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Interaction Notes
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Interaction of Electromagnetic Fields with an Object which has an Electromagnetic Symmetry Plane

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Abstract

For some objects of interest such as some aircraft the objects have an electromagnetic symmetry plane, at least approximately so. For such objects one can define mirror electromagnetic quantities based on a reflection through the symmetry plane and then define symmetric and antisymmetric parts of the various electromagnetic quantities (fields, currents, etc.). These symmetric and antisymmetric parts can be treated separately, both for calculations and for measurements. This symmetry decomposition can be used to separate various features of the electromagnetic interaction (such as resonances) into two separate categories, one associated with the symmetric part and the other associated with the antisymmetric part. This technique can then be used to partially simplify the understanding of electromagnetic interaction with an object with an electromagnetic symmetry plane.

I. Introduction

In trying to understand the interaction of electromagnetic fields with bodies of complex shape such as an airplane, a building, etc. one is often faced with very difficult electromagnetic boundary value problems indeed. The boundary value problems one might solve would in general be only rough approximations pertaining to some of the features of a real structure of interest. In electromagnetic pulse (EMP) interaction questions the real structures of concern may have numerous design features so that they cannot be considered as simple shapes like spheres, circular cylinders, etc. without introducing various inaccuracies, sometimes significant, in some of the coupling to various penetrations, etc.

Still, while the object of interest may be rather complex there may be certain symmetries in the object which can be used to simplify the analysis of the electromagnetic interaction. By use of such symmetry one can determine some features of the electromagnetic interaction and/or decompose the interaction problem into more than one piece, each piece being treated separately. This approach has found various applications as, for example, in quantum mechanics and it can be considered as an application of group theory.

In this note we consider some of the results which apply to objects with an electromagnetic symmetry plane, a very simple type of symmetry with a symmetry group corresponding to S_2, the symmetric group of degree 2. The fields, currents, etc. are split into symmetric and antisymmetric parts or modes which can be considered separately. In some cases of interest there will be some small number of interaction modes (for example resonances), each with its own current distribution etc., which are of dominant interest to the EMP interaction problem. Each of these interaction modes can be categorized as symmetric or antisymmetric or at least split into such parts. With this splitting one can then hope to reduce the number of these dominant interaction modes to be considered simultaneously because of two separate treatments of the symmetric and antisymmetric parts.

This splitting into symmetric and antisymmetric parts can be used not only for interaction calculations but also for interaction measurements of current etc. Furthermore EMP simulation tests can be configured so as to separate to some extent the symmetric and antisymmetric parts by making an appropriate symmetry plane of the simulator coincide with the symmetry plane of the object of interest.

Of course a real object of interest may only approximately have an electromagnetic symmetry plane in the sense that there are some small asymmetric perturbations of the electromagnetic characteristics (e.g. shape) of the object. In such cases the results for the separation of interaction modes into symmetric and antisymmetric parts are only approximate and there is in general some "coupling" between these parts or modes. However if this "coupling" is not too large then the approximate symmetry decomposition may still be useful for calculating, measuring, and categorizing the interaction characteristics of the object. On the other hand some electromagnetic objects (such as EMP simulators or electromagnetic field sensors) are purposely constructed to rather accurately have such symmetry planes (or even higher order symmetries) to simplify or give other desirable characteristics to their performance and to simplify the calculation and other understanding of their performance.

The type of symmetry that we consider in this note is simply reflection symmetry, one of the S_2 kinds of symmetry in 3 dimensions. This gives two independent interaction modes: symmetric and antisymmetric which might be thought of as some kind of parity in the electromagnetic wave functions and might also be referred to as even and odd. This type of symmetry is appropriate to various real objects of interest such as certain aircraft, ships, buildings, etc. In some cases other symmetries including higher order symmetries may be present which would permit even further decomposition of the interaction modes. Perhaps some other symmetries could be considered in future notes.

II. Reflection of Coordinates Through a Symmetry Plane

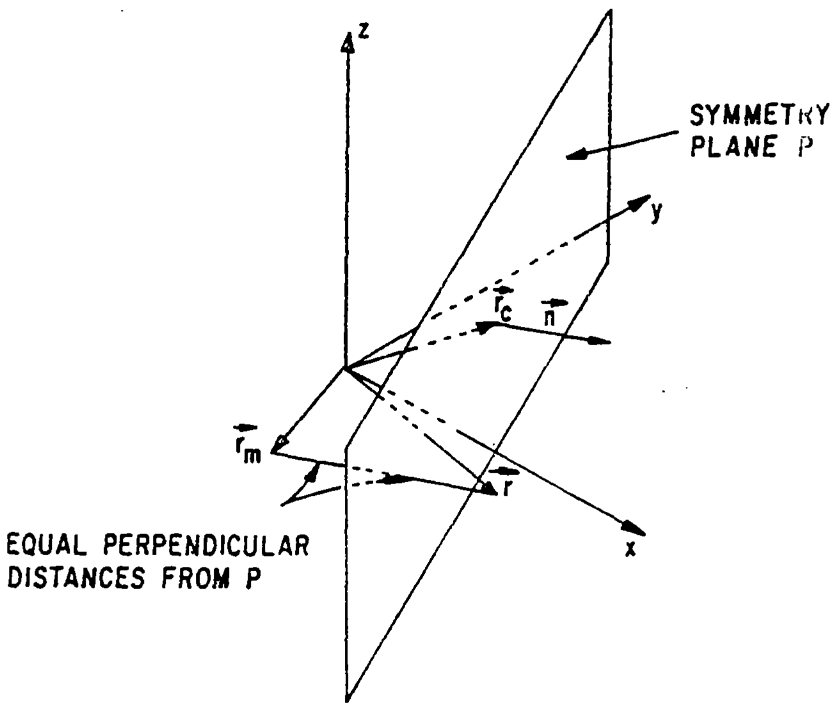
Consider then an electromagnetic symmetry plane, a surface which we call P as shown in figure 1. Such a plane can be specified by a surface normal unit vector $\vec{n}$ which is independent of position on P, and by some point $\vec{r} = \vec{r}_c$ which lies on P. The general position vector is[1]

$$\vec{r} = x\vec{e}_x + y\vec{e}_y + z\vec{e}_z \tag{2.1}$$

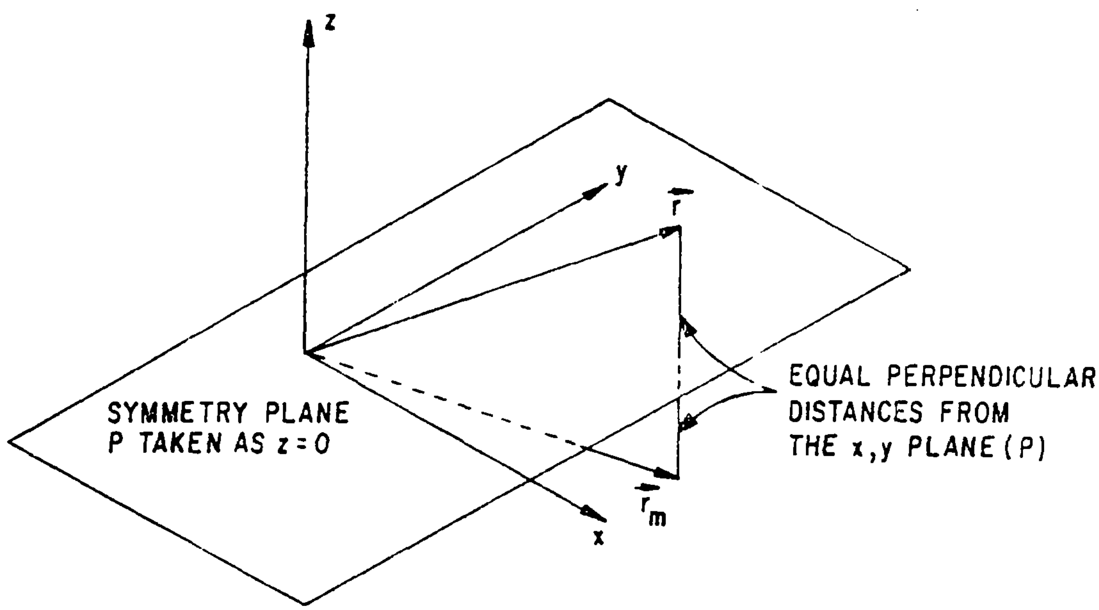
where we have used cartesian coordinates (x, y, z) with unit vectors denoted by $\vec{e}$ with appropriate subscripts. Figure 1A shows P with some general orientation $\vec{n}$ and containing some general $\vec{r}_c$. Figure 1B shows a special form for P where we have taken

$$\begin{aligned} \vec{r}_c &= \vec{0} \\ \vec{n} &= \vec{e}_z \end{aligned} \tag{2.2}$$

1. All units are rationalized MKSA.



A. SYMMETRY PLANE WITH ARBITRARY POSITION AND ORIENTATION



B. SYMMETRY PLANE TAKEN AS x, y PLANE

FIGURE 1. SYMMETRY PLANE WITH COORDINATES

This last choice is what we take for illustration with specific coordinates in this note. However, this does not reduce the generality of the results because this choice merely corresponds to a translation of the origin of the coordinates and a rotation of the coordinates to make the z axis perpendicular to P.

In considering objects with an electromagnetic symmetry plane P we define for each position $\vec{r}$ a reflection or mirror position $\vec{r}_m$ on the opposite side of P. For the simple case that P is the x, y plane as shown in figure 1B this simply involves changing the sign of z so that we could write

$$\begin{aligned} \vec{r} &\equiv (x,y,z) \equiv x\vec{e}_x+y\vec{e}_y+z\vec{e}_z \\ \vec{r}_m &= (x,y,-z) = x\vec{e}_x+y\vec{e}_y-z\vec{e}_z \end{aligned} \tag{2.3}$$

where z can of course be positive or negative. This can be generalized to the case shown in figure 1A by relating $\vec{r} - \vec{r}_c$ to $\vec{r}_m - \vec{r}_c$ with a general normal vector $\vec{n}$ as

$$\begin{aligned} \vec{n}\times(\vec{r}-\vec{r}_c) &= \vec{n}\times(\vec{r}_m-\vec{r}_c) \\ \vec{n}\cdot(\vec{r}-\vec{r}_c) &= -\vec{n}\cdot(\vec{r}_m-\vec{r}_c) \end{aligned} \tag{2.4}$$

which can be rewritten as

$$\begin{aligned} \vec{n}\times\vec{r}_m &= \vec{n}\times\vec{r} \\ \vec{n}\cdot\vec{r}_m &= 2\vec{n}\cdot\vec{r}_c-\vec{n}\cdot\vec{r} \end{aligned} \tag{2.5}$$

A general equation for $\vec{r}_m$ is then

$$\vec{r}_m-\vec{r}_c = \vec{r}-\vec{r}_c-2\vec{n}[\vec{n}\cdot(\vec{r}-\vec{r}_c)] \tag{2.6}$$

The projections of $\vec{r}$ and $\vec{r}_m$ on P are the same while they are on opposite sides of P and equidistant from P.

For convenience we introduce a reflection matrix which, for the case that P is the z = 0 plane, can be written as

$$\overleftrightarrow{R} \equiv (R_{ij}) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \tag{2.7}$$

so that for this case of P we have

$$\vec{r}_m = \overleftrightarrow{R}\cdot\vec{r} \ , \quad \vec{r} = \overleftrightarrow{R}\cdot\vec{r}_m \tag{2.8}$$

One might introduce a rotation matrix $\overleftrightarrow{U}$ for rotating the coordinates in the form

$$\vec{r}\,'-\vec{r}_c = \overleftrightarrow{U}\cdot\vec{r} \tag{2.9}$$

where this matrix is unitary with real coefficients and $\vec{r}\,' = (x', y', z')$ is a new cartesian coordinate system (say as in figure 1A). Note the shift of the $\vec{r}\,'$ coordinates by $\vec{r}_c$ so that we can get a new cartesian system from one where P is z = 0 by a rotation and a translation; P contains $\vec{r}\,' = \vec{r}_c$ in the new coordinate system. Because $\overleftrightarrow{U}$ is unitary we have

$$|\vec{r}\,'-\vec{r}_c| = |\vec{r}| \tag{2.10}$$

Note that the reflection matrix in equation 2.7 is its own inverse so that it is called an involutory matrix and we can write

$$\overleftrightarrow{R}^{-1} = \overleftrightarrow{R} \ , \quad \overleftrightarrow{R}\cdot\overleftrightarrow{R} \equiv \overleftrightarrow{R}^{2} = \overleftrightarrow{I} \tag{2.11}$$

where the identity matrix is

$$\overleftrightarrow{I} \equiv (\delta_{ij}) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{2.12}$$

The reflection matrix can be expressed in the $\vec{r}'$ coordinate system by applying equation 2.9 to $\vec{r}_m$ to give

$$\vec{r}_m' - \vec{r}_c = \vec{\vec{U}} \cdot \vec{r}_m \tag{2.13}$$

Then applying the unitary rotation matrix to equations 2.8 (either one) we have

$$\begin{aligned} \vec{r}_m' - \vec{r}_c &= \vec{\vec{U}} \cdot \vec{r}_m = \vec{\vec{U}} \cdot \vec{\vec{R}} \cdot \vec{r} \\ &= \left[\vec{\vec{U}} \cdot \vec{\vec{R}} \cdot \vec{\vec{U}}^{-1}\right] \cdot \vec{\vec{U}} \cdot \vec{r} = \left[\vec{\vec{U}} \cdot \vec{\vec{R}} \cdot \vec{\vec{U}}^{-1}\right] \cdot [\vec{r}' - \vec{r}_c] \\ &= \vec{\vec{R}}' \cdot [\vec{r}' - \vec{r}_c] \end{aligned} \tag{2.14}$$

where we have defined

$$\vec{\vec{R}}' \equiv \vec{\vec{U}} \cdot \vec{\vec{R}} \cdot \vec{\vec{U}}^{-1} \tag{2.15}$$

which is a similarity transformation of the reflection matrix. This can be considered the reflection matrix in the $\vec{r}'$ coordinate system which reflects the coordinates through P using the center $\vec{r}_c$ as in equation 2.14. Note that the rotated reflection matrix in equation 2.15 is also involutory (i.e. is its own inverse).

The two matrices $\vec{\vec{I}}$ and $\vec{\vec{R}}$ (also $\vec{\vec{I}}$ and $\vec{\vec{R}}'$) form a group with matrix multiplication as the operation. They comprise a representation of the symmetric group S_2.

III. Decomposition of Electromagnetic Quantities into Symmetric and Antisymmetric Parts

Maxwell's equations are written in the time domain as

$$\begin{aligned} &\nabla \times \vec{E}(\vec{r},t) = -\frac{\partial}{\partial t} \vec{B}(\vec{r},t) \ , \qquad \nabla \times \vec{H}(\vec{r},t) = \vec{J}(\vec{r},t) + \frac{\partial}{\partial t} \vec{D}(\vec{r},t) \\ &\nabla \cdot \vec{B}(\vec{r},t) = 0 \ , \qquad \nabla \cdot \vec{D}(\vec{r},t) = \rho(\vec{r},t) \end{aligned} \tag{3.1}$$

The constitutive relations plus Ohm's law can be written using Laplace transformed fields, etc. as

$$\tilde{\vec{D}}(\vec{r}) = \vec{\vec{\varepsilon}}(\vec{r}) \cdot \tilde{\vec{E}}(\vec{r})$$

$$\tilde{\vec{B}}(\vec{r}) = \vec{\vec{\mu}}(\vec{r}) \cdot \tilde{\vec{H}}(\vec{r}) \qquad (3.2)$$

$$\tilde{\vec{J}}(\vec{r}) = \vec{\vec{\sigma}}(\vec{r}) \cdot \tilde{\vec{E}}(\vec{r})$$

where the permittivity, permeability, and conductivity have been written as matrices and may be functions of s, the Laplace transform variable; for some cases we will take these parameters as scalars. The Laplace transform (two sided) of time-domain quantities is denoted by the addition of a tilde (~) above the symbol. Note that the current density in equations 3.2 is the conduction current density and does not include sources.

There are other related electromagnetic relations such as the equation of continuity

$$\nabla \cdot \vec{J}(\vec{r},t) + \frac{\partial}{\partial t} \rho(\vec{r},t) = 0 \qquad (3.3)$$

There are other commonly used electromagnetic quantities such as scalar and vector potentials which can be used to calculate the fields as

$$\vec{E}(\vec{r},t) = -\nabla \Phi(\vec{r},t) - \frac{\partial}{\partial t} \vec{A}(\vec{r},t)$$

$$\qquad (3.4)$$

$$\vec{B}(\vec{r},t) = \nabla \times \vec{A}(\vec{r},t)$$

For free space (permittivity ε_o, permeability μ_o, and zero conductivity) and some source current and charge densities the potentials can be written as

$$\Phi(\vec{r},t) = \frac{1}{\varepsilon_o} \int_V \frac{\rho\left(\vec{r}\,'',t-\frac{|\vec{r}-\vec{r}\,''|}{c}\right)}{4\pi|\vec{r}-\vec{r}\,''|} \, dV$$

$$\vec{A}(\vec{r},t) = \mu_o \int_V \frac{\vec{J}\left(\vec{r}\,'',t-\frac{|\vec{r}-\vec{r}\,''|}{c}\right)}{4\pi|\vec{r}-\vec{r}\,''|} \, dV \tag{3.5}$$

with the speed of light

$$c \equiv \frac{1}{\sqrt{\mu_o \varepsilon_o}} \tag{3.6}$$

where $\vec{r}\,''$ is the position vector for integration over the source volume V. The retarded time is used in equations 3.5 so that only outgoing waves are included; no incident wave (from infinity) is included in this formulation although other terms for an incident wave can be added.

Corresponding to the reflection of $\vec{r}$ to $\vec{r}_m$, the mirror position, one can make a reflection of the electromagnetic quantities. Here we wish to define mirror fields, mirror currents, etc. which are in some sense the reflection of the fields etc. from the mirror position $\vec{r}_m$ to the position $\vec{r}$ and also satisfy Maxwell's equations and related electromagnetic equations (equations 3.1 through 3.6 and others); such mirror fields etc. would then have all the characteristics of the original ones. Starting with the electric field define a mirror electric field in the same way the mirror position has been defined in equations 2.8. Using a subscript m we have a consistent set of mirror quantities as

$$\vec{E}_m(\vec{r},t) = \vec{\vec{R}}\cdot\vec{E}(\vec{r}_m,t) \ , \quad \vec{B}_m(\vec{r},t) = -\vec{\vec{R}}\cdot\vec{B}(\vec{r}_m,t)$$

$$\vec{D}_m(\vec{r},t) = \vec{\vec{R}}\cdot\vec{D}(\vec{r}_m,t) \ , \quad \vec{H}_m(\vec{r},t) = -\vec{\vec{R}}\cdot\vec{H}(\vec{r}_m,t)$$

$$\rho_m(\vec{r},t) = \rho(\vec{r}_m,t) \ , \quad \vec{J}_m(\vec{r},t) = \vec{\vec{R}}\cdot\vec{J}(\vec{r}_m, t) \tag{3.7}$$

$$\Phi_m(\vec{r},t) = \Phi(\vec{r}_m,t) \ , \quad \vec{A}_m(\vec{r},t) = \vec{\vec{R}}\cdot\vec{A}(\vec{r}_m,t)$$

One could reverse the signs of all these mirror quantities and still have a consistent set of quantities, but we prefer this convention because the scalar quantities reflect with no sign change. Note that if these mirror quantities are themselves reflected $\vec{r}_m$ returns to $\vec{r}$ and the original quantities are returned because the reflection matrix is its own inverse and the same applied to the negative of the reflection matrix.

For the reflection properties of equations 3.7 first consider the simple case of scalar ε, μ, and σ all independent of position, i.e. a uniform isotropic medium. If $\vec{E}_m$ has the form shown in equations 3.7 then $\vec{D}_m$ and $\vec{J}_m$ must also have the same form to satisfy one of the constitutive relations and Ohm's law; if $\vec{H}_m$ has the form shown (with minus sign) then $\vec{B}_m$ must have the same form because of the other constitutive relation. The curl equations are satisfied as can be seen by writing out the cartesian forms as

$$\nabla\times\vec{E}_m(\vec{r},t) = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_{m_x}(\vec{r},t) & E_{m_y}(\vec{r},t) & E_{m_z}(\vec{r},t) \end{vmatrix}$$

$$= \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ \frac{\partial}{\partial x_m} & \frac{\partial}{\partial y_m} & -\frac{\partial}{\partial z_m} \\ E_x(\vec{r}_m,t) & E_y(\vec{r}_m,t) & -E_z(\vec{r}_m,t) \end{vmatrix}$$

$$= -\overset{\leftrightarrow}{R} \cdot \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ \frac{\partial}{\partial x_m} & \frac{\partial}{\partial y_m} & \frac{\partial}{\partial z_m} \\ E_x(\vec{r}_m,t) & E_y(\vec{r}_m,t) & E_z(\vec{r}_m,t) \end{vmatrix}$$

$$= \frac{\partial}{\partial t} \overset{\leftrightarrow}{R}\cdot\vec{B}(\vec{r}_m,t)$$

$$= -\frac{\partial}{\partial t} \vec{B}_m(\vec{r},t) \qquad (3.8)$$

and similarly

$$\nabla\times\vec{H}_m(\vec{r},t) = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_{m_x}(\vec{r},t) & H_{m_y}(\vec{r},t) & H_{m_z}(\vec{r},t) \end{vmatrix}$$

$$= \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ \frac{\partial}{\partial x_m} & \frac{\partial}{\partial y_m} & -\frac{\partial}{\partial z_m} \\ -H_x(\vec{r}_m,t) & -H_y(\vec{r}_m,t) & H_z(\vec{r}_m,t) \end{vmatrix}$$

$$= \vec{\vec{R}} \cdot \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ \frac{\partial}{\partial x_m} & \frac{\partial}{\partial y_m} & \frac{\partial}{\partial z_m} \\ H_x(\vec{r}_m,t) & H_y(\vec{r}_m,t) & H_z(\vec{r}_m,t) \end{vmatrix}$$

$$= \vec{\vec{R}}\cdot\vec{J}(\vec{r}_m,t) + \frac{\partial}{\partial t}\,\vec{\vec{R}}\cdot\vec{D}(\vec{r}_m,t)$$

$$= \vec{J}_m(\vec{r},t) + \frac{\partial}{\partial t}\,\vec{D}_m(\vec{r},t) \qquad (3.9)$$

Thus the curl equations are satisfied. Note that the sign change in defining $\vec{H}_m$ (as compared to $\vec{E}_m$) is necessary to satisfy equations 3.8 and 3.9. This sign reversal of the magnetic field reflection formula is the same as is observed in coordinate inversion (or changing the sign of all three cartesian coordinates); the magnetic field is then often called a pseudo vector.

The charge density reflection is related to the current density reflection through the equation of continuity as

$$\frac{\partial}{\partial t} \rho_m(\vec{r},t) = -\nabla\cdot\vec{J}_m(\vec{r},t)$$

$$= -\frac{\partial}{\partial x} J_{m_x}(\vec{r},t) - \frac{\partial}{\partial y} J_{m_y}(\vec{r},t) - \frac{\partial}{\partial z} J_{m_z}(\vec{r},t)$$

$$= -\frac{\partial}{\partial x_m} J_x(\vec{r}_m,t) - \frac{\partial}{\partial y_m} J_y(\vec{r}_m,t) - \frac{\partial}{\partial z_m} J_z(\vec{r}_m,t)$$

$$= \frac{\partial}{\partial t} \rho(\vec{r}_m,t) \tag{3.10}$$

which is as in equations 3.7. The reflection characteristics of the scalar potential can be deduced from those of the charge density applied to one of equations 3.5; those of the vector potential are the same as for the current density as can be deduced from the second of equations 3.5. Alternatively one can manipulate the curl and gradient in equations 3.4 to deduce the reflection characteristics of the scalar and vector potentials from those of the electric and magnetic fields. Having defined a self consistent set of mirror fields etc. in equations 3.7 for our special cartesian coordinates for which P is the z = 0 plane, one can apply this to other cartesian systems such as $\vec{r}\,'$ by multiplying (dot product) on the left by the unitary rotation matrix $\overleftrightarrow{U}$ and convert $\overleftrightarrow{R}$ to $\overleftrightarrow{R}'$ as before. For the scalars nothing is needed.

Now we can define the symmetric and antisymmetric parts of the fields etc. The symmetric part or mode is defined as one half the sum of the original quantity plus the mirror quantity. The antisymmetric part or mode is one half the original quantity minus one half the mirror quantity. The symmetric part is denoted by the subscript sy; the antisymmetric part is denoted by the subscript as. One might also use the terms even and odd respectively and consider each of the parts as different parity states. The symmetric parts are

$$\vec{E}_{sy}(\vec{r},t) = \frac{1}{2}[\vec{E}(\vec{r},t)+\vec{E}_m(\vec{r},t)] = \frac{1}{2}[\vec{E}(\vec{r},t)+\overleftrightarrow{R}\cdot\vec{E}(\vec{r}_m,t)]$$

$$\vec{B}_{sy}(\vec{r},t) = \frac{1}{2}[\vec{B}(\vec{r},t)+\vec{B}_m(\vec{r},t)] = \frac{1}{2}[\vec{B}(\vec{r},t)-\overleftrightarrow{R}\cdot\vec{B}(\vec{r}_m,t)]$$

$$\vec{D}_{sy}(\vec{r},t) = \frac{1}{2}[\vec{D}(\vec{r},t)+\vec{D}_m(\vec{r},t)] = \frac{1}{2}[\vec{D}(\vec{r},t)+\overleftrightarrow{R}\cdot\vec{D}(\vec{r}_m,t)]$$

$$
\begin{aligned}
\vec{H}_{sy}(\vec{r},t) &= \tfrac{1}{2}[\vec{H}(\vec{r},t)+\vec{H}_m(\vec{r},t)] = \tfrac{1}{2}[\vec{H}(\vec{r},t)-\vec{\vec{R}}\cdot\vec{H}(\vec{r}_m,t)] \\
\rho_{sy}(\vec{r},t) &= \tfrac{1}{2}[\rho(\vec{r},t)+\rho_m(\vec{r},t)] = \tfrac{1}{2}[\rho(\vec{r},t)+\rho(\vec{r}_m,t)] \\
\vec{J}_{sy}(\vec{r},t) &= \tfrac{1}{2}[\vec{J}(\vec{r},t)+\vec{J}_m(\vec{r},t)] = \tfrac{1}{2}[\vec{J}(\vec{r},t)+\vec{\vec{R}}\cdot\vec{J}(\vec{r}_m,t)] \\
\Phi_{sy}(\vec{r},t) &= \tfrac{1}{2}[\Phi(\vec{r},t)+\Phi_m(\vec{r},t)] = \tfrac{1}{2}[\Phi(\vec{r},t)+\Phi(\vec{r}_m,t)] \\
\vec{A}_{sy}(\vec{r},t) &= \tfrac{1}{2}[\vec{A}(\vec{r},t)+\vec{A}_m(\vec{r},t)] = \tfrac{1}{2}[\vec{A}(\vec{r},t)+\vec{\vec{R}}\cdot\vec{A}(\vec{r}_m,t)]
\end{aligned}
\tag{3.11}
$$

The antisymmetric parts are

$$
\begin{aligned}
\vec{E}_{as}(\vec{r},t) &= \tfrac{1}{2}[\vec{E}(\vec{r},t)-\vec{E}_m(\vec{r},t)] = \tfrac{1}{2}[\vec{E}(\vec{r},t)-\vec{\vec{R}}\cdot\vec{E}(\vec{r}_m,t)] \\
\vec{B}_{as}(\vec{r},t) &= \tfrac{1}{2}[\vec{B}(\vec{r},t)-\vec{B}_m(\vec{r},t)] = \tfrac{1}{2}[\vec{B}(\vec{r},t)+\vec{\vec{R}}\cdot\vec{B}(\vec{r}_m,t)] \\
\vec{D}_{as}(\vec{r},t) &= \tfrac{1}{2}[\vec{D}(\vec{r},t)-\vec{D}_m(\vec{r},t)] = \tfrac{1}{2}[\vec{D}(\vec{r},t)-\vec{\vec{R}}\cdot\vec{D}(\vec{r}_m,t)] \\
\vec{H}_{as}(\vec{r},t) &= \tfrac{1}{2}[\vec{H}(\vec{r},t)-\vec{H}_m(\vec{r},t)] = \tfrac{1}{2}[\vec{H}(\vec{r},t)+\vec{\vec{R}}\cdot\vec{H}(\vec{r}_m,t)] \\
\rho_{as}(\vec{r},t) &= \tfrac{1}{2}[\rho(\vec{r},t)-\rho_m(\vec{r},t)] = \tfrac{1}{2}[\rho(\vec{r},t)-\rho(\vec{r}_m,t)] \\
\vec{J}_{as}(\vec{r},t) &= \tfrac{1}{2}[\vec{J}(\vec{r},t)-\vec{J}_m(\vec{r},t)] = \tfrac{1}{2}[\vec{J}(\vec{r},t)-\vec{\vec{R}}\cdot\vec{J}(\vec{r}_m,t)] \\
\Phi_{as}(\vec{r},t) &= \tfrac{1}{2}[\Phi(\vec{r},t)-\Phi_m(\vec{r},t)] = \tfrac{1}{2}[\Phi(\vec{r},t)-\Phi(\vec{r}_m,t)] \\
\vec{A}_{as}(\vec{r},t) &= \tfrac{1}{2}[\vec{A}(\vec{r},t)-\vec{A}_m(\vec{r},t)] = \tfrac{1}{2}[\vec{A}(\vec{r},t)-\vec{\vec{R}}\cdot\vec{A}(\vec{r}_m,t)]
\end{aligned}
\tag{3.12}
$$

Since the mirror quantities satisfy Maxwell's equations etc. and since we are only considering a linear problem (i.e. the permittivity, permeability, and conductivity are independent of the fields, etc.) then the symmetric and antisymmetric parts also satisfy Maxwell's and other associated equations. Note that by adding the symmetric and antisymmetric parts the original fields etc. are recovered; by subtracting the antisymmetric from the symmetric parts the mirror fields etc. are obtained. With the fields and related quantities written in terms of their symmetric and antisymmetric parts we have a separation in the sense that symmetric fields and potentials are associated only with symmetric current and charge densities, and similarly for the antisymmetric quantities. So far we have just considered the case of a uniform isotropic medium. Of course the purpose for the separation is for the convenient treatment of electromagnetic interaction with symmetric objects and this is considered in the next section in terms of requirements on the permittivity, permeability, and conductivity. In equations 3.11 and 3.12 the symmetric and antisymmetric parts have been written for the case that the symmetry plane P is the z = 0 plane, but just as with the mirror quantities these are simply generalized to a more general $\vec{r}\,'$ cartesian coordinate system by multiplying on the left by a unitary rotation matrix $\vec{\vec{U}}$ and converting $\vec{\vec{R}}$ to $\vec{\vec{R}}\,'$ as before.

Having decomposed the fields etc. into symmetric and antisymmetric parts one can consider the symmetry of these two parts with respect to the symmetry plane P. In terms of the cartesian coordinate system $\vec{r}$ where P is the z = 0 plane one can write symmetry relations between the symmetric parts at $\vec{r}$ and $\vec{r}_m$ and similarly for the antisymmetric parts. Comparing the symmetric parts as in equations 3.11 at $\vec{r}$ and the mirror position $\vec{r}_m$ we have symmetry relations

$$
\begin{aligned}
\vec{E}_{sy}(\vec{r}_m,t) &= \vec{\vec{R}}\cdot\vec{E}_{sy}(\vec{r},t) \ , & \vec{B}_{sy}(\vec{r}_m,t) &= -\vec{\vec{R}}\cdot\vec{B}_{sy}(\vec{r},t) \\
\vec{D}_{sy}(\vec{r}_m,t) &= \vec{\vec{R}}\cdot\vec{D}_{sy}(\vec{r},t) \ , & \vec{H}_{sy}(\vec{r}_m,t) &= -\vec{\vec{R}}\cdot\vec{H}_{sy}(\vec{r},t) \\
\rho_{sy}(\vec{r}_m,t) &= \rho_{sy}(\vec{r},t) \ , & \vec{J}_{sy}(\vec{r}_m,t) &= \vec{\vec{R}}\cdot\vec{J}_{sy}(\vec{r},t) \qquad (3.13) \\
\Phi_{sy}(\vec{r}_m,t) &= \Phi_{sy}(\vec{r},t) \ , & \vec{A}_{sy}(\vec{r}_m,t) &= \vec{\vec{R}}\cdot\vec{A}_{sy}(\vec{r},t)
\end{aligned}
$$

Comparing the antisymmetric parts as in equations 3.12 at $\vec{r}$ and the mirror position r_m we have the symmetry relations

$$
\begin{aligned}
\vec{E}_{as}(\vec{r}_m,t) &= -\vec{\vec{R}}\cdot\vec{E}_{as}(\vec{r},t) , & \vec{B}_{as}(\vec{r}_m,t) &= \vec{\vec{R}}\cdot\vec{B}_{as}(\vec{r},t) \\
\vec{D}_{as}(\vec{r}_m,t) &= -\vec{\vec{R}}\cdot\vec{D}_{as}(\vec{r},t) , & \vec{H}_{as}(\vec{r}_m,t) &= \vec{\vec{R}}\cdot\vec{H}_{as}(\vec{r},t) \\
\rho_{as}(\vec{r}_m,t) &= -\rho_{as}(\vec{r},t) , & \vec{J}_{as}(\vec{r}_m,t) &= -\vec{\vec{R}}\cdot\vec{J}_{as}(\vec{r},t) \\
\Phi_{as}(\vec{r}_m,t) &= -\Phi_{as}(\vec{r},t) , & \vec{A}_{as}(\vec{r}_m,t) &= -\vec{\vec{R}}\cdot\vec{A}_{as}(\vec{r},t)
\end{aligned}
\tag{3.14}
$$

Note that the signs in equations 3.13 and 3.14 are exactly opposite. This feature is a convenient one and allows one to better picture the spatial distribution of the symmetric and antisymmetric parts. Note the predominant (but not exclusive) use of the plus sign for the symmetry relations for the symmetric part, and conversely for the antisymmetric part. The magnetic field has exactly the opposite symmetry relations for its symmetric and antisymmetric parts as compared to the electric field.

While the symmetric parts have plus signs in the symmetry relations for the scalars as well as for the vectors other than the magnetic field, note that the reflection matrix $\vec{\vec{R}}$ used with the vectors reverses the sign of the z component (the component perpendicular to the symmetry plane P). Thus the vector symmetric parts have a variety of symmetry relations for the various vector components. The electric field, current density, and vector potential components parallel to P keep the same sign on reflection through P while the components of these vectors perpendicular to P reverse sign. The magnetic field is just the opposite in that the components parallel to P reverse sign while the component normal to P keeps the same sign. Note that the symmetric magnetic field is perpendicular to P for $\vec{r}$ unless it is discontinuous at P. The symmetric electric field, current density, and vector potential are parallel to P for $\vec{r}$ on P unless they are discontinuous at P.

The antisymmetric parts have minus signs for the scalars on reflection through P. Again the electric and magnetic fields have different signs on reflection and the reflection matrix gives different reflection properties to the vector components. The antisymmetric electric field, current density, and vector potential have their components parallel to P reverse sign on reflection and those perpendicular to P keep the same sign. The antisymmetric magnetic field has its components parallel to P keep the same sign on reflection and its component perpendicular to P reverse sign. The antisymmetric electric field, current density, and vector potential are perpendicular to P for $\vec{r}$ on P. The antisymmetric magnetic field is parallel to P for $\vec{r}$ on P.

The symmetry relations for the symmetric and antisymmetric parts in equations 3.13 and 3.14 have been written for cartesian coordinates $\vec{r}$ where the symmetry plane P is z = 0. For the scalars the expression of the relations in general cartesian coordinates $\vec{r}'$ is of the same form with $\vec{r}'$ replacing $\vec{r}$ and $\vec{r}_m'$ replacing $\vec{r}_m$. For the vectors multiplication on the left by a rotation matrix $\overleftrightarrow{U}$ converts the vectors to the more general coordinates and gives $\overleftrightarrow{R}'$ as the appropriate reflection matrix.

Now that the symmetric and antisymmetric parts are separated note that the symmetric electric field, current density, magnetic field, etc. go together and the same applies to the antisymmetric parts. The symmetric and antisymmetric parts can be treated separately, i.e. there is no cross coupling between these parts or modes. For calculations one can treat each part separately, and because of the symmetry relations (equations 3.13 and 3.14) the quantities need only be considered for a half space on one side of the symmetry plane P. This fact can be used, for example, to effectively reduce the number of zones into which an object of interest is divided for numerical electromagnetic calculations. By use of this symmetry decomposition measurements of various quantities such as current and charge can be made so as to explicitly display the symmetric and antisymmetric parts. Of course in this section we have only considered the simple case of a uniform, isotropic medium which only applies to portions of our region of interest, such as the free space (air), earth, water, etc. surrounding our object of interest which has the symmetry plane P. In the next section we consider a more general case consistent with the minimum requirements imposed by the simple medium used here.

IV. Object with an Electromagnetic Symmetry Plane

The reason for splitting the fields etc. into symmetric and antisymmetric parts is for convenience in treating electromagnetic problems involving an object with a symmetry plane P. For the case of a uniform isotropic medium this splitting gives symmetric and antisymmetric parts which have no cross coupling so that they can be treated separately. We would like this feature to hold for a more general medium which is perhaps both nonuniform and anisotropic. In particular we would like this decomposition to apply to some object situated in some medium such as free space, soil, water, etc. or combination of such media. This requirement poses the question of what is meant by the electromagnetic symmetry plane P. The object of interest situated in various media can be considered as one medium with nonuniform and possibly anisotropic characteristics. The symmetry plane P must apply to this complete medium which includes the object with its surroundings.

Given the symmetric parts (equations 3.11) and the antisymmetric parts (equations 3.12) as satisfying Maxwell's and other related equations, then by taking sums and differences of these parts both the original fields etc. and the mirror ones can be obtained and by linearity they satisfy Maxwell's equations etc. The criterion for what constitutes a medium with symmetry plane P is that the mirror quantities and thereby the symmetric and antisymmetric parts can be formed and all satisfy Maxwell's equations and the other related equations. The original fields, etc. of course must satisfy these equations including any variation of the parameters of the medium with position and/or frequency. Applying the original fields etc. to form the mirror quantities the medium properties only enter in certain relations between the quantities from equations 3.2. Thus the matrix permittivity, permeability, and conductivity have to satisfy relations at the positions $\vec{r}$ and $\vec{r}_m$ for the original fields and current density as

$$\begin{aligned}
\tilde{\vec{D}}(\vec{r}) &= \overset{\leftrightarrow}{\varepsilon}(\vec{r})\cdot\tilde{\vec{E}}(\vec{r}) \;, & \tilde{\vec{D}}(\vec{r}_m) &= \overset{\leftrightarrow}{\varepsilon}(\vec{r}_m)\cdot\tilde{\vec{E}}(\vec{r}_m) \\
\tilde{\vec{B}}(\vec{r}) &= \overset{\leftrightarrow}{\mu}(\vec{r})\cdot\tilde{\vec{H}}(\vec{r}) \;, & \tilde{\vec{B}}(\vec{r}_m) &= \overset{\leftrightarrow}{\mu}(\vec{r}_m)\cdot\tilde{\vec{H}}(\vec{r}_m) \qquad (4.1) \\
\tilde{\vec{J}}(\vec{r}) &= \overset{\leftrightarrow}{\sigma}(\vec{r})\cdot\tilde{\vec{E}}(\vec{r}) \;, & \tilde{\vec{J}}(\vec{r}_m) &= \overset{\leftrightarrow}{\sigma}(\vec{r}_m)\cdot\tilde{\vec{E}}(\vec{r}_m)
\end{aligned}$$

These same relations must hold for the mirror quantities which also satisfy these as well as Maxwell's equations, giving

$$\begin{aligned}
\tilde{\vec{D}}_m(\vec{r}) &= \overset{\leftrightarrow}{\varepsilon}(\vec{r})\cdot\tilde{\vec{E}}_m(\vec{r}) \;, & \tilde{\vec{D}}_m(\vec{r}_m) &= \overset{\leftrightarrow}{\varepsilon}(\vec{r}_m)\cdot\tilde{\vec{E}}_m(\vec{r}_m) \\
\tilde{\vec{B}}_m(\vec{r}) &= \overset{\leftrightarrow}{\mu}(\vec{r})\cdot\tilde{\vec{H}}_m(\vec{r}) \;, & \tilde{\vec{B}}_m(\vec{r}_m) &= \overset{\leftrightarrow}{\mu}(\vec{r}_m)\cdot\tilde{\vec{H}}_m(\vec{r}_m) \qquad (4.2) \\
\tilde{\vec{J}}_m(\vec{r}) &= \overset{\leftrightarrow}{\sigma}(\vec{r})\cdot\tilde{\vec{E}}_m(\vec{r}) \;, & \tilde{\vec{J}}(\vec{r}_m) &= \overset{\leftrightarrow}{\sigma}(\vec{r}_m)\cdot\tilde{\vec{E}}(\vec{r}_m)
\end{aligned}$$

Note that we are using the cartesian coordinates $\vec{r}$ where the symmetry plane P is $z = 0$ for this development.

Equations 3.7 give the relations between the mirror quantities and the original ones. Now take the mirror quantities evaluated at $\vec{r}_m$ in equations 4.2 and multiply these three equations on the left by $\overleftrightarrow{R}$ or $-\overleftrightarrow{R}$ as appropriate. This converts them to equations for the original quantities as

$$\tilde{\vec{D}}(\vec{r}) = \overleftrightarrow{R}\cdot\tilde{\vec{D}}_m(\vec{r}_m) = [\overleftrightarrow{R}\cdot\overleftrightarrow{\varepsilon}(\vec{r}_m)\cdot\overleftrightarrow{R}]\cdot[\overleftrightarrow{R}\cdot\tilde{\vec{E}}_m(\vec{r}_m)] = [\overleftrightarrow{R}\cdot\overleftrightarrow{\varepsilon}(\vec{r}_m)\cdot\overleftrightarrow{R}]\cdot\tilde{\vec{E}}(\vec{r})$$

$$\tilde{\vec{B}}(\vec{r}) = -\overleftrightarrow{R}\cdot\tilde{\vec{B}}_m(\vec{r}_m) = -[\overleftrightarrow{R}\cdot\overleftrightarrow{\mu}(\vec{r}_m)\cdot\overleftrightarrow{R}]\cdot[\overleftrightarrow{R}\cdot\tilde{\vec{H}}_m(\vec{r}_m)] = [\overleftrightarrow{R}\cdot\overleftrightarrow{\mu}(\vec{r}_m)\cdot\overleftrightarrow{R}]\cdot\tilde{\vec{H}}(\vec{r})$$

$$\tilde{\vec{J}}(\vec{r}) = \overleftrightarrow{R}\cdot\tilde{\vec{J}}_m(\vec{r}_m) = [\overleftrightarrow{R}\cdot\overleftrightarrow{\sigma}(\vec{r}_m)\cdot\overleftrightarrow{R}]\cdot[\overleftrightarrow{R}\cdot\tilde{\vec{E}}_m(\vec{r}_m)] = [\overleftrightarrow{R}\cdot\overleftrightarrow{\sigma}(\vec{r}_m)\cdot\overleftrightarrow{R}]\cdot\tilde{\vec{E}}(\vec{r}) \tag{4.3}$$

which use the fact that $\overleftrightarrow{R}$ is its own inverse. Now compare these results with the forms in equations 4.1 for the original quantities at r. Depending on the form of the fields incident on our object of interest (due to particular source locations) there are various types of resulting electric and magnetic field distributions possible. Let us then in general require

$$\overleftrightarrow{\varepsilon}(\vec{r}) = \overleftrightarrow{R}\cdot\overleftrightarrow{\varepsilon}(\vec{r}_m)\cdot\overleftrightarrow{R}$$

$$\overleftrightarrow{\mu}(\vec{r}) = \overleftrightarrow{R}\cdot\overleftrightarrow{\mu}(\vec{r}_m)\cdot\overleftrightarrow{R} \tag{4.4}$$

$$\overleftrightarrow{\sigma}(\vec{r}) = \overleftrightarrow{R}\cdot\overleftrightarrow{\sigma}(\vec{r}_m)\cdot\overleftrightarrow{R}$$

and use this as our definition of what is required for an object of interest together with any surrounding media of significance to have a symmetry plane P. There may be cases that various of the matrix components do not need to have this symmetry relationship because certain fields or just particular field components are zero so that for various $\vec{r}$, $\vec{r}_m$ combinations various of the matrix elements are multiplied by zero. An example where the fields are zero is the case of the fields inside a perfectly conducting shield with initially zero fields and with no sources interior to the shield. There may be various cases in which various features of the permittivity, permeability, and conductivity have only a small (perhaps negligible) effect on the fields, currents, etc. over large parts (or even most) of the volume of interest. If these features which give small effect over most of the object are not symmetric as required in equations 4.4 one might still use the symmetry decomposition of the fields etc. as a convenient approximation.

The results of equations 4.4 can be extended to the more general cartesian coordinate system $\vec{r}'$ (as before) by multiplying on the left by a rotation matrix $\overleftrightarrow{U}$. However in the $\vec{r}$ system the results are comparatively simple due to the simple form of the reflection matrix $\overleftrightarrow{R}$ as in equation 2.7. In order to see some of the details of the symmetry of the permittivity, permeability, and conductivity matrices in the $\vec{r}$ system one can multiply out the right sides of equations 4.4 and explicitly display the resulting matrices in terms of their components; this gives

$$\overleftrightarrow{\varepsilon}(\vec{r}) \equiv \begin{pmatrix} \varepsilon_{xx}(\vec{r}) & \varepsilon_{xy}(\vec{r}) & \varepsilon_{xz}(\vec{r}) \\ \varepsilon_{yx}(\vec{r}) & \varepsilon_{yy}(\vec{r}) & \varepsilon_{yz}(\vec{r}) \\ \varepsilon_{zx}(\vec{r}) & \varepsilon_{zy}(\vec{r}) & \varepsilon_{zz}(\vec{r}) \end{pmatrix}$$

$$= \overleftrightarrow{R}\cdot\overleftrightarrow{\varepsilon}(\vec{r}_m)\cdot\overleftrightarrow{R} = \begin{pmatrix} \varepsilon_{xx}(\vec{r}_m) & \varepsilon_{xy}(\vec{r}_m) & -\varepsilon_{xz}(\vec{r}_m) \\ \varepsilon_{yx}(\vec{r}_m) & \varepsilon_{yy}(\vec{r}_m) & -\varepsilon_{yz}(\vec{r}_m) \\ -\varepsilon_{zx}(\vec{r}_m) & -\varepsilon_{zy}(\vec{r}_m) & \varepsilon_{zz}(\vec{r}_m) \end{pmatrix}$$

$$\overleftrightarrow{\mu}(\vec{r}) \equiv \begin{pmatrix} \mu_{xx}(\vec{r}) & \mu_{xy}(\vec{r}) & \mu_{xz}(\vec{r}) \\ \mu_{yx}(\vec{r}) & \mu_{yy}(\vec{r}) & \mu_{yz}(\vec{r}) \\ \mu_{zx}(\vec{r}) & \mu_{zy}(\vec{r}) & \mu_{zz}(\vec{r}) \end{pmatrix}$$

$$= \overleftrightarrow{R}\cdot\overleftrightarrow{\mu}(\vec{r}_m)\cdot\overleftrightarrow{R} = \begin{pmatrix} \mu_{xx}(r_m) & \mu_{xy}(r_m) & -\mu_{xz}(r_m) \\ \mu_{yx}(r_m) & \mu_{yy}(r_m) & -\mu_{yz}(r_m) \\ -\mu_{zx}(r_m) & -\mu_{zy}(r_m) & \mu_{zz}(r_m) \end{pmatrix} \tag{4.5}$$

$$\overleftrightarrow{\sigma}(\vec{r}) \equiv \begin{pmatrix} \sigma_{xx}(\vec{r}) & \sigma_{xy}(\vec{r}) & \sigma_{xz}(\vec{r}) \\ \sigma_{yx}(\vec{r}) & \sigma_{yy}(\vec{r}) & \sigma_{yz}(\vec{r}) \\ \sigma_{zx}(\vec{r}) & \sigma_{zy}(\vec{r}) & \sigma_{zz}(\vec{r}) \end{pmatrix}$$

$$= \overleftrightarrow{R}\cdot\overleftrightarrow{\sigma}(\vec{r}_m)\cdot\overleftrightarrow{R} = \begin{pmatrix} \sigma_{xx}(\vec{r}_m) & \sigma_{xy}(\vec{r}_m) & -\sigma_{xz}(\vec{r}_m) \\ \sigma_{yx}(\vec{r}_m) & \sigma_{yy}(\vec{r}_m) & -\sigma_{yz}(\vec{r}_m) \\ -\sigma_{zx}(\vec{r}_m) & -\sigma_{zy}(\vec{r}_m) & \sigma_{zz}(\vec{r}_m) \end{pmatrix}$$

Note that the xz, zx, yz, and zy components of the permittivity, permeability, and conductivity matrices all change sign on reflection while the remaining five components of each matrix keep the same sign.

The symmetry requirements for the object and surrounding media are the general ones in equations 4.5. In some special cases it may be more appropriate to deal with other parameters instead of the volume ones, as in the case of perfectly conducting bodies or perfectly conducting surfaces. For cases that the object of interest or part of it is perfectly conducting (or can be approximated as such) then the details of the conductivity matrix need not be considered for such parts of the object. One only needs the shape to be symmetric with respect to P so that reflecting $\vec{r}$ to $\vec{r}_m$ reproduces the same perfectly conducting parts. For typical cases that the permittivity, permeability, and conductivity are scalars the results of equations 4.4 reduce to the simple symmetry relations

$$\varepsilon(\vec{r}) = \varepsilon(\vec{r}_m) \;, \quad \mu(\vec{r}) = \mu(\vec{r}_m) \;, \quad \sigma(\vec{r}) = \sigma(\vec{r}_m) \tag{4.6}$$

which again can be typically viewed as a geometrical property of the object. For example, a plastic part with a certain ε on one side of P has to have a mirror part with the same ε on the other side of P.

In the common case of highly conducting metal surfaces it is usually convenient to introduce a surface current density $\vec{J}_s$ (amps/meter) and a surface charge density ρ_s (coulombs/meter^2). Corresponding to equations 3.7 the mirror quantities are

$$\rho_{s_m}(\vec{r},t) = \rho_s(\vec{r}_m,t) \;, \quad \vec{J}_{s_m}(\vec{r},t) = \overleftrightarrow{R}\cdot\vec{J}_s(\vec{r},t) \tag{4.7}$$

The symmetric and antisymmetric parts corresponding to equations 3.11 and 3.12 are

$$\rho_{s_{sy}}(\vec{r},t) = \frac{1}{2}[\rho_s(\vec{r},t)+\rho_{s_m}(\vec{r},t)] = \frac{1}{2}[\rho_s(\vec{r},t)+\rho_s(\vec{r}_m,t)]$$

$$\rho_{s_{as}}(\vec{r},t) = \frac{1}{2}[\rho_s(\vec{r},t)-\rho_{s_m}(\vec{r},t)] = \frac{1}{2}[\rho_s(\vec{r},t)-\rho_s(\vec{r}_m,t)]$$

$$\vec{J}_{s_{sy}}(\vec{r},t) = \frac{1}{2}[\vec{J}_s(\vec{r},t)+\vec{J}_{s_m}(\vec{r},t)] = \frac{1}{2}[\vec{J}_s(\vec{r},t)+\vec{\vec{R}}\cdot\vec{J}_s(\vec{r}_m,t)] \tag{4.8}$$

$$\vec{J}_{s_{as}}(\vec{r},t) = \frac{1}{2}[\vec{J}_s(\vec{r},t)-\vec{J}_{s_m}(\vec{r},t)] = \frac{1}{2}[\vec{J}_s(\vec{r},t)-\vec{\vec{R}}\cdot\vec{J}_s(\vec{r}_m,t)]$$

These parts have symmetry relations corresponding to equations 3.13 and 3.14 of the form

$$\rho_{s_{sy}}(\vec{r}_m,t) = \rho_{s_{sy}}(\vec{r},t) \ , \quad \rho_{s_{as}}(\vec{r}_m,t) = -\rho_{s_{as}}(\vec{r},t)$$

$$\vec{J}_{s_{sy}}(\vec{r}_m,t) = \vec{\vec{R}}\cdot\vec{J}_{s_{sy}}(\vec{r},t) \ , \quad \vec{J}_{s_{as}}(\vec{r}_m,t) = -\vec{\vec{R}}\cdot\vec{J}_{s_{as}}(\vec{r},t) \tag{4.9}$$

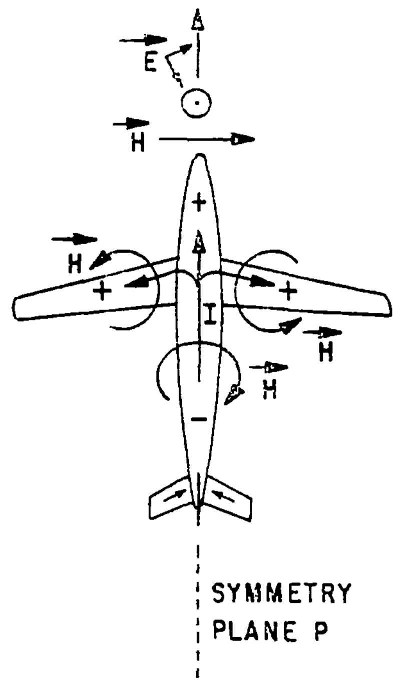
The surface charge density and surface current density on a surface can be related to displacement field ($\vec{D}$) and magnetic field ($\vec{H}$) respectively which are adjacent to the surface of interest. These fields can be used in designing sensors to measure the surface charge density and surface current density.

In addition to the surface charge density and surface current density one might consider some more macroscopic parameters such as charge and current. For use with decomposition with respect to the symmetry plane P one might consider the charge in some volume described by a range of the coordinates $\vec{r}$; one would then need to consider the charge in the mirror volume associated with the coordinates $\vec{r}_m$ ranging over the mirror positions corresponding to all $\vec{r}$ in the original volume. Similarly one might consider a current I as the surface integral of the current density $\vec{J}$ over some particular surface and define a mirror surface by transforming $\vec{r}$ to $\vec{r}_m$ for all points on the surface. Note that while current is normally considered as a scalar it does have a direction, i.e. it passes through a surface in a particular direction if the current is non zero. When confined to a narrow conducting path such as a wire the current can be considered as a localized quantity with a direction parallel to the path idealized as a line.

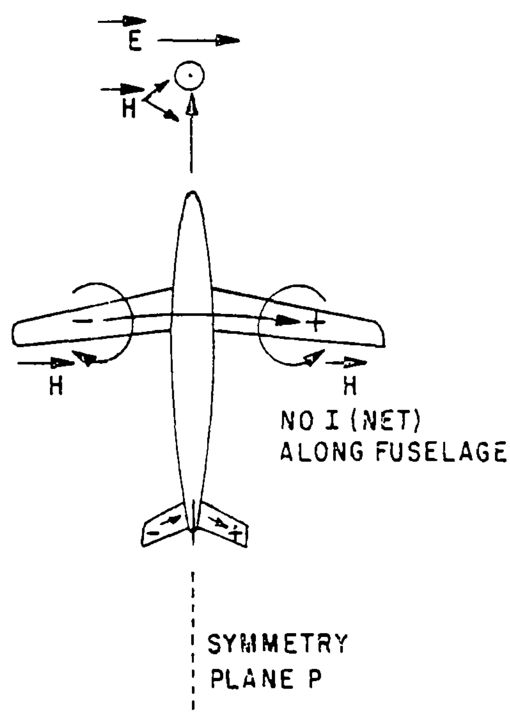
As an example to illustrate some of the features of the symmetry decomposition into symmetric and antisymmetric parts consider the example in figure 2. Here we have a somewhat generalized airplane with a conducting skin and a vertical symmetry plane passing through the nose and tail with one wing on each side of this symmetry plane P. Of course there may be some asymmetries in a real aircraft related to various things such as antenna locations, wheel well positions, electrical cable routing, etc.; these asymmetries are assumed to have negligible effects on the overall charge and current flow on the exterior of the aircraft. There is assumed to be some form of incident wave and this is split into symmetric and antisymmetric parts. Some of the reflection symmetries for the fields, current, and charge are shown for the symmetric part in figure 2A and for the antisymmetric part in figure 2B. The current I is defined as the total current passing through each open surface (say planar) which intersects the fuselage or wing in a manner perpendicular to the long dimension of the fuselage or wing. With this definition note that for the antisymmetric part the net current on the fuselage is zero rather conveniently. There is a significant current on the wings which can have a significant resonance at a frequency where the total wing length is of the order of a half wavelength. On the other hand the symmetric part has a significant fuselage current which also goes out onto the wings and tail structure. It can also have a significant resonance with a half wavelength of the order of the fuselage length with corrections for loading by the wings and tail structure. Thus we have identified a resonance associated with each of the two parts of the symmetry decomposition. By making this symmetry decomposition then one can avoid mixing these two different resonant modes and thereby look at their variation with the various parameters of the aircraft separately. Note in figure 2 how the charge distribution varies over the aircraft for the symmetric and antisymmetric parts. Also note how for both symmetric and antisymmetric parts the magnetic field (resultant, not just incident) is perpendicular to the surface current density in its immediate vicinity as well as parallel to the assumed highly conducting skin. For our present example we have considered some of the features of an aircraft far removed (compared to its size) from earth or any other electromagnetic medium other than free space (air).

V. Decomposition of an Incident Plane Wave into Symmetric and Antisymmetric Parts

Now that a general field distribution has been split into symmetric and antisymmetric parts with respect to the symmetry



A. SYMMETRIC PART : TOP VIEW



B. ANTISYMMETRIC PART : TOP VIEW

FIGURE 2. EXAMPLE OF AN OBJECT WITH AN ELECTROMAGNETIC SYMMETRY PLANE : AN AIRPLANE

plane P let us consider an example. Specifically consider the symmetric and antisymmetric parts of a plane wave in a uniform, isotropic medium. This simple example can be used to describe the symmetry decomposition with respect to P of a plane wave incident on an object of interest with such a symmetry plane where the object is placed in a uniform isotropic medium such as free space. This plane wave does not represent the total fields but only the incident fields. However, we can still treat the symmetric and antisymmetric incident fields separately to calculate the associated symmetric and antisymmetric total fields respectively.

In the Laplace transform domain we can express a general plane wave in a uniform isotropic medium in the form[2]

$$\tilde{\vec{E}}(\vec{r}) = \tilde{\vec{E}}_o e^{-\vec{\gamma}\cdot\vec{r}}$$

$$\tilde{\vec{H}}(\vec{r}) = \frac{Y}{\gamma}\,\vec{\gamma}\times\tilde{\vec{E}}_o e^{-\vec{\gamma}\cdot\vec{r}} \tag{5.1}$$

with the constraint

$$\vec{\gamma}\cdot\tilde{\vec{E}}_o = 0 \tag{5.2}$$

The wave admittance is

$$Y \equiv \frac{1}{Z} = \left[\frac{\sigma+s\varepsilon}{s\mu}\right]^{1/2} \tag{5.3}$$

and the propagation vector is

$$\vec{\gamma} \equiv i\vec{K} = \gamma\vec{e}_1 = iK\vec{e}_1 \tag{5.4}$$

where the propagation constant is

$$\gamma \equiv iK = [s\mu(\sigma+s\varepsilon)]^{1/2} \tag{5.5}$$

2. P. C. Clemmow, The Plane Wave Spectrum Representation of Electromagnetic Fields, Pergamon, 1966, chapter II.

The unit vector $\vec{e}_1$ is the direction of propagation and $\tilde{\vec{E}}_o$ is a vector which is independent of $\vec{r}$ but may be a function of s. Note that equations 5.1 and 5.2 can also be written as

$$\tilde{\vec{E}}(\vec{r}) = \tilde{\vec{E}}_o e^{-\gamma \vec{e}_1 \cdot \vec{r}}$$

$$\tilde{\vec{H}}(\vec{r}) = Y\vec{e}_1 \times \tilde{\vec{E}}_o e^{-\gamma \vec{e}_1 \cdot \vec{r}} \qquad (5.6)$$

$$\vec{e}_1 \cdot \tilde{\vec{E}}_o = 0$$

One can see that this type of plane wave satisfies Maxwell's equations and the constitutive relations plus Ohm's law (equations 3.1 and 3.2). The curl of the electric field is

$$\nabla \times \tilde{\vec{E}}(\vec{r}) = \nabla \times \left[\tilde{\vec{E}}_o e^{-\gamma \vec{e}_1 \cdot \vec{r}}\right] = \nabla\left[e^{-\gamma \vec{e}_1 \cdot \vec{r}}\right] \times \tilde{\vec{E}}_o$$

$$= -\gamma \vec{e}_1 \times \tilde{\vec{E}}_o e^{-\gamma \vec{e}_1 \cdot \vec{r}} = -s\mu Y \vec{e}_1 \times \tilde{\vec{E}}_o e^{-\gamma \vec{e}_1 \cdot \vec{r}}$$

$$= -s\mu \tilde{\vec{H}}(\vec{r}) \qquad (5.7)$$

The curl of the magnetic field is

$$\nabla \times \tilde{\vec{H}}(\vec{r}) = \nabla \times \left[Y\vec{e}_1 \times \tilde{\vec{E}}_o e^{-\gamma \vec{e}_1 \cdot \vec{r}}\right]$$

$$= Y\left\{\vec{e}_1 \nabla \cdot \left[\tilde{\vec{E}}_o e^{-\gamma \vec{e}_1 \cdot \vec{r}}\right] - [\vec{e}_1 \cdot \nabla]\tilde{\vec{E}}_o e^{-\gamma \vec{e}_1 \cdot \vec{r}}\right\}$$

$$= Y\left\{\vec{e}_1\left[\tilde{\vec{E}}_o \cdot \nabla e^{-\gamma \vec{e}_1 \cdot \vec{r}}\right] - \tilde{\vec{E}}_o\left[\vec{e}_1 \cdot \nabla e^{-\gamma \vec{e}_1 \cdot \vec{r}}\right]\right\}$$

$$= Y\gamma e^{-\gamma \vec{e}_1 \cdot \vec{r}}\left\{-[\vec{e}_1 \cdot \tilde{\vec{E}}_o]\vec{e}_1 + [\vec{e}_1 \cdot \vec{e}_1]\tilde{\vec{E}}_o\right\}$$

$$= [\sigma+s\varepsilon]\tilde{\vec{E}}_o e^{-\gamma\vec{e}_1\cdot\vec{r}} = [\sigma+s\varepsilon]\tilde{\vec{E}}(\vec{r}) \qquad (5.8)$$

provided

$$\vec{e}_1\cdot\tilde{\vec{E}}_o = 0 \;, \quad \vec{e}_1\cdot\vec{e}_1 = 1 \qquad (5.9)$$

The first of equations 5.9 is the restriction on $\tilde{\vec{E}}_o$ stated in equations 5.6 and the second of equations 5.9 is the requirement that $\vec{e}_1$ be a special kind of unit vector.

Note that $\vec{e}_1$ can be complex and the second of equations 5.9 can be written as

$$\begin{aligned} &\mathrm{Re}[\vec{e}_1]\cdot\mathrm{Re}[\vec{e}_1] - \mathrm{Im}[\vec{e}_1]\cdot\mathrm{Im}[\vec{e}_1] = 1 \\ &\mathrm{Re}[\vec{e}_1]\cdot\mathrm{Im}[\vec{e}_1] = 0 \\ &\vec{e}_1 = \mathrm{Re}[\vec{e}_1] + i\mathrm{Im}[\vec{e}_1] \end{aligned} \qquad (5.10)$$

Complex $\vec{e}_1$ can be used to describe various types of plane waves which are often not considered as plane waves. An example is a surface bound wave near an impedance plane.

Other convenient terms are found by taking the components of the propagation vector along the three cartesian axes (x, y, z) giving

$$\begin{aligned} \gamma_x &\equiv iK_x = \vec{\gamma}\cdot\vec{e}_x = i\vec{K}\cdot\vec{e}_x \\ \gamma_y &\equiv iK_y = \vec{\gamma}\cdot\vec{e}_y = i\vec{K}\cdot\vec{e}_y \\ \gamma_z &\equiv iK_z = \vec{\gamma}\cdot\vec{e}_z = i\vec{K}\cdot\vec{e}_z \end{aligned} \qquad (5.11)$$

With this breakout of the components of the propagation vector we can write

$$e^{-\vec{\gamma}\cdot\vec{r}} = e^{-i\vec{K}\cdot\vec{r}} = e^{-\gamma_x x} e^{-\gamma_y y} e^{-\gamma_z z} = e^{-iK_x x} e^{-iK_y y} e^{-iK_z z} \tag{5.12}$$

Since $\tilde{\vec{E}}_o$ and $\vec{e}_1$ do not depend on the coordinates then the only coordinate dependence is contained in the exponential factor. As shown in equation 5.12 this term factors into separate terms containing the three cartesian coordinates separately so our plane wave fits the classical separation of variables approach.

Again let the symmetry plane P be the plane z = 0 so that we have the position vector and its image

$$\begin{aligned} \vec{r} &= (x,y,z) = x\vec{e}_x + y\vec{e}_y + z\vec{e}_z \\ \vec{r}_m &= (x,y,-z) = x\vec{e}_x + y\vec{e}_y - z\vec{e}_z \end{aligned} \tag{5.13}$$

Corresponding to the original plane wave in equations 5.1 or 5.6 we can form a mirror plane wave by using equations 3.7 to give

$$\begin{aligned} \tilde{\vec{E}}_m(\vec{r}) &= \vec{\vec{R}}\cdot\tilde{\vec{E}}(\vec{r}_m) = \vec{\vec{R}}\cdot\tilde{\vec{E}}_o e^{-\gamma\vec{e}_1\cdot\vec{r}_m} \\ \tilde{\vec{H}}_m(\vec{r}) &= -\vec{\vec{R}}\cdot\tilde{\vec{H}}(\vec{r}_m) = -Y\vec{\vec{R}}\cdot[\vec{e}_1\times\tilde{\vec{E}}_o]e^{-\gamma\vec{e}_1\cdot\vec{r}_m} \end{aligned} \tag{5.14}$$

Note that

$$\vec{\vec{R}}\cdot[\vec{e}_1\times\tilde{\vec{E}}_o] = \vec{\vec{R}}\cdot\begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ e_{1_x} & e_{1_y} & e_{1_z} \\ \tilde{E}_{o_x} & \tilde{E}_{o_y} & \tilde{E}_{o_z} \end{vmatrix} = -\begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ e_{1_x} & e_{1_y} & -e_{1_z} \\ \tilde{E}_{o_x} & \tilde{E}_{o_y} & -\tilde{E}_{o_z} \end{vmatrix}$$

$$= -[\vec{\vec{R}}\cdot\vec{e}_1]\times[\vec{\vec{R}}\cdot\tilde{\vec{E}}_o] \tag{5.15}$$

For convenience one could define

$$\tilde{\vec{H}}_o \equiv Y\vec{e}_1 \times \tilde{\vec{E}}_o \tag{5.16}$$

so that

$$\tilde{\vec{H}}(\vec{r}) = \tilde{\vec{H}}_o e^{-\gamma \vec{e}_1 \cdot \vec{r}}$$

$$\tilde{\vec{H}}_m(\vec{r}) = -\vec{\vec{R}} \cdot \tilde{\vec{H}}_o e^{-\gamma \vec{e}_1 \cdot \vec{r}_m} \tag{5.17}$$

Having the mirror fields equations 3.11 can be used to find the symmetric parts as

$$\tilde{\vec{E}}_{sy}(\vec{r}) = \frac{1}{2}[\tilde{\vec{E}}(\vec{r}) + \tilde{\vec{E}}_m(\vec{r})] = \frac{1}{2} e^{-[\gamma_x x + \gamma_y y]} \left\{ \tilde{\vec{E}}_o e^{-\gamma_z z} + \vec{\vec{R}} \cdot \tilde{\vec{E}}_o e^{\gamma_z z} \right\}$$

$$= e^{-[\gamma_x x + \gamma_y y]} \left\{ \tilde{E}_{o_x} \cosh(\gamma_z z) \vec{e}_x + \tilde{E}_{o_y} \cosh(\gamma_z z) \vec{e}_y - \tilde{E}_{o_z} \sinh(\gamma_z z) \vec{e}_z \right\}$$

$$\tilde{\vec{H}}_{sy}(\vec{r}) = \frac{1}{2}[\tilde{\vec{H}}(\vec{r}) + \tilde{\vec{H}}_m(\vec{r})] = \frac{1}{2} e^{-[\gamma_x x + \gamma_y y]} \left\{ \tilde{\vec{H}}_o e^{-\gamma_z z} - \vec{\vec{R}} \cdot \tilde{\vec{H}}_o e^{\gamma_z z} \right\} \tag{5.18}$$

$$= e^{-[\gamma_x x + \gamma_y y]} \left\{ -\tilde{H}_{o_x} \sinh(\gamma_z z) \vec{e}_x - \tilde{H}_{o_y} \sinh(\gamma_z z) \vec{e}_y + \tilde{H}_{o_z} \cosh(\gamma_z z) \vec{e}_z \right\}$$

Similarly from equations 3.12 the antisymmetric parts are

$$\tilde{\vec{E}}_{as}(\vec{r}) = \frac{1}{2}[\tilde{\vec{E}}(\vec{r}) - \tilde{\vec{E}}_m(\vec{r})] = \frac{1}{2} e^{-[\gamma_x x + \gamma_y y]} \left\{ \tilde{\vec{E}}_o e^{-\gamma_z z} - \vec{\vec{R}} \cdot \tilde{\vec{E}}_o e^{\gamma_z z} \right\}$$

$$= e^{-[\gamma_x x + \gamma_y y]} \left\{ -\tilde{E}_{o_x} \sinh(\gamma_z z) \vec{e}_x - \tilde{E}_{o_y} \sinh(\gamma_z z) \vec{e}_y + \tilde{E}_{o_z} \cosh(\gamma_z z) \vec{e}_z \right\}$$

$$\tilde{\vec{H}}_{as}(\vec{r}) = \frac{1}{2}[\tilde{\vec{H}}(\vec{r})-\tilde{\vec{H}}_m(\vec{r})] = \frac{1}{2}\, e^{-[\gamma_x x+\gamma_y y]}\left\{\tilde{\vec{H}}_o e^{-\gamma_z z}+\vec{\vec{R}}\cdot\tilde{\vec{H}}_o e^{\gamma_z z}\right\} \tag{5.19}$$

$$= e^{-[\gamma_x x+\gamma_y y]}\left\{\tilde{H}_{o_x}\cosh(\gamma_z z)\vec{e}_x+\tilde{H}_{o_y}\cosh(\gamma_z z)\vec{e}_y-\tilde{H}_{o_z}\sinh(\gamma_z z)\vec{e}_z\right\}$$

The $\cosh(\gamma_z z)$ and $\sinh(\gamma_z z)$ terms are even and odd respectively in z and contain the symmetry characteristics with respect to P consistent with equations 3.13 and 3.14.

Now that an incident plane wave (Laplace transformed) in a uniform isotropic medium has been split into symmetric and antisymmetric parts let us consider the special case of free space ($\varepsilon = \varepsilon_o$, $\mu = \mu_o$, $\sigma = 0$) in the time domain. Also constrain $\vec{e}_1$ to be a real unit vector independent of s (or frequency). The propagation constant is then

$$\gamma = iK = \frac{s}{c} \tag{5.20}$$

Then equations 5.6 become in the time domain

$$\begin{aligned}
\vec{E}(\vec{r},t) &= \vec{E}_o\left(t-\frac{\vec{e}_1\cdot\vec{r}}{c}\right) \\
\vec{H}(\vec{r},t) &= \vec{H}_o\left(t-\frac{\vec{e}_1\cdot\vec{r}}{c}\right) \\
\vec{H}_o(t) &= Y_o\vec{e}_1\times\vec{E}_o(t) \\
\vec{e}_1\cdot\vec{E}_o(t) &= 0 \\
Y_o &= \sqrt{\frac{\varepsilon_o}{\mu_o}} \\
c &= \frac{1}{\sqrt{\mu_o\varepsilon_o}}
\end{aligned} \tag{5.21}$$

The mirror quantities in the time domain are then

$$\vec{E}_m(\vec{r},t) = \vec{\vec{R}}\cdot\vec{E}_o\left(t-\frac{\vec{e}_1\cdot\vec{r}_m}{c}\right)$$

$$\vec{H}_m(\vec{r},t) = -\vec{\vec{R}}\cdot\vec{H}_o\left(t-\frac{\vec{e}_1\cdot\vec{r}_m}{c}\right) \qquad (5.22)$$

For convenience we can define

$$\vec{e}_{1_m} \equiv \vec{\vec{R}}\cdot\vec{e}_1$$

$$\vec{E}_{o_m}(t) = \vec{\vec{R}}\cdot\vec{E}_o(t) \qquad (5.23)$$

$$\vec{H}_{o_m}(t) = -\vec{\vec{R}}\cdot\vec{H}_o(t)$$

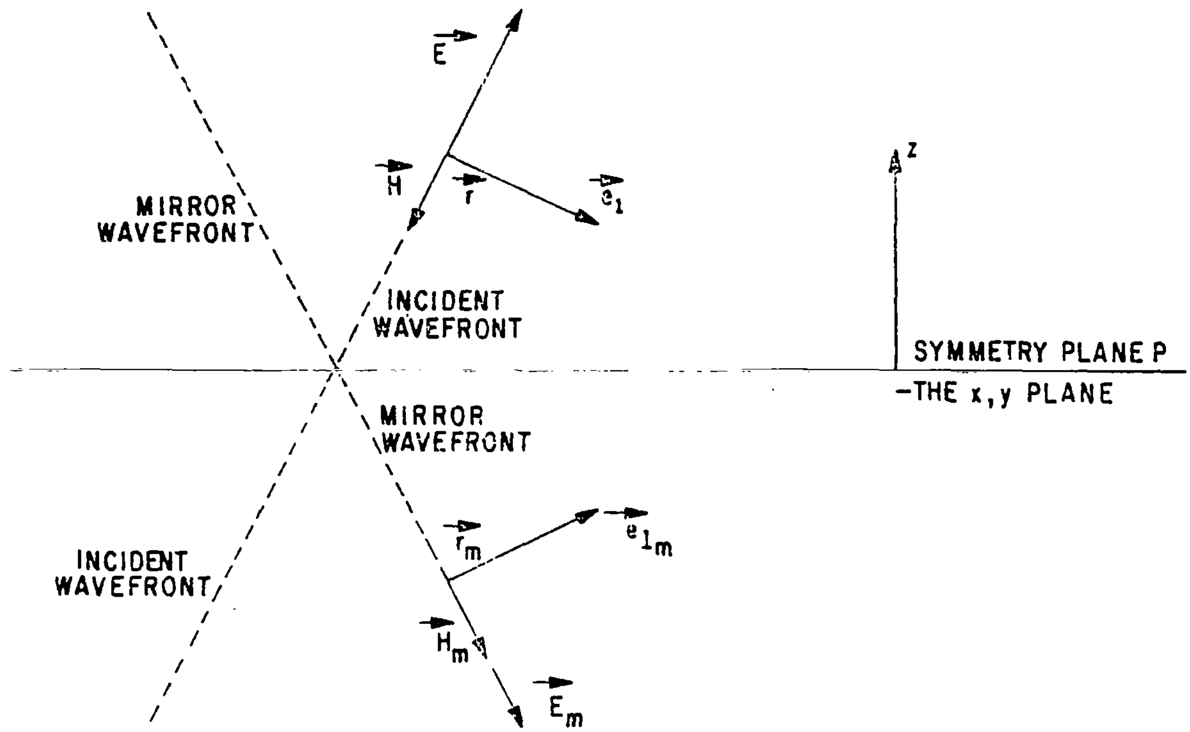
so that

$$\vec{H}_{o_m}(t) = -Y_o\vec{\vec{R}}\cdot[\vec{e}_1\times\vec{E}_o(t)] = Y_o[\vec{\vec{R}}\cdot\vec{e}_1]\times[\vec{\vec{R}}\cdot\vec{E}_o(t)]$$

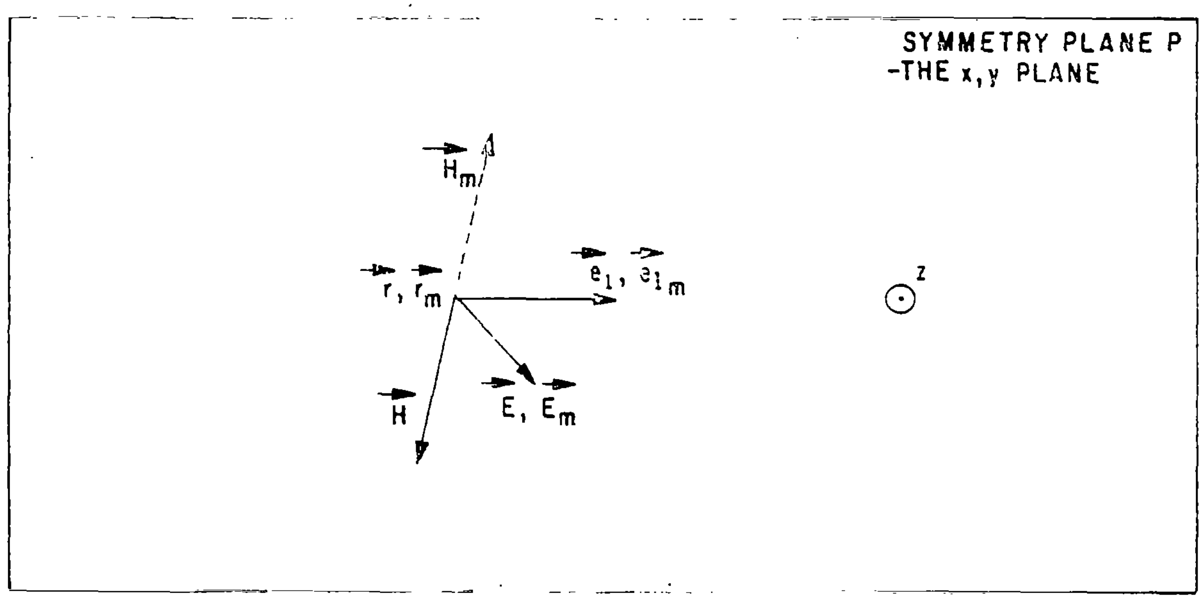
$$= Y_o\vec{e}_{1_m}\times\vec{E}_{o_m}(t) \qquad (5.24)$$

which can be compared directly to the corresponding equation in equations 5.21. The original fields and the mirror fields are illustrated for this uniform plane wave in figure 3.

The symmetric parts of this uniform plane wave in the time domain are



A. VIEW PARALLEL TO THE SYMMETRY PLANE P AND PERPENDICULAR TO THE DIRECTION OF PROPAGATION $\vec{e}_1$



B. VIEW PERPENDICULAR TO THE SYMMETRY PLANE P FROM POSITIVE z

FIGURE 3. SYMMETRY DECOMPOSITION OF AN INCIDENT PLANE WAVE WITH FIXED REAL PROPAGATION DIRECTION $\vec{e}_1$

$$\vec{E}_{sy}(\vec{r},t) = \frac{1}{2}[\vec{E}(\vec{r},t)+\vec{E}_m(\vec{r},t)]$$

$$= \frac{1}{2}\left[\vec{E}_o\left(t-\frac{\vec{e}_1\cdot\vec{r}}{c}\right)+\vec{E}_{o_m}\left(t-\frac{\vec{e}_{1_m}\cdot\vec{r}}{c}\right)\right]$$

(5.25)

$$\vec{H}_{sy}(\vec{r},t) = \frac{1}{2}[\vec{H}(\vec{r},t)+\vec{H}_m(\vec{r},t)]$$

$$= \frac{1}{2}\left[\vec{H}_o\left(t-\frac{\vec{e}_1\cdot\vec{r}}{c}\right)+\vec{H}_{o_m}\left(t-\frac{\vec{e}_{1_m}\cdot\vec{r}}{c}\right)\right]$$

The antisymmetric parts are

$$\vec{E}_{as}(\vec{r},t) = \frac{1}{2}[\vec{E}(\vec{r},t)-\vec{E}_m(\vec{r},t)]$$

$$= \frac{1}{2}\left[\vec{E}_o\left(t-\frac{\vec{e}_1\cdot\vec{r}}{c}\right)-\vec{E}_{o_m}\left(t-\frac{\vec{e}_{1_m}\cdot\vec{r}}{c}\right)\right]$$

(5.26)

$$\vec{H}_{as}(\vec{r},t) = \frac{1}{2}[\vec{H}(\vec{r},t)-\vec{H}_m(\vec{r},t)]$$

$$= \frac{1}{2}\left[\vec{H}_o\left(t-\frac{\vec{e}_1\cdot\vec{r}}{c}\right)-\vec{H}_{o_m}\left(t-\frac{\vec{e}_{1_m}\cdot\vec{r}}{c}\right)\right]$$

Note that for the case of free space and real $\vec{e}_1$ the frequency domain form with $s = i\omega$ shows uniform field amplitudes with x and y but a sinusoidal variation of the field amplitudes with z for both symmetric and antisymmetric parts. The time domain parts as in equations 5.25 and 5.26 can be thought of as two interesting pulsed waves with planar wave fronts as are quite distinct in the case of a step function type of $\vec{E}_o$ with time independent polarization.

Having the symmetric and antisymmetric parts of an incident plane wave then for an object with symmetry plane P one can consider the symmetric and antisymmetric parts separately. Taking one of these parts only half of the object need be considered because the fields, etc. on one half are directly applicable to the other half. This fact can be used for reducing computational difficulty and for increasing numerical accuracy. Furthermore various of the electromagnetic features such as resonant modes will each be associated with one or the other of the symmetric and antisymmetric parts. This will tend to give a separation of the electromagnetic features into different parts which can be used to reduce the complexity of understanding the electromagnetic response of the object.

VI. Use of Symmetry-Plane Decomposition in Measurements of Electromagnetic Interaction with an Object

The decomposition of fields, currents, etc. into symmetric and antisymmetric parts based on an electromagnetic symmetry plane P has another use besides simplifying some of the calculation and understanding of electromagnetic interaction with a body which has such a symmetry plane. In particular this decomposition can be used as an experimental technique. Since various features of the electromagnetic interaction are associated with one of the two parts (symmetric or antisymmetric), then if one measures the symmetric and antisymmetric parts of some electromagnetic quantity (such as current, charge, etc.) one can have some separation of the various interaction modes such as the different resonances directly in the data. This could prove to be useful in making the data more easily understandable.

In order to measure the symmetric and antisymmetric parts of an electromagnetic quantity of interest one must measure the quantity at $\vec{r}$ and the mirror quantity at $\vec{r}_m$ and then take the sum and difference to find the symmetric and antisymmetric parts respectively. This can be done as part of a data analysis effort by mathematically combining the results from $\vec{r}$ and $\vec{r}_m$; for such data reduction one must know the time relation between the two time domain experimental waveforms, or the phase relation if frequency-domain techniques are used. Alternatively one could directly sum and difference signals from the two measurement positions and display and/or record the symmetric and antisymmetric parts. How this is done depends on the object on which the electromagnetic interaction is being measured. If the object of interest is highly conducting (at least in part) such that there exists a highly conducting current path (idealized as perfectly conducting) between the $\vec{r}$ and $\vec{r}_m$ pair of interest, then this conducting path can be utilized to "hide" conducting signal cables from $\vec{r}$ and $\vec{r}_m$ symmetrically laid to arrive at a common comparison position on the symmetry plane P. This could be a rather convenient technique involving symmetrically positioned sensors and cables leading to a centrally placed recording system such as an oscilloscope (with shielded enclosure, power supply, etc.) where summing and differencing is also performed. For some practical objects of interest (such as metallic aircraft) conducting paths for routing signal cables from the sensor to a position on the symmetry plane and on the metallic conductors of the body are commonplace giving numerous cable routing geometries. In other cases one might telemeter the signals from $\vec{r}$ and $\vec{r}_m$ (via microwave techniques, optical techniques, etc.) to some remote position where the telemetry signals can be demodulated and combined to form the symmetric and antisymmetric parts.

For illustration let us briefly consider what might be a typical application of this experimental technique. In a previous section (equations 4.7 through 4.9) we have considered the symmetric and antisymmetric parts of two quantities useful for experimental purposes, namely the surface charge density and the surface current density. These two quantities are appropriate when considering interaction of electromagnetic fields with objects with large metal surfaces. The surface charge density on one side of the surface is equal to the displacement vector component $\vec{n} \cdot \vec{D}$ perpendicular to the surface. An electric type sensor on the surface then can measure the surface charge density. The surface current density on one side of the surface (for sufficiently large conductivity and thickness of the metal skin) is equal to the tangential magnetic field $\vec{n} \times \vec{H}$ on that side of the surface; the surface current density, magnetic field, and normal vector to the surface $\vec{n}$ are all mutually perpendicular. A magnetic type sensor on the surface can then be used to measure the surface current density. Of course a metallic skin such as on an aircraft might have charge and current on both sides of this skin (idealized as a surface). If one wishes the net surface current density and/or surface charge density (both sides) then appropriate sensors on both sides can be used to sum the results from both sides to obtain the total coulombs/meter^2 or amps/meter on (or in) the skin. Other related quantities such as total current through some surface (such as a total fuselage or wing current) or the total charge on some large area of the object could also be measured with other types of sensors.

For a pictorial example refer to figure 4. Here we show an aircraft. Note in figure 4B that such an aircraft might even be sitting on the earth (runway, taxiway, conducting ground plane, etc.); with the symmetry plane of the aircraft perpendicular to the earth surface (as well as perpendicular to any stratification layers in the earth near the aircraft) the symmetry-plane decomposition of the fields etc. still applies. The aircraft might also be connected to the earth ("grounded") provided such connection(s) were also symmetric with respect to P. Figure 4 shows an aircraft with surface current density sensors (perhaps using the magnetic field) located at symmetrical positions with symmetrical orientations on top of the wings. Signal transmission lines (cables) have their outer conductors in electrical contact with the metallic wings and fuselage so as to appear as small perturbations in the conducting wing and

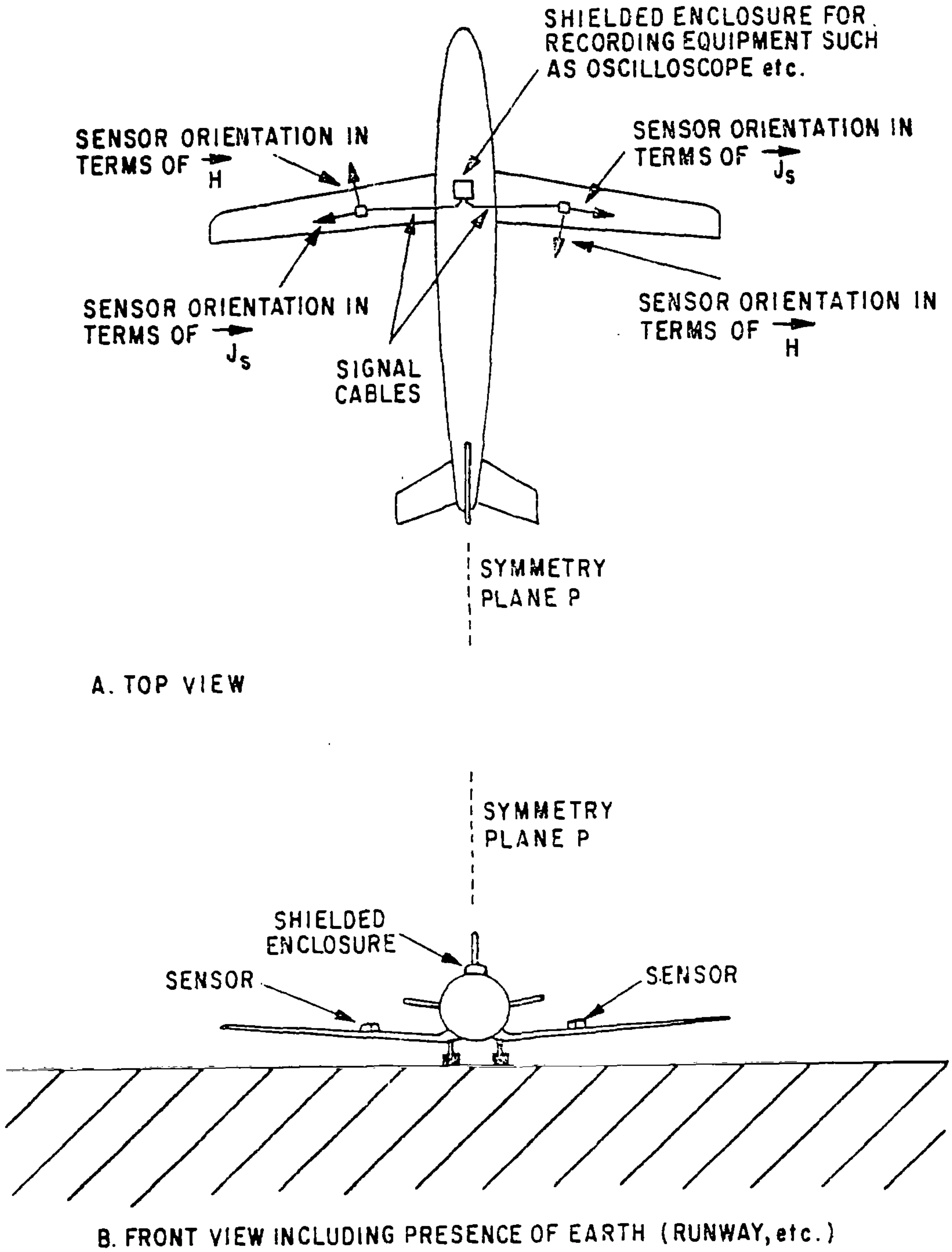


FIGURE 4. EXAMPLE OF THE MEASUREMENT OF SYMMETRIC AND ANTISYMMETRIC PARTS: SURFACE CURRENT DENSITY ON AIRPLANE WINGS

fuselage surfaces. These transmission lines are routed along symmetrical paths to a shielded oscilloscope enclosure centered on P; for example the shielded enclosure might be placed as a bulge on top of the fuselage as shown in figure 4. Alternatively the signal transmission lines might enter the fuselage at symmetrical positions with respect to P using special feed-through panels and from there go to the oscilloscope (shielded if needed) inside the fuselage.

The illustration in figure 4 is only typical. One might have surface charge density sensors near the wingtips, surface current density sensors on opposite sides of the fuselage or on opposite engines slung under the wings, etc. One might measure voltages (negative line integral of the electric field) between symmetrically positioned pairs of locations or measure total current on symmetrically located conducting structures. The possibilities are quite large.

VII. Symmetry Plane of Object Made Coincident with a Symmetry Plane of a Simulator

In measuring the interaction of electromagnetic fields with an object one often uses some kind of simulator to produce a desired field distribution in the vicinity of the object. Such a simulator might be simulating various aspects of the nuclear EMP in either time or frequency domain. For various reasons, such as the large size of the simulator structure or the desire to maximize field strengths, the simulator structure is not always far away from the object compared to the object dimensions. For typical simulators the fields scattered from the object can then interact with the simulator structure and be rescattered back to the object, thereby altering the currents etc. on the object. For a simulator not significantly larger than the object placed inside it then the fields in the absence of the object cannot be considered in an accurate sense as the incident fields. This is an important question in simulator design and has been considered in several notes.

In this note we are considering the splitting of the electromagnetic interaction into symmetric and antisymmetric parts. This splitting has required that the object be symmetric about a plane P. If the fields scattered from the object are rescattered back to the object to a significant extent by the simulator then it is necessary that the symmetry be preserved in this rescattering. This requires that the conductor positions, impedances, etc. associated with the simulator also be symmetric with respect to the same symmetry plane P as is the object. The fields, currents, etc. can be a combination of symmetric and antisymmetric parts so the simulator sources (generators) do not need to be symmetrically positioned if the associated source impedances and geometric structures are placed at the appropriate mirror positions. This symmetry requirement on the simulator then restricts somewhat the geometry of simulation tests if one wants the coupling between symmetric and antisymmetric parts on the object to be negligible. Alternatively one can make the simulator large enough compared to the object or far enough away from it that the rescattering of the fields from the simulator back to the object is negligible.

An interesting approach to simulation experiments on an object with a symmetry plane P would be to not only make the simulator have the same symmetry plane, but also make the sources be placed so as to give either a symmetric or an antisymmetric field distribution (but not both). This would give only the corresponding symmetric or antisymmetric current, charge, etc. on the object. Thus we have a way to experimentally separate the symmetric and antisymmetric parts by producing them separately. Compare this technique to the one discussed in section VI in which the two parts are separately measured in a situation in which both parts are present. Depending on the simulator design the production of only one of the two parts may restrict somewhat the flexibility (angle of incidence, polarization, etc.) in the variations of the field distributions that are achievable.

As a first example consider a symmetric object such as an airplane symmetrically positioned in a symmetrical parallel plate transmission line as shown in figure 5. The basic requirement is that the symmetry plane P of the airplane also be a symmetry plane of the simulator including the sources with their source impedances. Figure 5A shows a configuration which excites only the symmetric parts; the symmetry plane P is parallel to the incident electric field and divides each of the plates into two equal parts. Note that for this type of symmetrical excitation the object (airplane) can still be rotated to various orientations as long as the symmetry plane P is not moved. Figure 5A also shows a dielectric stand for the airplane also symmetrically centered on P. Figure 5B shows a configuration which excites only the antisymmetric part; the symmetry plane P is perpendicular to the incident electric field and is spaced halfway between two symmetrically positioned equal plates. Again the object (airplane) can be rotated as long as the symmetry plane P is not moved. Note that a dielectric stand (like a large railroad trestle) and even the presence of a conducting dielectric such as soil etc. can also be included as long as they are symmetric with respect to P. This general type of simulator has application to testing things like airplanes in their inflight configuration; some of the features of this kind of simulator are discussed in another note.[3] Here we wish to point out that this type of simulator

3. Capt Carl E. Baum, Sensor and Simulation Note 82, Some Considerations Concerning a Horizontally Polarized Transmission-Line Simulator, April 1969.

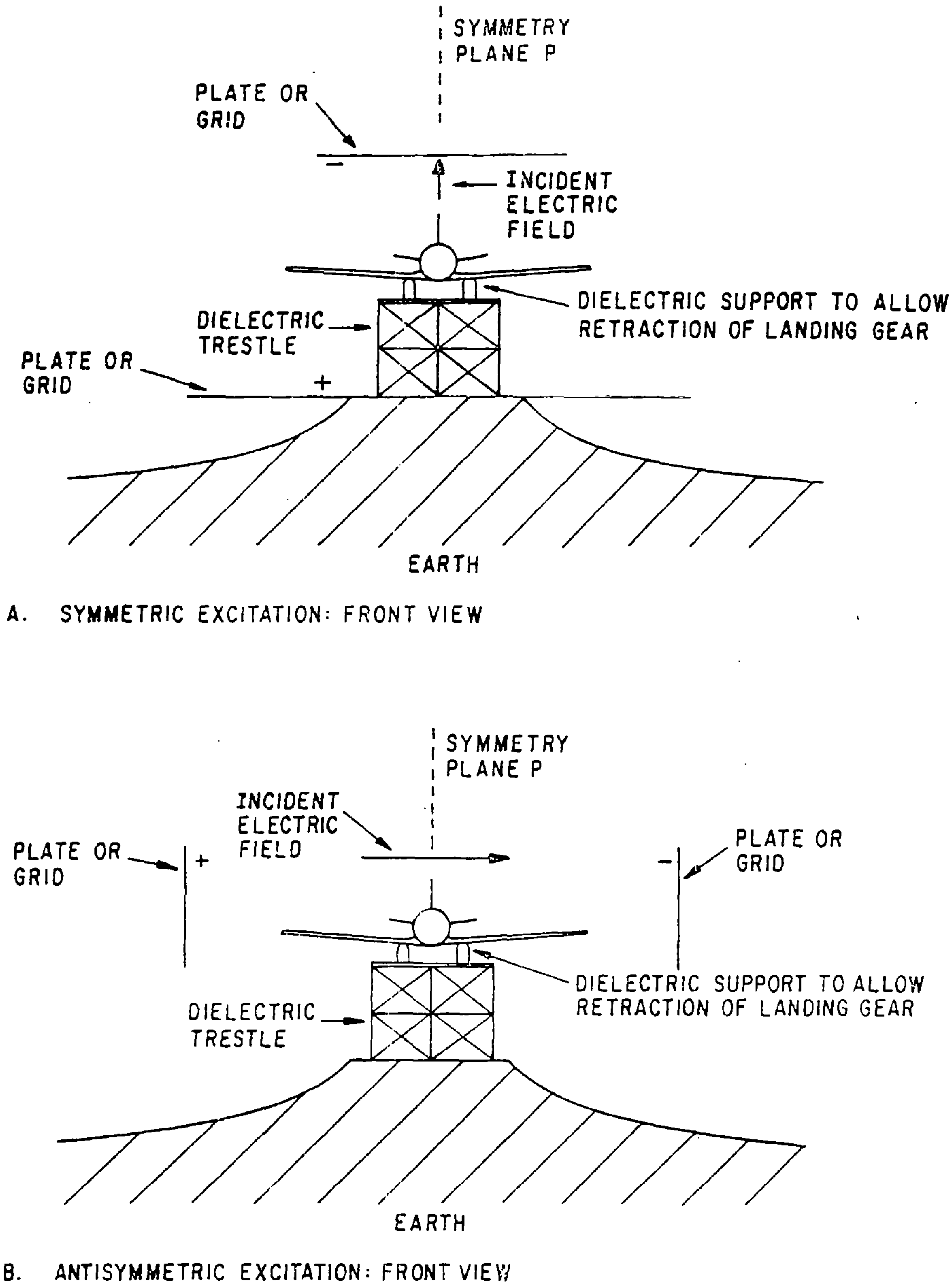
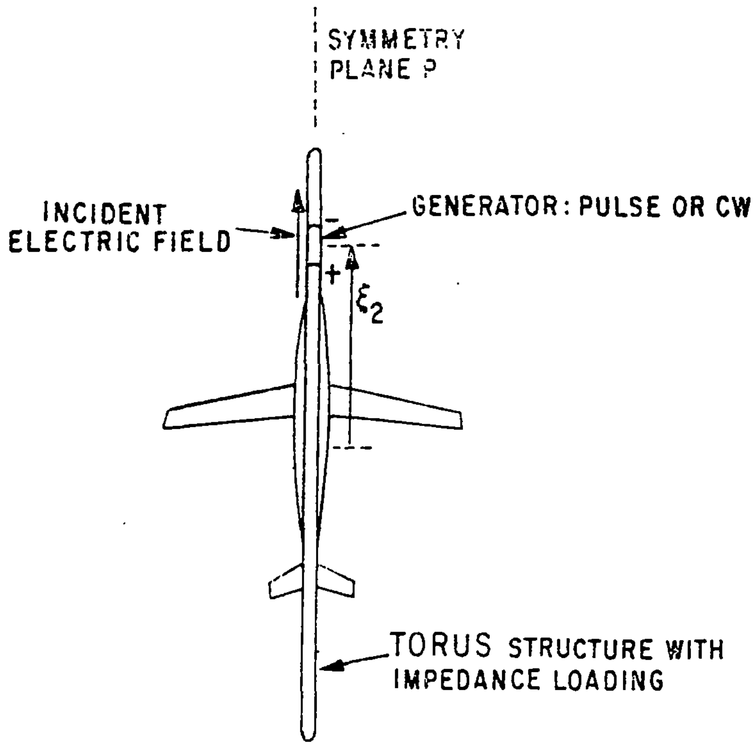


FIGURE 5. SYMMETRICAL OBJECT SUCH AS AN AIRPLANE SYMMETRICALLY PLACED IN A PARALLEL PLATE SIMULATOR

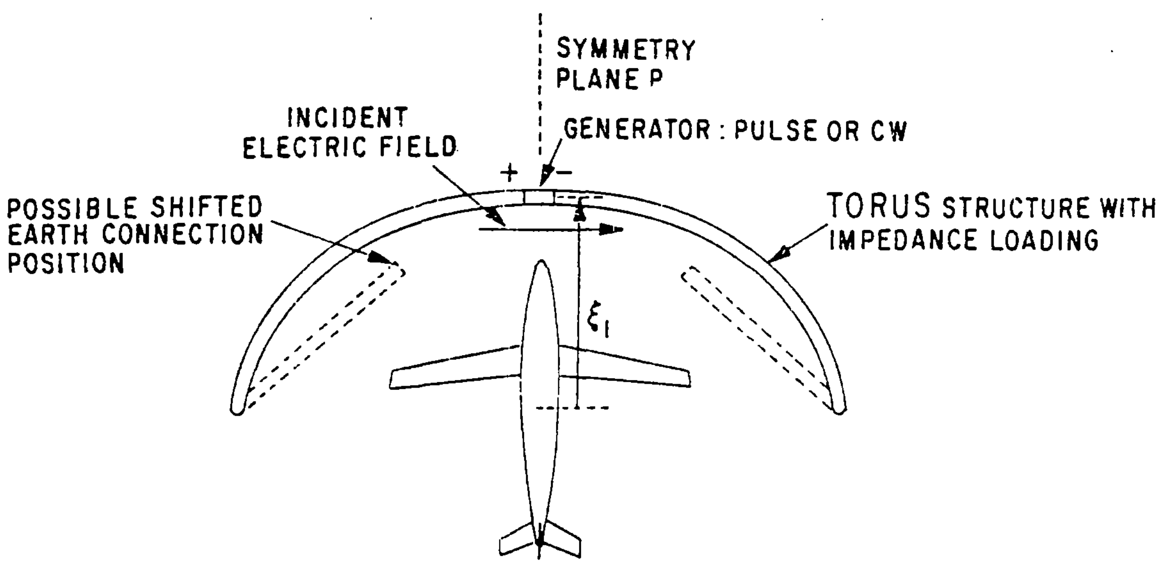
can also be used to excite only the antisymmetric part, at least for restricted directions of incidence. Note also that the same dielectric support (trestle) can be used for both symmetric and antisymmetric excitation by reconfiguring the conducting plates (or wire grids) that guide the fields into the two different geometries shown in figure 5. These options (and even other plate configurations with respect to the dielectric support) give a simulation facility of this type considerable flexibility.

The parallel plate or TEM transmission line type of simulators are applicable to testing objects like aircraft in their inflight mode. Another interesting class of simulators is the hybrids useful for testing objects near the ground with a simulated "high altitude" type of EMP as it would be incident onto the earth's surface. A particularly good type of hybrid is the TORUS[4]; it can give an essentially complete set of angles of incidence and polarization, good high and low frequency performance, and has a convenient geometry for electromagnetic calculations. With this type of simulator one might be testing say an aircraft on the ground surface (runway, etc.). Besides using various angles of incidence and polarization for more general tests one might configure the object and the TORUS so as to produce only one of the two parts: symmetric or antisymmetric. Figure 6A shows a configuration for producing only a symmetric part. The TORUS is in its vertical configuration and its major radius lies on the symmetry plane P, so for an airplane the fuselage and the TORUS structure can be roughly considered as lying in the same plane if the structures are thought of as lines. Note that the angle of the pulse generator along the half circle is arbitrary in producing the symmetric part and this gives some flexibility. However the direction of incidence is basically parallel to the plane P and the polarization of the incident electric field is parallel to P. Figure 6B shows a configuration for producing only an antisymmetric part. The "plane" of the TORUS is perpendicular to P and the pulse generator and TORUS structure are symmetrically placed to extend on both sides of P. By varying ξ_1 (the angle of bend of the TORUS structure from the vertical) the direction of incidence can be varied while still parallel to P (for fields near P). However the polarization of the electric field near P is restricted to be perpendicular to P. Thus for both of these TORUS configurations the direction of incidence can be varied, though only parallel to the vertical plane P near P, while the polarization near P is either parallel or perpendicular to P. Note for the antisymmetric case that one could move the generator away from P if a second identical generator were placed in the TORUS structure at the symmetrical position with respect to

4. Capt Carl E. Baum, Sensor and Simulation Note 94, Some Considerations Concerning a Simulator with the Geometry of a Half Toroid Joined to a Ground or Water Surface, November 1969.



A. SYMMETRIC EXCITATION WITH $\xi_1 = 0$: TOP VIEW



B. ANTISYMMETRIC EXCITATION WITH $\xi_2 = 0$: TOP VIEW

FIGURE 6. SYMMETRICAL OBJECT SUCH AS AN AIRPLANE SYMMETRICALLY PLACED IN A TORUS SIMULATOR

P and with polarity such that both generators drive current around the TORUS in the same direction. There are various other modifications of TORUS that one might use to get more general angles of incidence and polarization for each part (symmetric or antisymmetric) separately, such as by combining two such simulator structures, perhaps even intersecting. However this introduces various complications which make electromagnetic analysis much more difficult. Note that while we have shown the TORUS placed over an aircraft parked on the ground there are various other objects such as symmetrical buildings which have an approximate electromagnetic symmetry plane P and are of interest.

VIII. Summary

The presence of an approximate electromagnetic symmetry plane in an object of interest then has various implications regarding electromagnetic interaction with the object. The fields, currents, etc. all have corresponding mirror quantities which also satisfy Maxwell's equations and other associated equations (including boundary conditions); this is derived from a reflection of the fields etc. as well as the object through the electromagnetic symmetry plane P. Combining the original quantities with the mirror quantities in sum and difference fashion gives the symmetric and antisymmetric parts of the electromagnetic quantities. Each of these parts can be treated independently as there is no cross coupling between them. Furthermore each of these parts has a convenient set of symmetry relations on reflection through the symmetry plane P which simplifies somewhat the treatment of the separate parts. Since the two parts can be treated separately various features of the interaction of fields with such a symmetric object can be considered in the analysis of one of the two parts. The complexity of the electromagnetic interaction can then be reduced somewhat by removing from the analysis various features associated with the other part not considered in the calculation. For example the lowest frequency resonances can be associated with the different parts thereby perhaps reducing the number of these lowest frequency resonances to be considered with each part separately.

This symmetry-plane decomposition applies not only to calculating the electromagnetic interaction with a symmetrical object of interest but to measurements of this interaction as well. By use of symmetrically positioned sensors with summing and differencing of the signals from the sensors the symmetrical and antisymmetrical parts can be measured. Alternatively by appropriately configuring an EMP simulator so as to have the same symmetry plane as the object of interest and produce only a symmetric or an antisymmetric field distribution then only the corresponding part is excited on the object of interest. Depending on the simulator design, however, the excitation of only one part of the general field distribution may restrict somewhat the flexibility of the simulator in simulating all the features of a desired EMP (direction of incidence, polarization, etc.).

This symmetry decomposition of the electromagnetic quantities into symmetric and antisymmetric parts can prove rather useful for understanding the electromagnetic interaction with a complex but symmetrical object such as an aircraft. However such real objects are in general not perfectly symmetric with respect to a symmetry plane. Thus there can in general be some coupling between symmetric and antisymmetric parts, although this coupling can be quite small. One may have to quantify this coupling question for real not-quite-symmetric objects of interest.