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RECEIVING PROPERTIES OF BALANCED FOUR-WIRE TRANSMISSION LINES EXCITED BY PLANE-WAVE ELECTRIC FIELDS

by

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SUMMARY

The response of a balanced four-wire transmission line, terminated at the ends in arbitrary values of impedance, to an unwanted signal polarized parallel to the conductors, is discussed. The wires are equally spaced on the periphery of a circle (so that the cross section is a square), and the diagonal conductors are electrically paralleled at the ends of the line. Sufficient data are presented to permit two and four conductor transmission lines to be compared on the basis of electrical noise pickup.

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Introduction

Balanced four-wire transmission lines are frequently used at communication receiving stations to connect balanced receiving antennas to the balanced inputs of radio receivers. From experience it is known that four-wire lines, consisting of four conductors fixed at the corners of a square, with diagonal conductors electrically paralleled at the ends of the line, discriminate more against unwanted signal pick-up than comparable two-wire lines. The purpose of the present investigation is to obtain formulas for the currents in the four-wire line terminating impedances when the incident plane-wave electric field is directed parallel to the axes of the line conductors. Using the results reported in an earlier paper[1] for two-conductor transmission lines, it then becomes feasible to compare the performances of two- and four-wire lines on a sound analytical basis.

The Short-Circuit Current in a Four-Wire Transmission-Line Excited by a Plane-Wave Electric Field Directed Parallel to the Conductors

Figure 1 illustrates the four-wire transmission line to be analyzed. Conductors (1) and (4) have their centers at $y = \pm c/2$, where the upper sign applies to wire No. 1. Conductors (2) and (3) are centered at $x = \mp c/2$. All wires are parallel to the z-axis. The spacing between adjacent wires, measured center-to-center is b. Evidently, $b = c/\sqrt{2}$. All conductors are of the same radius a. For clarity, i.e., to exhibit the short-circuit current $I_{sc}(h) = I_{sc}(-h)$, one transmission-line loop (wires 2 and 3) is shown displaced from the other loop (wires 1 and 4) in the drawing. The length of the line is 2h. The inequalities $h \gg a$, $\beta a \ll 1$, are assumed to hold.

The interfering signal $\hat{z}E_z^{inc}$ arrives at the aximuth angle Φ. Φ is measured from the positive x-axis.

[1]C. W. Harrison, Jr., "Receiving Characteristics of Two-Wire Lines Excited by Uniform and Non-Uniform Electric Fields," SC-R-64-164 of May 6, 1964. A copy of this report may be obtained from the Technical Information Department of the Sandia Corporation

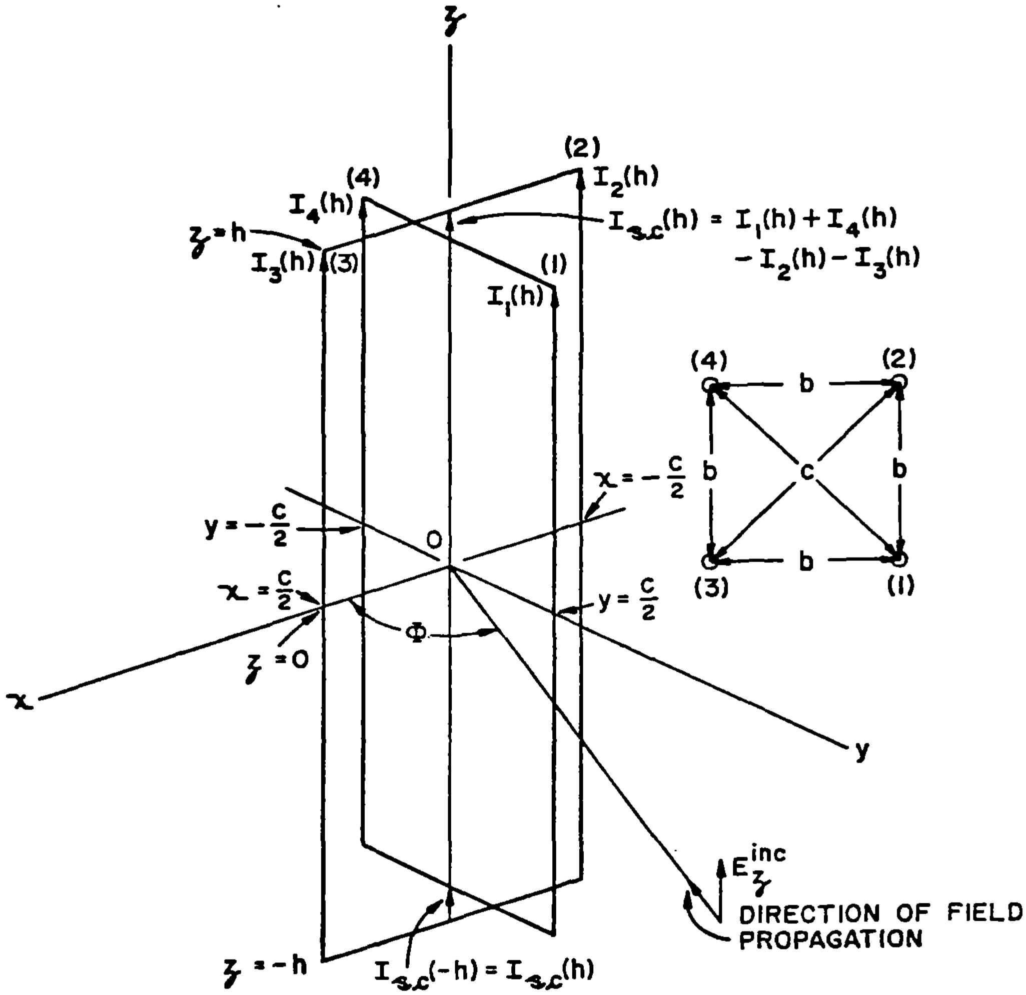

Figure 1. Coordinate System of Four-Wire Transmission-Line with Conductors Fixed at the Corners of a Square, and Diagonal Wires Paralleled at the Ends of the Line

The simultaneous integral equations applying to conductors 1-4, in sequence, are[2,3,4,5]

$$J_d(z) + I_1(z)\lambda_a + I_2(z)\lambda_b + I_3(z)\lambda_b + I_4(z)\lambda_c = F_1(z) \tag{1}$$

$$J_d(z) + I_1(z)\lambda_b + I_2(z)\lambda_a + I_3(z)\lambda_c + I_4(z)\lambda_b = F_2(z) \tag{2}$$

$$J_d(z) + I_1(z)\lambda_b + I_2(z)\lambda_c + I_3(z)\lambda_a + I_4(z)\lambda_b = F_3(z) \tag{3}$$

$$J_d(z) + I_1(z)\lambda_c + I_2(z)\lambda_b + I_3(z)\lambda_b + I_4(z)\lambda_a = F_4(z) \tag{4}$$

where

$$J_d(z) = \int_{-h}^{h} \sum_{n=1}^{4} I_n(z')K_d(z, z')dz' \tag{5}$$

$$\left.\begin{aligned} \lambda_a &= 2\,\ell n\left(\frac{d}{a}\right) \\ \lambda_b &= 2\,\ell n\left(\frac{d}{b}\right) \\ \lambda_c &= 2\,\ell n\left(\frac{d}{c}\right) \end{aligned}\right\} \tag{6}$$

$$F_1(z) = -j\frac{4\pi}{\zeta_o}\left(C_1 \cos\beta z + U\,e^{j\frac{\beta c}{2}\sin\Phi}\right) \tag{7}$$

$$F_2(z) = -j\frac{4\pi}{\zeta_o}\left(C_2 \cos\beta z + U\,e^{-j\frac{\beta c}{2}\cos\Phi}\right) \tag{8}$$

$$F_3(z) = -j\frac{4\pi}{\zeta_o}\left(C_3 \cos\beta z + U\,e^{j\frac{\beta c}{2}\cos\Phi}\right) \tag{9}$$

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[2] C. W. Harrison, Jr., and R. W. P. King, "Folded Dipoles and Loops," IRE Transactions on Antennas and Propagation, Vol. AP-9 No. 2, pp 171-187, March 1961.

[3] C. W. Harrison, Jr., and R. W. P. King, "Theory of Coupled Folded Antennas," IRE Transactions on Antennas and Propagation, Vol. AP-8 No. 2, pp 131-135, March 1960.

[4] C. W. Harrison, Jr., "Antenna Coupling Error in Direction Finders," Journal of Research, National Bureau of Standards-D. Radio Propagation Vol. 65D, No. 4, pp 363-369, July-August 1961.

[5] C. W. Harrison, Jr., "Missile with Attached Umbilical Cable as a Receiving Antenna," IEEE Transactions on Antennas and Propagation, Vol. AP-11, No. 5, pp 587-588, September 1963.

$$F_4(z) = -j\frac{4\pi}{\zeta_o}\left(C_4 \cos \beta z + U e^{-j\frac{\beta c}{2}\sin\Phi}\right) \tag{10}$$

$I_n(z)$ are the currents in conductors 1-4 inclusive. The currents are assumed to flow in the positive z direction. C_1, C_2, C_3, and C_4 are constants of integration. $\beta = 2\pi/\lambda$ is the wave number. $U = -E_z^{inc}/\beta$. $\zeta_o = 120\pi$ ohms is the resistance of space. d is the equivalent radius of the four-wire line. Although not needed in the present analysis, $d = \sqrt[4]{ab^3\sqrt{2}}$. $K_d(z, z') = \exp(-j\beta R)/R$; $R = \sqrt{(z - z')^2 + d^2}$.

The following transmission-line equations are easily derived from (1)-(4):

$$I_1(z) - I_4(z) = F_a(z) = \frac{F_1(z) - F_4(z)}{\lambda_{ac}} \tag{11}$$

$$I_2(z) - I_3(z) = F_b(z) = \frac{F_2(z) - F_3(z)}{\lambda_{ac}} \tag{12}$$

$$I_1(z) - I_2(z) = F_c(z) = \frac{\left[F_1(z) - F_2(z)\right]\lambda_{ab} - \left[F_3(z) - F_4(z)\right]\lambda_{bc}}{\lambda_{ab}^2 - \lambda_{bc}^2} \tag{13}$$

$$I_3(z) - I_4(z) = F_d(z) = \frac{\left[F_3(z) - F_4(z)\right]\lambda_{ab} - \left[F_1(z) - F_2(z)\right]\lambda_{bc}}{\lambda_{ab}^2 - \lambda_{bc}^2} \tag{14}$$

$$I_1(z) - I_3(z) = F_e(z) = \frac{\left[F_1(z) - F_3(z)\right]\lambda_{ab} - \left[F_2(z) - F_4(z)\right]\lambda_{bc}}{\lambda_{ab}^2 - \lambda_{bc}^2} \tag{15}$$

$$I_2(z) - I_4(z) = F_f(z) = \frac{\left[F_2(z) - F_4(z)\right]\lambda_{ab} - \left[F_1(z) - F_3(z)\right]\lambda_{bc}}{\lambda_{ab}^2 - \lambda_{bc}^2} \tag{16}$$

Here

$$\left.\begin{aligned}
\lambda_{ab} &= \lambda_a - \lambda_b = 2\,\ell n\left(\frac{b}{a}\right) \\
\lambda_{bc} &= \lambda_b - \lambda_c = \ell n\, 2 \\
\lambda_{ac} &= \lambda_a - \lambda_c = \ell n\left(\frac{2b^2}{a^2}\right) \\
\lambda_{ab}^2 - \lambda_{bc}^2 &= \ell n\left(\frac{b^2}{2a^2}\right)\ell n\left(\frac{2b^2}{a^2}\right)
\end{aligned}\right\} \tag{17}$$

The problem is to find

$$I_{sc}(h) = I_{sc}(-h) = I_1(h) + I_4(h) - I_2(h) - I_3(h) \tag{18}$$

where $I_{sc}(h)$ is the current flowing in the short-circuited transmission-line terminals. Evidently from (11), (16), (13), and (12),

$$\left.\begin{aligned} I_1(h) &= F_a(h) + I_4(h) \\ I_4(h) &= -F_\ell(h) + I_2(h) \\ -I_2(h) &= F_c(h) - I_1(h) \\ -I_3(h) &= F_b(h) - I_2(h) \end{aligned}\right\} \tag{19}$$

so that

$$I_{sc}(h) = F_b(h) + F_c(h) - F_\ell(h). \tag{20}$$

Further, inspection of Figure 1 shows that there are no potential differences between wires at $z = \pm h$. Accordingly,

$$\left.\frac{\partial F_a(z)}{\partial z}\right]_{z=\pm h} = \left.\frac{\partial F_b(z)}{\partial z}\right]_{z=\pm h} = \left.\frac{\partial F_c(z)}{\partial z}\right]_{z=\pm h} = 0. \tag{21}$$

From (21), (11)-(13), and (7)-(10), it follows that

$$C_1 = C_2 = C_3 = C_4. \tag{22}$$

One now constructs $I_{sc}(h)$ from (20), using (12), (13), (16), and (7)-(10). The result is

$$I_{sc}(h) = -j\frac{8\pi U}{\zeta_o \ell n\left(\frac{2b^2}{a^2}\right)}\left\{\left[\cos\left(\frac{\beta c}{2}\sin\Phi\right) - \cos\left(\frac{\beta c}{2}\cos\Phi\right) + j\sin\left(\frac{\beta c}{2}\cos\Phi\right)\right]\frac{2\,\ell n\,\frac{b}{a}}{\ell n\left(\frac{b^2}{2a^2}\right)}\right.$$

$$\left. + \left[\cos\left(\frac{\beta c}{2}\sin\Phi\right) - \cos\left(\frac{\beta c}{2}\cos\Phi\right) - j\sin\left(\frac{\beta c}{2}\cos\Phi\right)\right]\frac{\ell n\,2}{\ell n\left(\frac{b^2}{2a^2}\right)} - j\sin\left(\frac{\beta c}{2}\cos\Phi\right)\right\}. \tag{23}$$

Since $\beta c << 1$, the approximations $\sin x \sim x$ and $\cos x \sim 1 - \frac{x^2}{2}$ may be used to simplify (23). When these substitutions have been made it is found that $I_{sc}(h)$ is given by

$$I_{sc}(h) \sim -j\frac{\pi}{2}\frac{U\beta^2c^2\cos 2\Phi}{\zeta_o \ell n\left(\frac{b}{a\sqrt{2}}\right)}. \tag{24}$$

But the characteristic impedance of a four-wire transmission line is

$$Z_{c4} = \frac{\zeta_o}{2\pi}\ell n\left(\frac{b}{a\sqrt{2}}\right). \tag{25}$$

Using (25) and remembering that $U = -E_z^{inc}/\beta$ and $c = b\sqrt{2}$, the final expression for $I_{sc}(h)$ becomes

$$I_{sc}(h) \sim j\frac{E_z^{inc}}{2Z_{c4}}\beta b^2 \cos 2\Phi. \tag{26}$$

Note that when $\Phi = n\frac{\pi}{4}$, n odd, (23) (as well as (26)) gives $I_{sc}(h) \equiv 0$.

Terminated Two and Four-Wire Transmission-Lines

The analysis of a four-wire line, containing load impedances Z_{h4} at $z = h$ and Z_{-h4} at $z = -h$, $(Z_{h4} \neq Z_{-h4})$, proceeds along the lines set forth in Reference 1. Here an arbitrarily terminated two-wire transmission line is analyzed by applying the superposition and compensation theorems. In the interest of brevity only the results of each analysis will be given here.

For a four-wire line:

$$I_4(h) \sim -\frac{E_z^{inc}}{2D_4}\beta b^2 \cos 2\Phi\left[Z_{c4}\sin 2\beta h + jZ_{-h4}(1 - \cos 2\beta h)\right] \tag{27}$$

$$I_4(-h) \sim -\frac{E_z^{inc}}{2D_4}\beta b^2 \cos 2\Phi\left[Z_{c4}\sin 2\beta h + jZ_{h4}(1 - \cos 2\beta h)\right] \tag{28}$$

$$D_4 = Z_{c4}(Z_{h4} + Z_{-h4})\cos 2\beta h + j\left(Z_{h4}Z_{-h4} + Z_{c4}^2\right)\sin 2\beta h. \tag{29}$$

Here $I_4(h)$ is the current in the impedance Z_{h4} and $I_4(-h)$ is the current in the impedance Z_{-h4}.

For a two-wire line:

$$I_2(h) \sim -j \frac{E_z^{inc} b \sin \Phi}{D_2} \left[Z_{c2} \sin 2\beta h + jZ_{-h2}(1 - \cos 2\beta h) \right] \tag{30}$$

$$I_2(-h) \sim -j \frac{E_z^{inc} b \sin \Phi}{D_2} \left[Z_{c2} \sin 2\beta h + jZ_{h2}(1 - \cos 2\beta h) \right] \tag{31}$$

$$D_2 = Z_{c2}(Z_{h2} + Z_{-h2}) \cos 2\beta h + j(Z_{h2} Z_{-h2} + Z_{c2}^2) \sin 2\beta h \tag{32}$$

where $I_2(h)$ and $I_2(-h)$ are the currents in the terminating impedances Z_{h2} and Z_{-h2}, respectively, and Z_{c2}, the characteristic impedance of the two-wire transmission line, is given by

$$Z_{c2} = 120 \ln\left(\frac{b}{a}\right). \tag{33}$$

Equations (27)-(32) make it possible to compare the radio-frequency noise pickup of terminated two- and four-wire transmission lines, when the polarization of the interfering signal is parallel to the axes of the line wires.

Example: Let the two- and four-wire transmission lines be constructed of A.W. G. No. 12 wire (radius $a = 0.0404$ inches) spaced distance $b = 1.3$ inches apart. Assume that $Z_{h2} = Z_{-h2} = Z_{c2}$, and $Z_{h4} = Z_{-h4} = Z_{c4}$. Let $E_z^{inc} = 1$ volt/m at $f = 12$ mc/sec.; $\Phi = 30°$; and $\beta h = 3\pi/4$. Then, $Z_{c2} = 416.54$ ohms and $Z_{c4} = 187.48$ ohms. It follows that for this situation $I_2(h) = 0.02802$ ma in 416.54 ohms, so that $V_{h2} = 11.67$ mv noise for $E_z^{inc} = 1$ v/m. Also, $I_4(h) = 0.2583$ μa in 187.48 ohms, so that $V_{h4} = 48.43$ μv noise for $E_z^{inc} = 1$ v/m. Hence $|I_4(h)/I_2(h)| = 9.219 \times 10^{-3}$. These figures give some idea of the efficacy of four-wire over two-wire lines for connecting balanced antennas to the balanced input of radio receivers at communication frequencies.

Conclusion

One may conclude that in general a balanced four-wire transmission line is vastly superior to the comparable two-wire line from the point of view of radio-frequency noise pickup.