partially saturated ferromagnetic sheath.

Formulation

The coaxial cable model to be considered (Fig. 1) is the same model previously analyzed. In addition to the assumption that the impressed current is a causal function of time ($i_T(t) = 0$, if $t \leq 0$), it will also be assumed initially that during the time interval $[0,t]$ the impressed current has only one polarity.

An idealized magnetization curve (Fig. 2) is assumed, following Agarwal [2], leading to a simple description of the flux distribution within the magnetic material. At time $t = 0$, the impressed current is zero; the magnetic flux density is zero throughout the material. As the impressed current increases, a wave of flux enters the sheath from the outside surface. The point $p(t)$ (Fig. 3) is the depth that the flux wave has penetrated into the material at time t. The magnetic flux density is B_S at any point closer to the outside surface than p, ($a_1 > r > a_1 - p$), and is zero for all points closer to the inside surface than p, ($a_1 - p > r > a_2$).

Manuscript received November 26, 1969; revised April 27, 1970. This work was supported by the U.S. Air Forces Weapons Laboratory and conducted under the auspices of the U.S. Atomic Energy Commission.

The author is with Sandia Laboratories, Albuquerque, N. Mex. 87115.

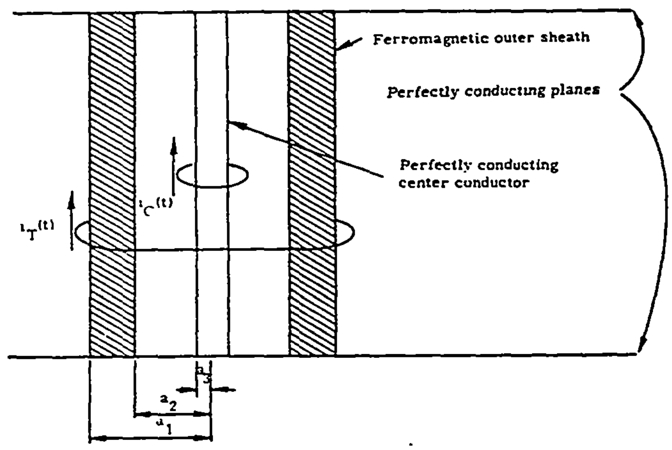

Fig. 1. Ferromagnetic shield model.

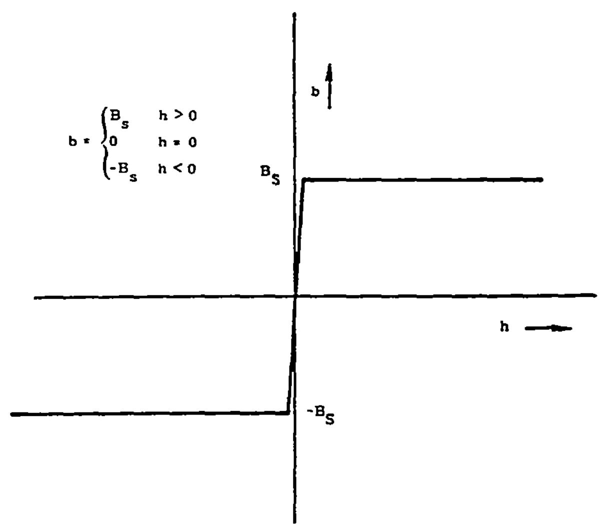

Fig. 2. Idealized magnetization characteristics.

Analysis

Previously [1], it was shown that the shielding problem reduces to finding the field distribution within the material subject to the boundary conditions

$$i_T(t) = 2\pi a_1 h_\phi(t,a_1) \tag{1}$$

and

$$i_C(t) = 2\pi a_2 h_\phi(t,a_2) \tag{2}$$

$$e_z(t,a_2) = \frac{\mu_0}{2\pi} \frac{\partial i_C(t)}{\partial t} \ln \frac{a_2}{a_3}. \tag{3}$$

The simple description of the flux distribution within the material allows a simple determination of the remaining field components.

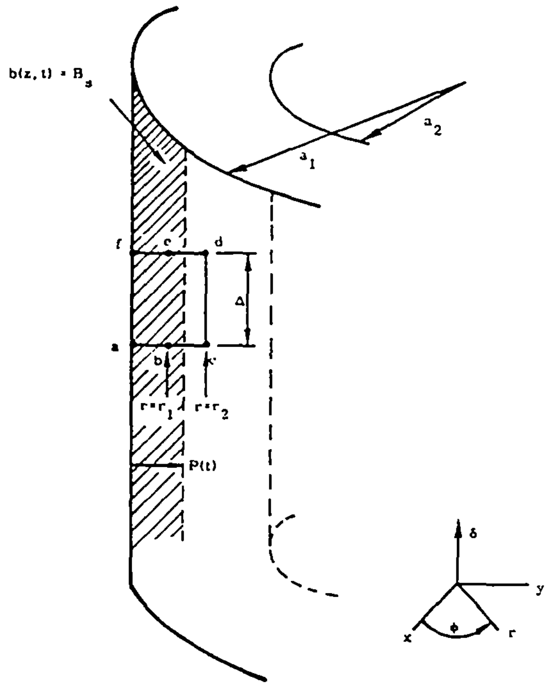

Fig. 3. Flux penetration.

The analysis begins with Faraday's law

$$\oint_L e \cdot dL = -\frac{\partial}{\partial t} \int_S b \cdot dS. \quad (4)$$

The path L circumscribes the surface S in the normal manner. Due to the symmetry of the problem, the electric field e has only a z component, while the magnetic field h and the magnetic flux density b have only a ϕ component. Integrating around the path $abefa$ (Fig. 3) yields

$$[e_z(r_1) - e_z(a_1)]\,\Delta = 0 \quad (5)$$

since $b_\phi(r,t) = B_S$ throughout this region. Therefore, the electric field is independent of position in the region $a_1 > r > (a_1 - p)$. Integrating around the path $acdfa$ (Fig. 3) yields

$$[e_z(r_2) - e_z(a_1)]\,\Delta = -B_S \frac{\partial p}{\partial t} \cdot \Delta. \quad (6)$$

Since the flux wave has not yet reached r_2, $e_z(r_2) = 0$, and accordingly,

$$\begin{aligned} e_z(r) &= B_S \frac{\partial p}{\partial t}, \quad (a_1 - p) \le r \le a_1 \\ &= 0, \quad (a_1 - p) > r > a_2. \end{aligned} \quad (7)$$

The magnetic field in the material is obtained using Ampere's law,

$$\nabla \times h = \sigma e. \quad (8)$$

Here the displacement current has been neglected. For the model considered, the curl equation reduces to

$$\frac{1}{r}\left[\frac{\partial (r h_\phi)}{\partial r}\right] = \sigma e_z \quad (9)$$

substituting (7) into (9) and integrating, yields

$$r h_\phi = \frac{r^2 \sigma B_S}{2} \frac{\partial p}{\partial t} + K_1, \quad (a_1 - p) < r < a_1. \quad (10)$$

The constant of integration K_1 can be evaluated using the boundary condition at the outside surface of the sheath (1), yielding

$$h_\phi = \frac{1}{r}\left[\frac{i_T(t)}{2\pi} - \frac{(a_1^2 - r^2)\sigma B_S}{2} \frac{\partial p}{\partial t}\right], \quad (a_1 - p) < r < a_1. \quad (11)$$

The depth that the flux wave has penetrated p can be determined by noting that at $r = a_1 - p$ the magnetic field is zero

$$0 = \frac{i_T(t)}{2\pi} - (2a_1 p - p^2) \frac{\sigma B_S}{2} \frac{\partial p}{\partial t}. \quad (12)$$

Integration with respect to t yields

$$a_1 p^2 - \frac{p^3}{3} = \frac{1}{\pi \sigma B_S} \int_0^t i_T(t)\, dt, \quad p < (a_1 - a_2). \quad (13)$$

Since $i_T(t)$ is the known driving current, (13) may be used to determine p. From p, the entire field distribution is determined using (7), (11), and (12).

Partly Saturated Ferromagnetic Shield

An example can illustrate the field distribution predicted by the theory developed so far.

Consider the application of a half sine pulse of current to the annealed-steel coaxial cable previously studied [1, Table 1]. The impressed current pulse is defined by

$$\begin{aligned} i_T(t) &= A \sin 2\pi f_0 t, \quad 0 < t < 1/2f_0 \\ i_T(t) &= 0, \quad \text{otherwise.} \end{aligned} \quad (14)$$

For the amplitude and frequency chosen ($A = 10$ amperes and $f_0 = 1$ kHz), the sheath does not saturate all the way through at any time, and so the equations developed so far are adequate to describe the electromagnetic field within the material during the pulse. The conductivity σ and saturation flux density B_S were taken to be 10^7 mho/m and 1.53 Wb/m^2, and the electric and magnetic fields at three points within the material were predicted (Fig. 4). For comparison, results were also obtained using the finite difference solution previously described. During the pulse, the field components near the outside surface are well represented by the simplified theory. Within the material the approximation is not as good, since, in the diminished magnetic field, the validity of the assumed magnetic characteristics of the material is reduced. At the end of the pulse, the finite difference solution predicts a reversal in the electric field polarity as the magnetic field intensity is reduced. The simplified theory predicts that the magnetic field everywhere within the saturated region simultaneously jumps from B to zero. From (4) the resulting electric field would be impulsive in nature and directly proportional to

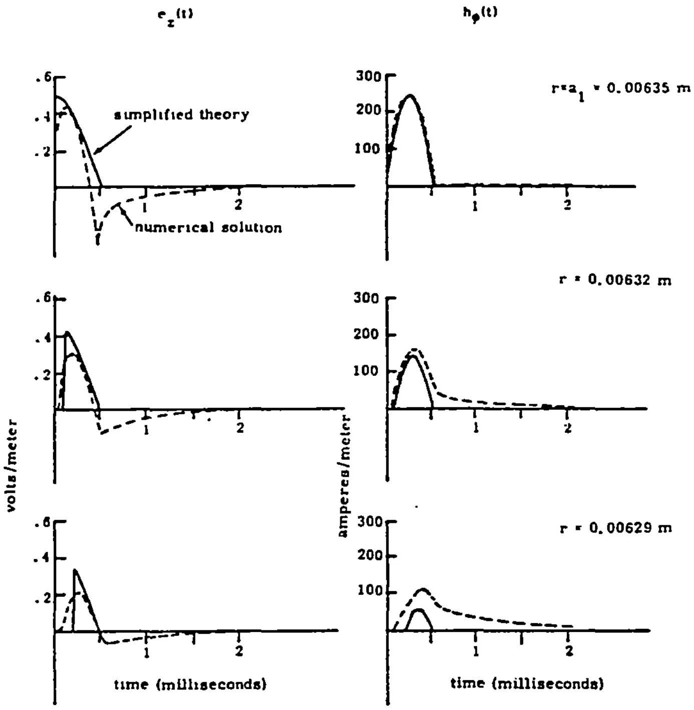

Fig. 4. Predicted fields within ferromagnetic sheath.

the distance from the point of observation to the edge of the saturated region.

$$e_z = -B_S[r - (a_1 - p)]\delta(t - 1/2f_0). \quad (15)$$

The electric field given by (15) is only a first-order iteration of equations (4) and (9); however, there is conceptual agreement between this result and the results predicted by the numerical solution. No attempt will be made to incorporate this correction into the simplified theory.

In summary, if the field level is small enough that the shield is not saturated all the way through, the simplified theory yields a reasonable approximation of the fields in the material during the pulse. It should be noted that since the field predicted by the simplified theory is reduced to zero at the end of the pulse, the final state is the same as the initial state. Accordingly, the predicted response to an oscillatory waveform is just the response to each half-cycle in turn. So far we have assumed that the sheath never saturates all the way through, and, consequently, this theory predicts that no current ever flows on the inside conductor. As mentioned in the Introduction, this erroneous prediction of perfect shielding is the result of the simplified magnetic parameters used; the numerical solution must be employed to find the shielding of a partially saturated sheath.

At higher levels of excitation, the sheath will saturate throughout the material, and even the simplified theory can be used to predict the current flowing on the center conductor of the cable. While this is not a very important consideration for shield design, this case is included in the next section for completeness.

Fully Saturated Ferromagnetic Shield

The theory developed yields a depth of wave penetration proportional to the integral of the impressed current (13). In the previous section, attention was limited to the case where the maximum penetration depth was less than the thickness of the sheath. In this section we consider the other condition, when, during some time interval, the value of p determined by (13) exceeds the thickness of the sheath.

From (4) it is apparent that during this time interval, the electric field is independent of position within the material since the flux density within the material is constant. The value of the electric field with the material is equal to the electric field at a_2 as given by (3)

$$e_z(r) = \frac{\mu_0}{2\pi}\frac{\partial i_C(t)}{\partial t}\ln\left(\frac{a_2}{a_3}\right), \quad a_1 \geq r \geq a_2. \quad (16)$$

The magnetic field within the material is obtained from (9):

$$\frac{1}{r}\frac{\partial(rh_\phi)}{\partial r} = \sigma e_z \quad (17)$$

so that

$$rh_\phi = \frac{\sigma e_z r^2}{2} + K_2. \quad (18)$$

The value of the constant K_2 may be determined from the other boundary conditions, (1) and (2):

$$\frac{i_T(t)}{2\pi} = a_1 h_\phi(a_1) = \frac{\sigma a_1^2}{2}\frac{\mu_0}{2\pi}\ln\left(\frac{a_2}{a_3}\right)\frac{\partial i_C}{\partial t} + K_2 \quad (19)$$

$$\frac{i_C(t)}{2\pi} = a_2 h_\phi(a_2) = \frac{\sigma a_2^2}{2}\frac{\mu_0}{2\pi}\ln\left(\frac{a_2}{a_3}\right)\frac{\partial i_C}{\partial t} + K_2 \quad (20)$$

so that

$$i_T(t) = i_C(t) + [\sigma(\pi a_1^2 - \pi a_2^2)]\left[\frac{\mu_0}{2\pi}\ln\frac{a_2}{a_3}\right]\frac{\partial i_C}{\partial t}. \quad (21)$$

The form of this final equation relating the impressed current to the current on the center conductor is interesting, for the two factors multiplying the time derivative are the conductance of the outer sheath and the inductance of the inner transmission line, so that

$$i_T(t) = i_C(t) + GL_c\frac{\partial i_C}{\partial t}. \quad (22)$$

An equivalent lumped element circuit that corresponds to this equation is a parallel RL circuit (Fig. 5).

To illustrate the accuracy of this approximation at low frequencies, the response of the annealed-steel coaxial cable [1, table I] was considered. A damped sine-wave impressed current was assumed ($i_T(t) = 1.2678Ae^{-f_0 t}\sin 2\pi f_0 t$), and for each of the three values of f_0 considered, the peak current was taken large enough to assure that the shield saturated all the way through (Fig. 6). For comparison, results were also obtained using the numerical solution described in [1]. The comparison reveals that the simplified theory will yield a reasonable estimate of the shielding provided by a fully saturated sheath at low frequencies.

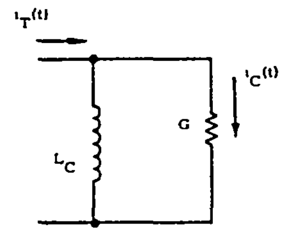

Fig. 5. Lumped equivalent circuit of a fully saturated cable shield.

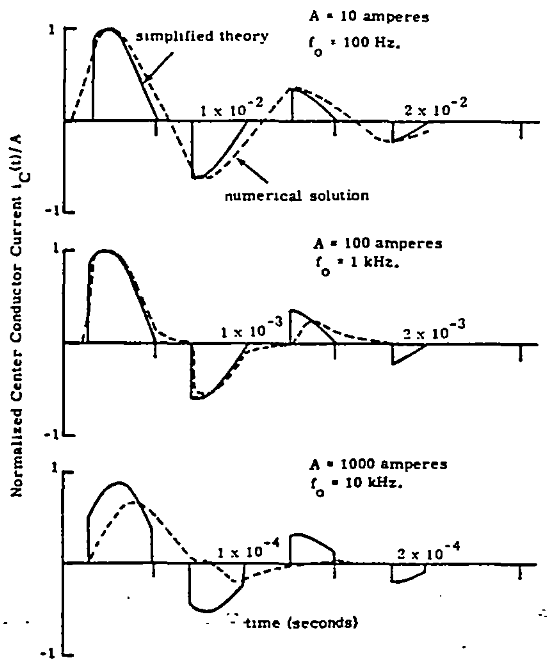

Fig. 6. Shielding characteristics of fully saturated sheath. Solid lines are results of simplified theory. Dashed lines are results of numerical solution.

Ferromagnetic Shield Design

Ferromagnetic materials make excellent cable shields; however, if intense excitation is expected, care must be taken to assure that the saturation properties of the material are accounted for in the selection of a thickness for the sheath. A simple way to do this is to use (13) to determine the maximum depth of penetration for each monopolarity portion of the expected impressed current. Add to the largest of these thicknesses the thickness of a sheath (determined by linear analysis) that will sufficiently attenuate a current of the waveshape of the expected interference and of amplitude. $2\pi a_1 h_c = I_{max}$ (h_c is a material constant; see [1, fig. 2]). Adding these two thicknesses together will yield a slight overdesign. Once the shield dimensions have been selected, a more accurate estimate of the expected attenuation can be obtained using the numerical solution.

If the magnetic characteristics exhibit a high relative μ for low levels of magnetic field, only the depth of penetration of the saturation flux will be significant, so it may be worthwhile to devote a little more space to this subject. The value of $p(t)$ can be obtained from (13) exactly or by iteration; however, it may also be noted that $a_1 \gg p$ normally, and hence,

$$a_1 p^2 \doteq \frac{1}{\pi \sigma B_S} \int_0^t i_T(t)\, dt. \tag{23}$$

For an impressed current that is sinusoidal, the maximum penetration occurs at the end of each half-cycle

$$a_1 p_{\max}{}^2 = \frac{1}{\pi \sigma B_S} \int_0^{1/2f_0} A \sin 2\pi f_0 t\, dt \tag{24}$$

or

$$a_1 p_{\max}{}^2 = \frac{A}{\pi \sigma B_S \pi f_0}. \tag{25}$$

Hence

$$p_{\max} = \frac{1}{\pi} \sqrt{\frac{A}{\sigma B_S f_0 a_1}}. \tag{26}$$

This equation exhibits the basic dependence of the maximum depth of penetration for both amplitude and frequency of the applied excitation. For a given thickness of sheath, if the frequency of the applied excitation is doubled, then the amplitude needed to saturate that sheath will also be doubled. This type of response is also illustrated in Fig. 6.

Conclusion

The simplified theory was developed to aid in the design of ferromagnetic cable shields. Equations (13) and (26) relate the shield properties of the sheath to material parameters and to the characteristics of the anticipated impressed excitation. The form of these equations is simple enough to make them useful in the design of ferromagnetic cable shields for applications where intense low frequency environments could cause sheath saturation.

References

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Vol. EMC-12, No. 3, August 1970
pp. 138-141