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PROPAGATION CONSTANTS OF AN INSULATED CONDUCTOR BURIED IN A STRATIFIED MEDIUM: THE AIR-EARTH AND SOIL-ROCK INTERFACES

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CHAPTER 1

INTRODUCTION

The analysis of buried insulated conductors has most often been performed by considering a line source of alternating current immersed in an infinite, homogeneous environment[1]. Recently, the effects of the presence of the air-earth interface lying above the cable position were studied by analyzing the reflected electric and magnetic fields[2]. The present report extends the methods developed in Reference 2 to the case where the cable is buried below the air-earth interface and above a second interface formed by the presence of bedrock (the soil-rock interface). The cable geometry is as shown in Figure 1-1.

The electromagnetic fields in the neighborhood of this buried cable can be considered as arising from three components:

1) the primary fields due to excitation of the conductor,

2) the secondary fields due to reflections of the primary fields from the air-earth interface, and

3) the secondary fields due to reflections of the primary fields from the soil-rock interface.

Further, one should formally acknowledge the presence of additional components due to multiple reflections between the two interfaces, that is, there is some contribution from waves which are reflected from the air-earth interface and then reflected from the soil-rock interface, etc. However, the magnitude of these multiply reflected fields is dependent upon the square (and higher powers) of the reflection coefficients,

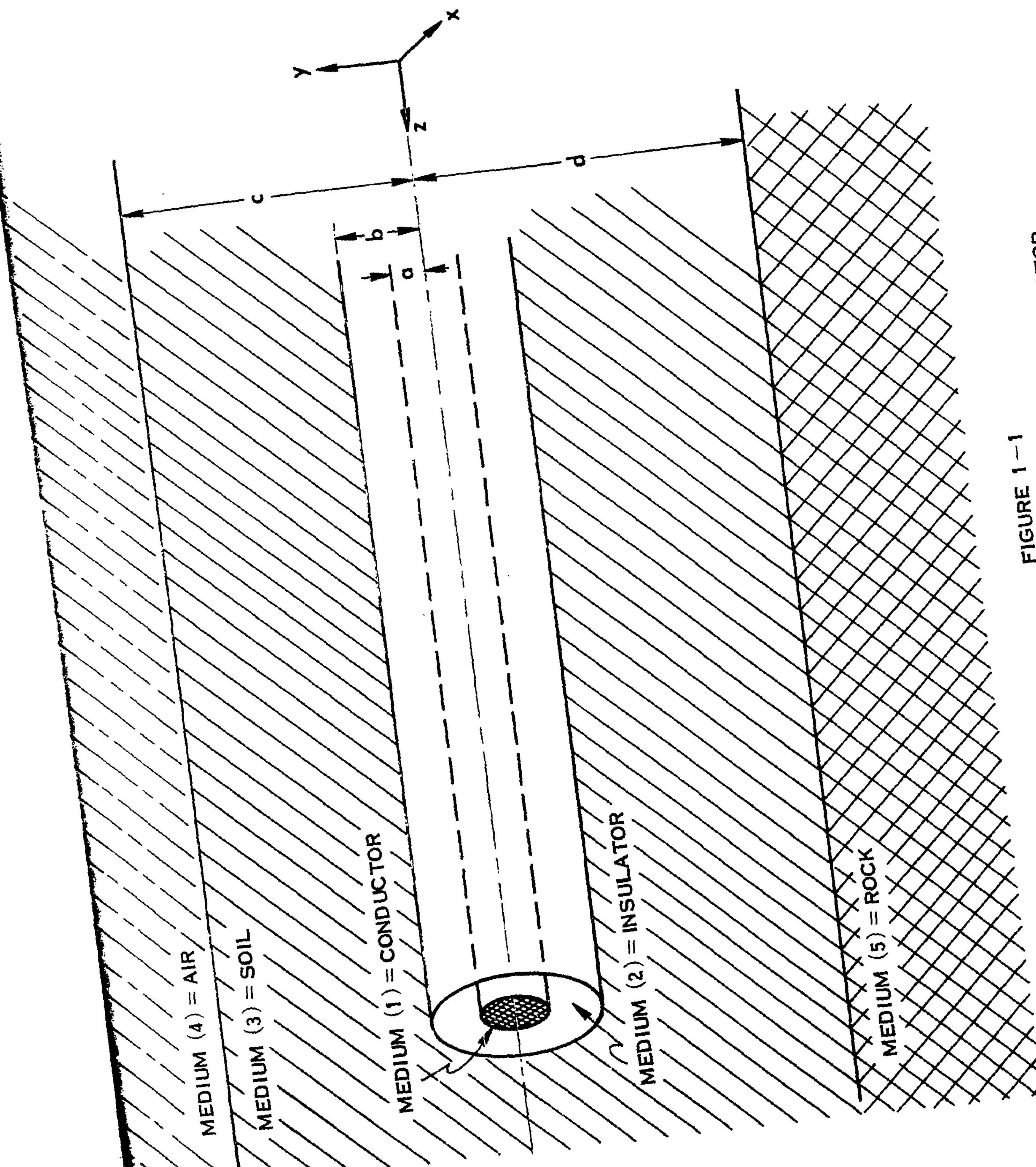
x
y
z
a
b
c
d
MEDIUM (4) = AIR
MEDIUM (3) = SOIL
MEDIUM (1) = CONDUCTOR
MEDIUM (2) = INSULATOR
MEDIUM (5) = ROCK

FIGURE 1-1

and since the magnitude of the singly reflected fields is rather small, the contribution of the multiply reflected fields is very small and will be neglected throughout this analysis.

There is a burial geometry, however, where multiple reflections cannot be neglected. If the cable is buried in the rock, with both the air-earth interface and the soil-rock interface above it, then the problem of reflections from the two interfaces is similar to the problem of optical reflection from a thin film and the present single reflection approach is inadequate. The general reflection theory developed in Reference 2 could be adapted to include these multiple reflections, however that project would be of sufficient magnitude to warrant a separate report.

The succeeding portions of this report are organized as follows: in Chapter 2, the reflections from the soil-rock interface are analyzed in the frequency range 10^{-2} Hz $\le f \le 10^{8}$ Hz for a variety of parameters, and in Chapter 3 the theory is applied to the reflections from both the air-earth interface and the soil-rock interface for several geometries.

CHAPTER 2

REFLECTIONS FROM THE SOIL-ROCK INTERFACE

It is assumed that the reader is familiar with both the notation and the content of Reference 2 which contains the development of the general theory of the reflection of inhomogeneous cylindrical waves from a plane interface separating isotropic, homogeneous (but otherwise arbitrary) conducting media.

By way of review, recall that the primary electric and magnetic fields of an infinitely long insulated conductor buried in an infinite earth can be written with axial position and time dependence $e^{ihz-i\omega t}$, where h, the axial propagation constant, is to be determined. Assuming fundamental transverse magnetic modes of cable excitation, the primary electric and magnetic fields in the conductor, insulator, and soil are written in terms of Bessel functions of order zero and order one (Equations (2-14) - (2-22) in Reference 2). Application of appropriate boundary conditions at the surfaces $r = a$ and $r = b$ leads to the following two simultaneous equations to be solved for the propagation constant h:

$$\frac{\mu_1\lambda_1}{k_1^2}\left[\frac{J_0(\lambda_1 a)}{J_1(\lambda_1 a)}\right] = \frac{\mu_2\lambda_2}{k_2^2}\left[\frac{\beta J_0(\lambda_2 a) + N_0(\lambda_2 a)}{\beta J_1(\lambda_2 a) + N_1(\lambda_2 a)}\right] \qquad (2\text{-}1)$$

and

$$\frac{\mu_2\lambda_2}{k_2^2}\left[\frac{\beta J_0(\lambda_2 b) + N_0(\lambda_2 b)}{\beta J_1(\lambda_2 b) + N_1(\lambda_2 b)}\right] = \frac{\mu_3\lambda_3}{k_3^2}\left[\frac{H_0^{(1)}(\lambda_3 b)}{H_1^{(1)}(\lambda_3 b)}\right] \qquad (2\text{-}2)$$

where

$$k_j^{\,2} = \mu_j \omega(\epsilon_j \omega + i\sigma_j)$$

$$\lambda_j^{\,2} = k_j^{\,2} - h^2 \ .$$

ϵ_j, μ_j, and σ_j are the permittivity, permeability, and conductivity of the jth medium, and β is an unknown coefficient which is to be eliminated.

Figure 2-1 shows the propagation constants obtained by solution of Equations (2-1) and (2-2) over the frequency range 10^{-2} Hz $\le f \le 10^{10}$ Hz for the geometrical and electrical parameters listed in Table 2-1. These are the propagation constants for the insulated conductor buried in an infinite earth, called h_∞.

The reflection theory developed in Reference 2 showed that the electric and magnetic fields reflected from the interface separating medium (3) and medium (j) are taken into account by adding an integral to the righthand side of Equation (2-2) which becomes

$$\frac{\mu_2\lambda_2}{k_2^{\,2}}\left[\frac{\beta J_0(\lambda_2 b) + N_0(\lambda_2 b)}{\beta J_1(\lambda_2 b) + N_1(\lambda_2 b)}\right] =$$

$$\frac{\mu_3\lambda_3}{k_3^{\,2}}\left[\frac{H_0^{(1)}(\lambda_3 b) + J_0(\lambda_3 b)\cdot F(k_3,k_j,h,D)/\lambda_3^{\,2}}{H_1^{(1)}(\lambda_3 b) + J_1(\lambda_3 b)\cdot F(k_3,k_j,h,D)/\lambda_3^{\,2}}\right] . \qquad (2\text{-}3)$$

TABLE 2-1

ELECTRICAL AND GEOMETRICAL PARAMETERS FOR THE BURIED INSULATED CONDUCTOR

MEDIUM	CONDUCTIVITY (mho/m)	PERMITTIVITY ($\epsilon_o = 8.854 \times 10^{-12}$ farad/m)	PERMEABILITY ($\mu_o = 4\pi \times 10^{-7}$ heavy/m)
(1) Copper	5.8×10^7	ϵ_o	μ_o
(2) Insulator	0	$4\epsilon_o$	μ_o
(3) Soil	10^{-2}	$4\epsilon_o$	μ_o

Radius of central conductor, a = 0.01794 m

Outer radius of insulator, b = 0.03588 m

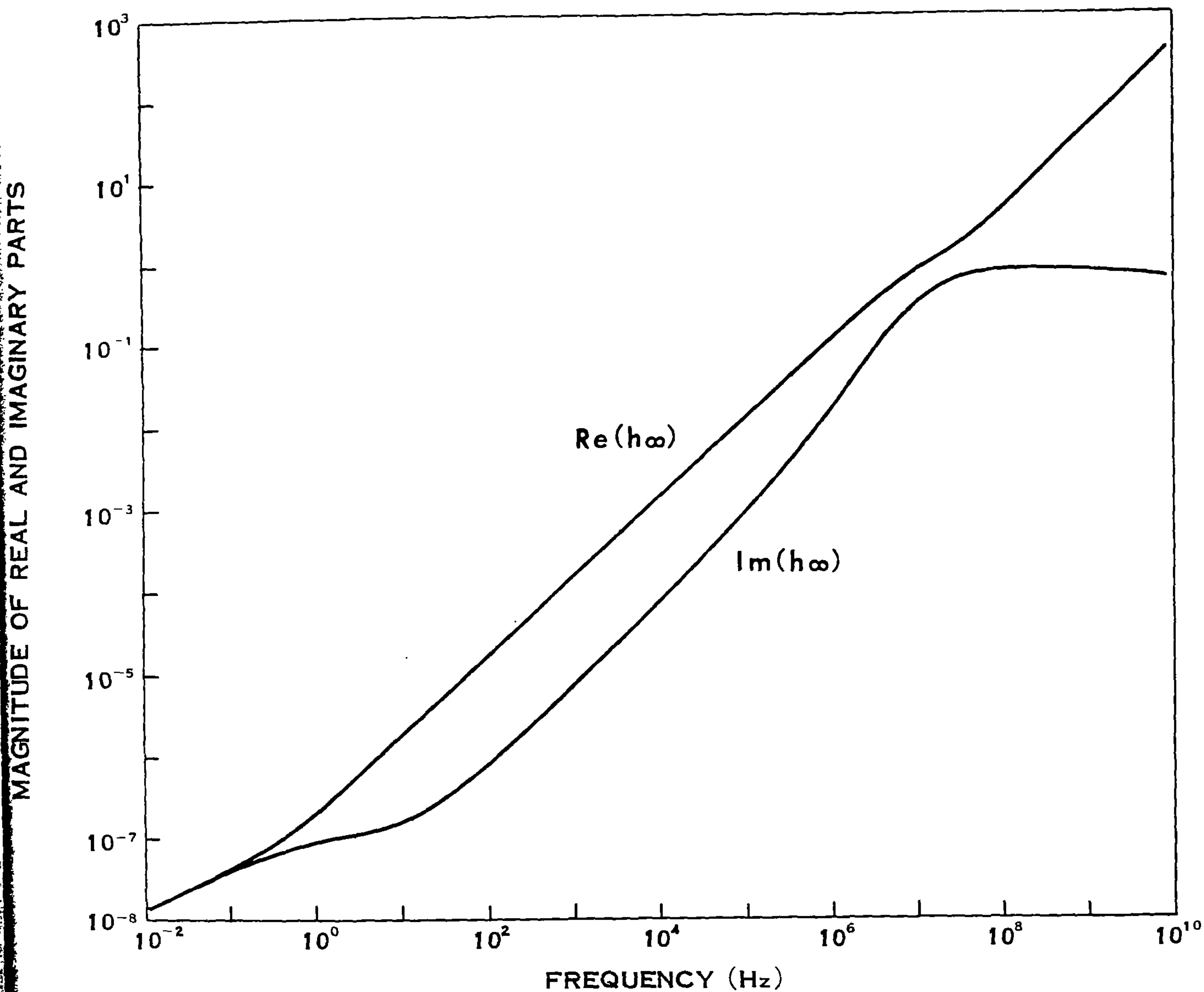

FIGURE 2-1

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Here D represents the distance between the cable and the interface, and the integral F is given by (see Equation (3-45) in Reference 2)

$$F(k_3, k_j, h, D) = \frac{1}{\pi}\int_{-\infty}^{\infty} e^{i2D\sqrt{k_3^2-h^2-\xi^2}} \left\{ k_3^2 \, \xi^2 \, f_{R_\perp} - h^2 (k_3^2-h^2-\xi^2)\, f_{R_\parallel} \right\} \frac{d\xi}{(\xi^2+h^2)\sqrt{k_3^2-h^2-\xi^2}} \; . \qquad (2\text{-}4)$$

The Fresnel coefficients are

$$f_{R_\perp} = \frac{\mu_j \sqrt{k_3^2-h^2-\xi^2} - \mu_3 \sqrt{k_j^2-h^2-\xi^2}}{\mu_j \sqrt{k_3^2-h^2-\xi^2} + \mu_3 \sqrt{k_j^2-h^2-\xi^2}} \qquad (2\text{-}5)$$

and

$$f_{R_\parallel} = \frac{\mu_3 \, k_j^2 \sqrt{k_3^2-h^2-\xi^2} - \mu_j \, k_3^2 \sqrt{k_j^2-h^2-\xi^2}}{\mu_3 \, k_j^2 \sqrt{k_3^2-h^2-\xi^2} + \mu_j \, k_3^2 \sqrt{k_j^2-h^2-\xi^2}} \; . \qquad (2\text{-}6)$$

Thus the propagation constant for a buried insulated conductor in the presence of a single plane interface may be determined by the simultaneous solution of Equations (2-1) and (2-3).

The propagation constant h has been determined from Equations (2-1) and (2-3) for several values of the cable-interface separation distance, and for a variety of different electrical properties of the rock. Figure 2-2 shows the fractional change in h, relative to h_∞, at a cable-interface separation distance of 1000 m (all parameters not shown on the figure are as listed in Table 2-1). Figure 2-3 includes similar results obtained at 100 m separation (also see

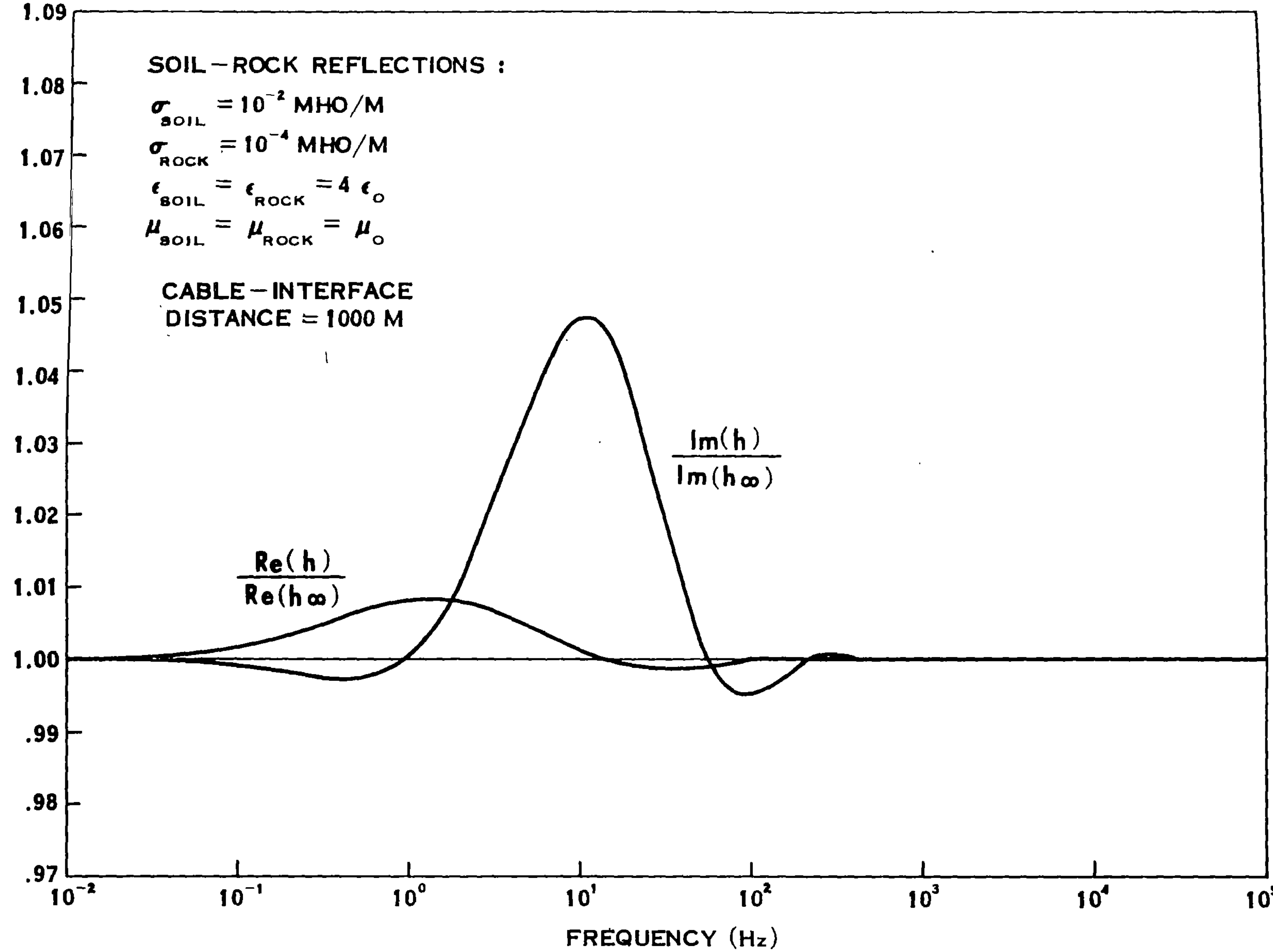

FIGURE 2–2

FRACTIONAL CHANGE IN THE PROPAGATION CONSTANT

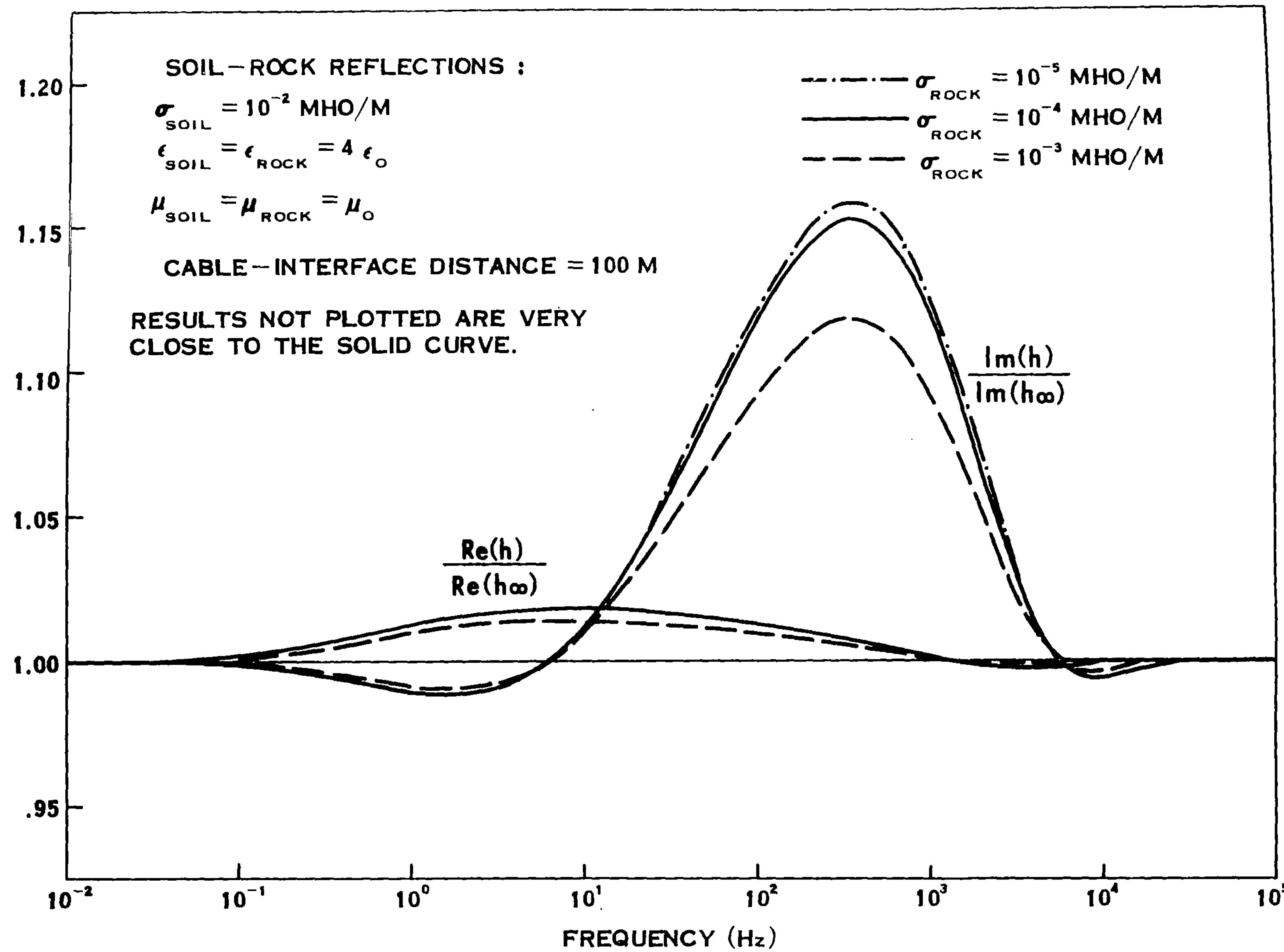

FIGURE 2–3

FRACTIONAL CHANGE IN THE PROPAGATION CONSTANT

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Figure 3-3 for 28 m separation). The form of these results is identical to the form of the results obtained by considering reflections from the air-earth interface (see Figure 4-7 through Figure 4-10 in Reference 2) namely:

a) the imaginary part of h is lowered by a few percent at low frequencies, and is dramatically increased near some characteristic frequency,

b) the real part of h is slightly increased at low frequencies,

c) at very low and very high frequencies no variation in h is observed, and

d) both the magnitude of the changes in the real and imaginary parts of h, and the characteristic frequencies at which these changes occur are reduced with increased cable-interface separation distance.

The change in the propagation constant h has been studied as a function of the electrical properties of the reflecting medium. Since it is generally acknowledged that bedrock is of lower conductivity than soil, Figure 2-3 presents the fractional change in h for rock conductivities of 10^{-3}, 10^{-4}, and 10^{-5} mho/m. As is to be expected, the reflection effects are enhanced as the rock conductivity is lowered, since this corresponds to greater and greater differences in the electrical properties of the two media. These variations in the rock conductivity affect only the magnitude of the changes in h, the shape and location of the effects are not substantially altered.

Choosing the somewhat arbitrary conductivity $\sigma_5 = 10^{-4}$ mho/m, changes in the propagation constant were studied as a function of the rock permittivity (at a cable-interface separation of 100 m). It was observed that for $\epsilon_5 = \epsilon_o$, $\epsilon_5 = 4\epsilon_o$, and $\epsilon_5 = 16\epsilon_o$, there were no appreciable differences in the calculated propagation constant. This lack of sensitivity is understandable since the rock permittivity enters the reflection coefficients only through the constant $k_5^2 = \mu_5\omega(\epsilon_5\omega + i\sigma_5)$. The first term in parenthesis for the frequencies where reflections are important ($f \leq 10^4$ Hz) is very much smaller than the second term, and small variations in the dielectric constant do not affect k_5 since the term containing ϵ_5 is negligible to begin with. Thus the propagation constant is more or less independent of the rock permittivity for low to moderate frequencies, as long as this medium has a reasonable conductivity.

CHAPTER 3

THE INSULATED CONDUCTOR BURIED BELOW AIR AND ABOVE ROCK

In the case where the insulated conductor is buried below the air-earth interface and above the soil-rock interface, the reflections from each interface must be separately evaluated. These reflections can be calculated independently through the integral F to arrive at the total electric and magnetic fields near the cable. In the notation of Figure 1-1 and Chapter 2, the propagation constant h is determined by simultaneous solution of the following two equations:

$$\frac{\mu_1\lambda_1}{k_1^2}\left[\frac{J_0(\lambda_1 a)}{J_1(\lambda_1 a)}\right] = \frac{\mu_2\lambda_2}{k_2^2}\left[\frac{\beta J_0(\lambda_2 a) + N_0(\lambda_2 a)}{\beta J_1(\lambda_2 a) + N_1(\lambda_2 a)}\right] \qquad (2\text{-}1)$$

and

$$\frac{\mu_2\lambda_2}{k_2^2}\left[\frac{\beta J_0(\lambda_2 b) + N_0(\lambda_2 b)}{\beta J_1(\lambda_2 b) + N_1(\lambda_2 b)}\right]$$

$$= \frac{\mu_3\lambda_3}{k_3^2}\left[\frac{H_0^{(1)}(\lambda_3 b) + J_0(\lambda_3 b)\left\{F(k_3,k_4,h,c) + F(k_3,k_5,h,d)\right\}/\lambda_3^2}{H_1^{(1)}(\lambda_3 b) + J_1(\lambda_3 b)\left\{F(k_3,k_4,h,c) + F(k_3,k_5,h,d)\right\}/\lambda_3^2}\right] \qquad (3\text{-}1)$$

The propagation constant h has been determined from Equations (2-1) and (3-1) for three cable positions between air and rock separated by 30 m (c+d = 30 m). Figure 3-1 shows the results, plotted as the fractional changes in the real and imaginary parts of h relative to h_∞, for the geometry of

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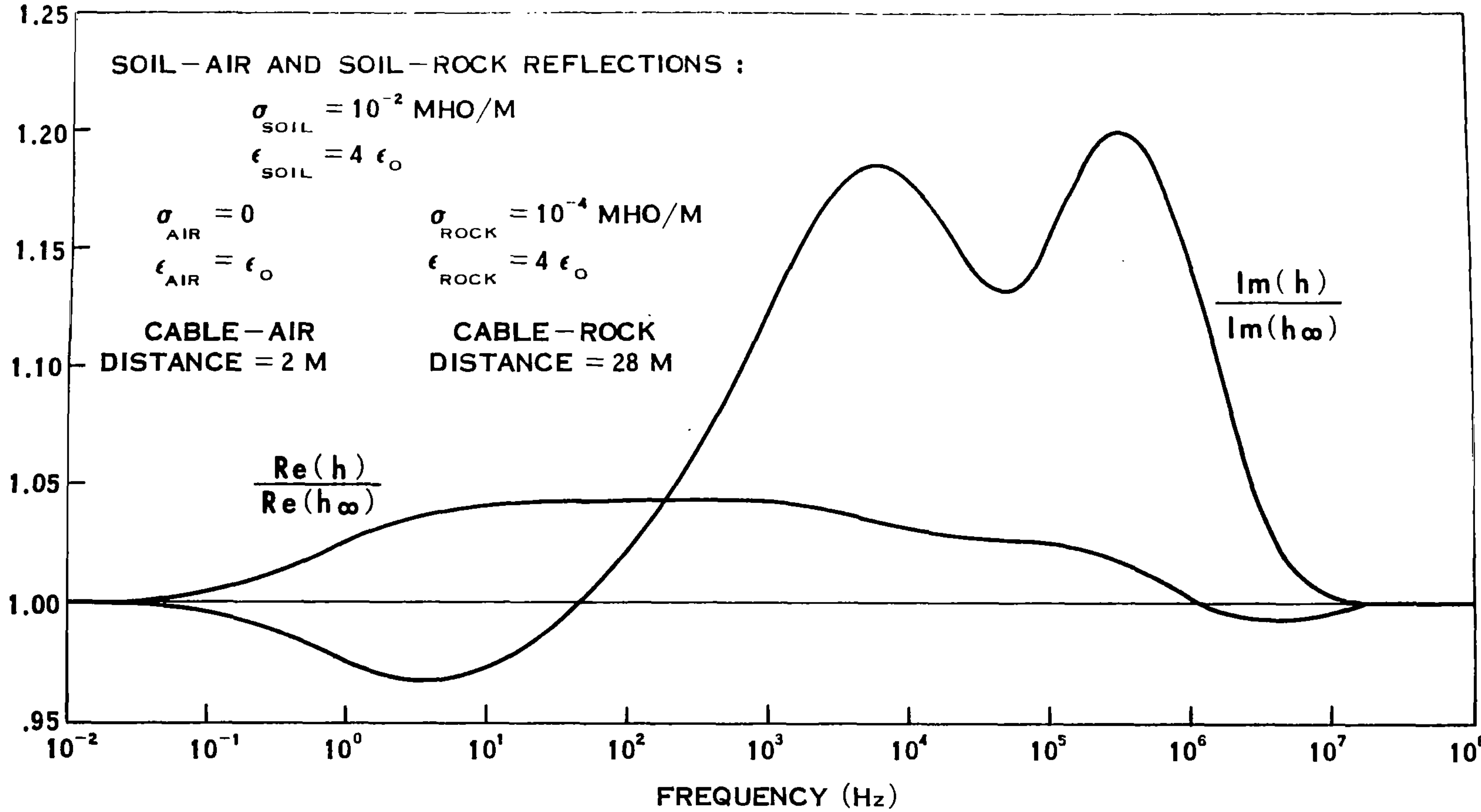

FIGURE 3-1

FRACTIONAL CHANGE IN THE PROPAGATION CONSTANT

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cable-air distance c = 2 m and cable-rock distance d = 28 m. All permeabilities are taken as μ_o, and any other parameter not shown on the figure is as listed in Table 2-1. In order to clarify these results, Figure 3-2 shows the effects due to reflections only from the air-earth interface, and Figure 3-3 shows the results due to the presence of the soil-rock interface alone. The total result of Figure 3-1 is seen to arise from a simple addition of these two independent effects with apparently no interference. The lack of interference between the two terms is to be expected since at the frequencies where reflections are important neither interface is removed from the cable by more than a small fraction of a wavelength. The interference effects that must be present at very high frequencies are hidden since the reflected fields there are too small to significantly affect the propagation constant.

Figure 3-4 shows the fractional change in the propagation constant for the geometry where the cable is 10 m below the air and 20 m above the rock. The results for the case where the cable is located 20 m below the air and 10 m above the rock are virtually identical to these and have not been included as a separate figure. As in the previous example, the net effect of the two reflections is just the algebraic sum of the two individual effects. In the present case however, each reflection gives rise to an enhancement of the imaginary part of h at nearly the same frequency, yielding a resultant curve with a single broad peak rather than the two distinct peaks previously observed.

In summary, the reflection theory developed in Reference 2 has been applied to the determination of the propagation constant of an insulated conductor buried in soil below the air-earth interface and above the soil-rock interface. The change in the

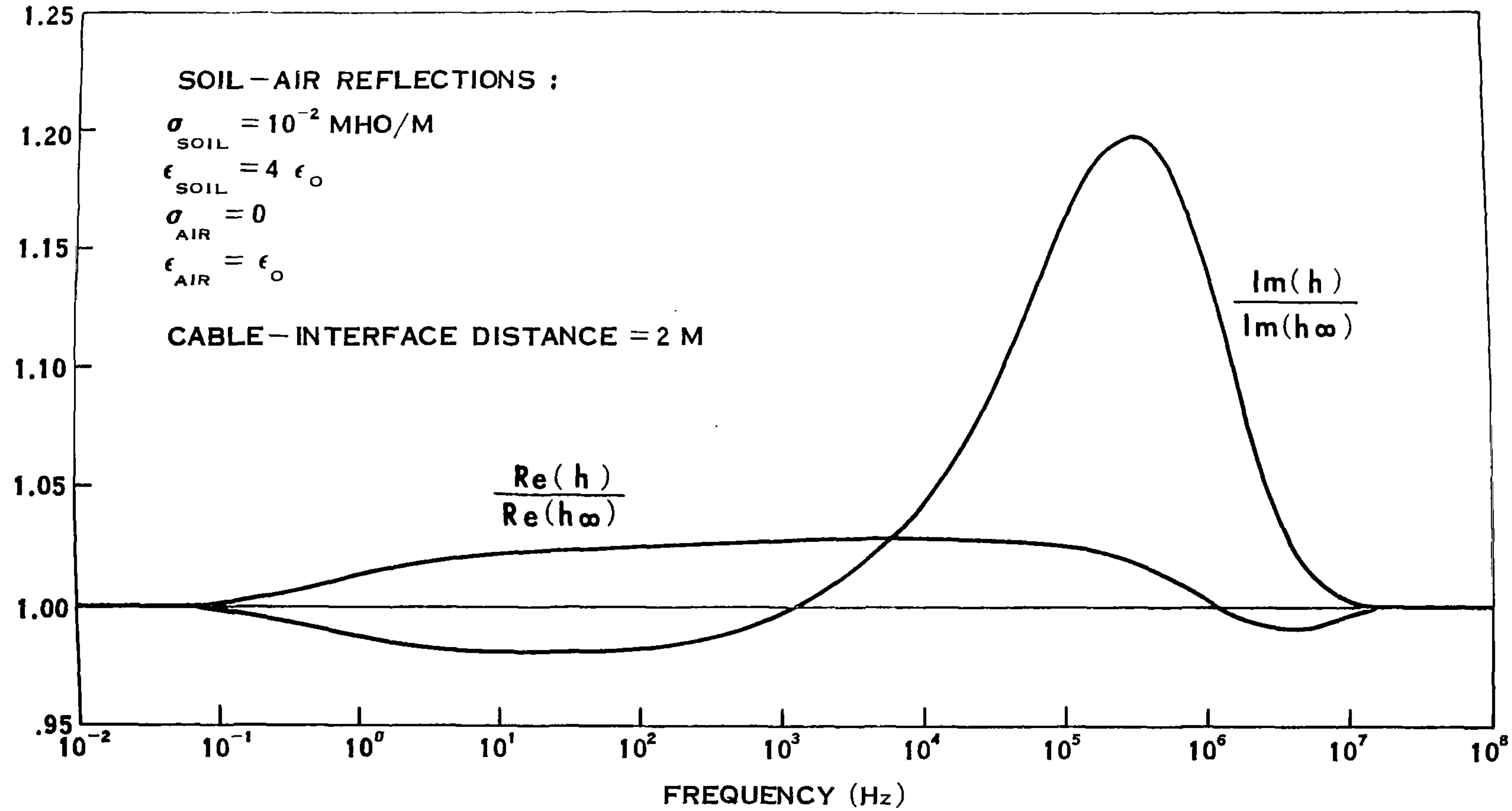

FIGURE 3 – 2

FRACTIONAL CHANGE IN THE PROPAGATION CONSTANT

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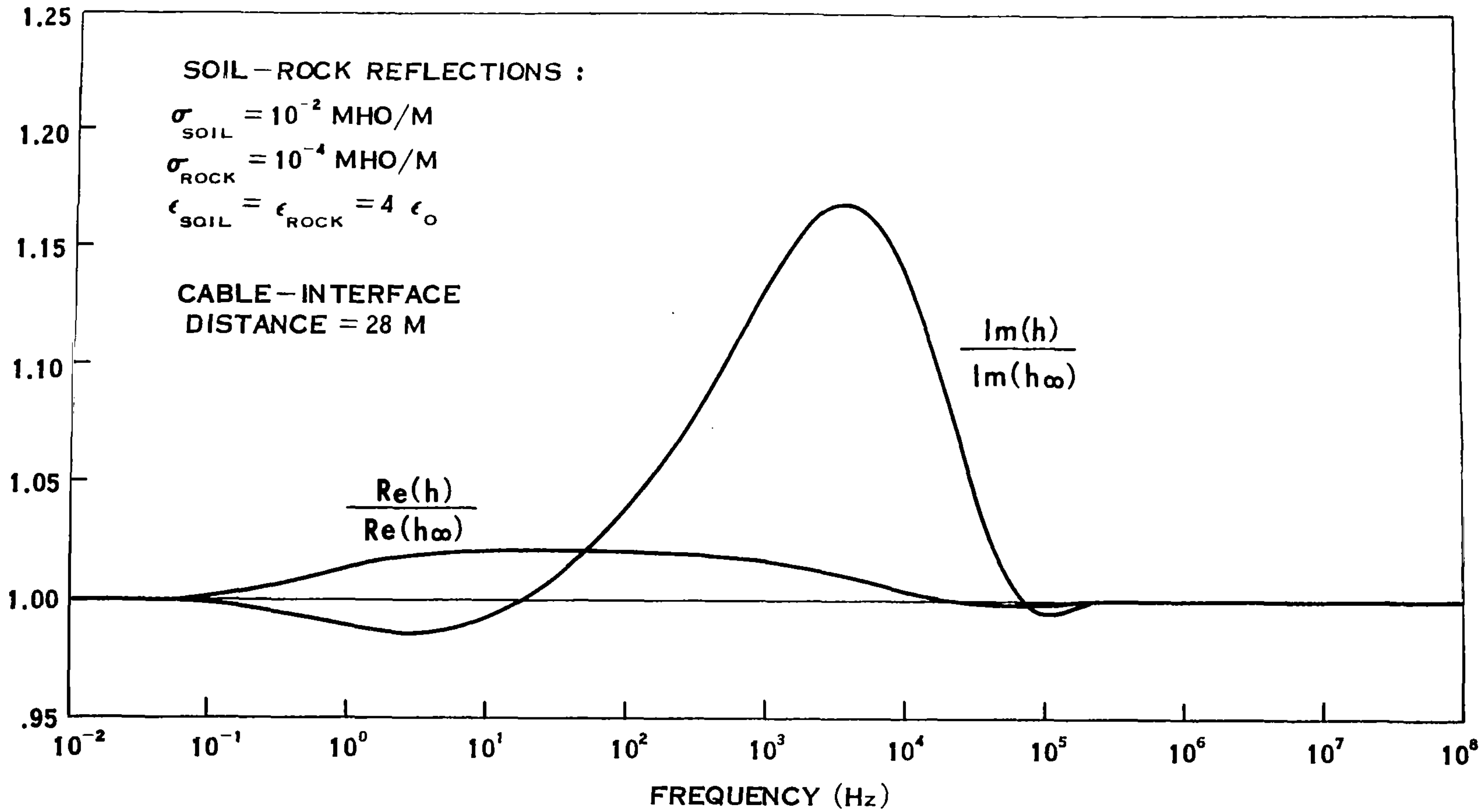

FIGURE 3 – 3

FRACTIONAL CHANGE IN THE PROPAGATION CONSTANT

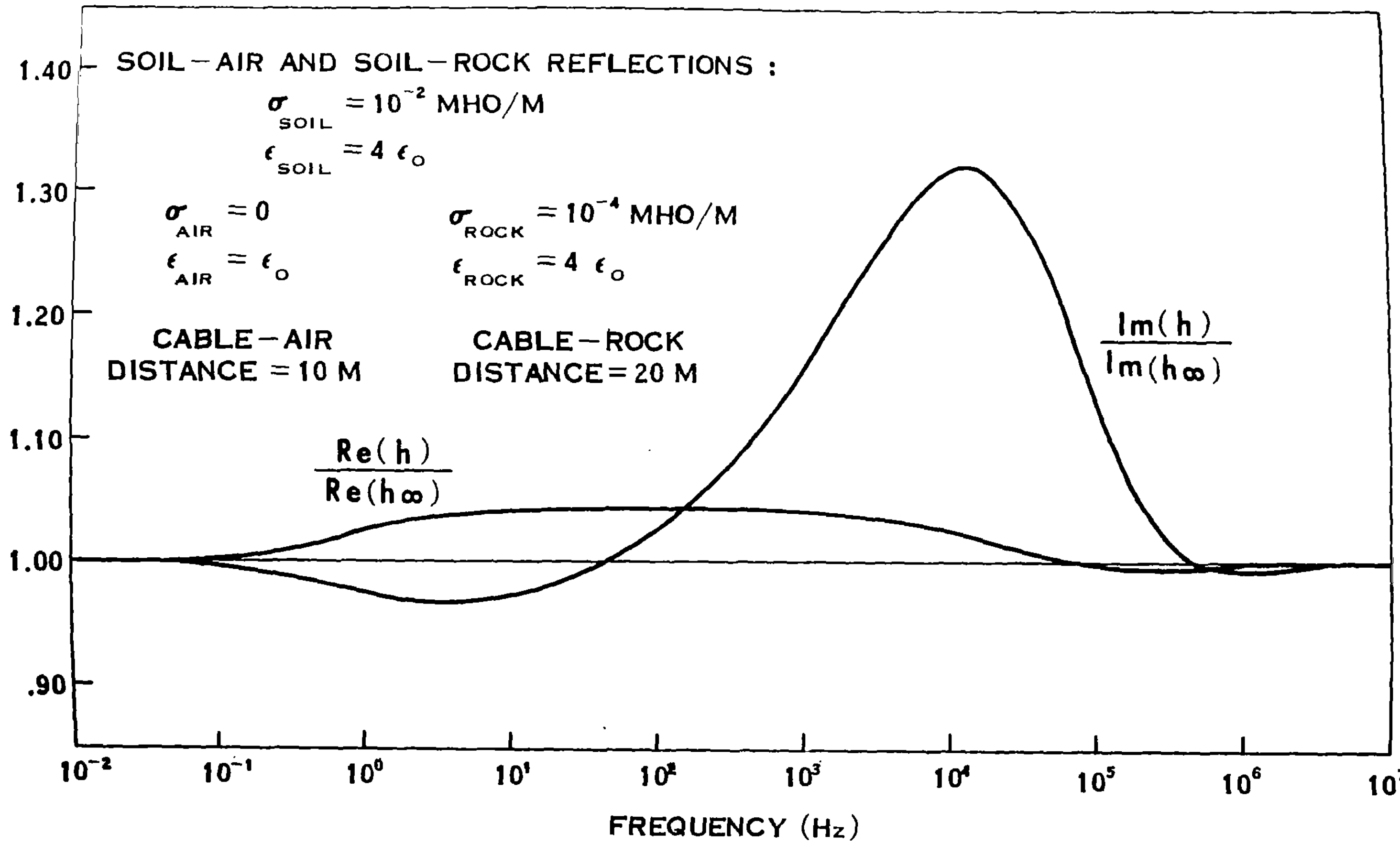

FIGURE 3 − 4

FRACTIONAL CHANGE IN THE PROPAGATION CONSTANT

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axial propagation constant h for such a geometry relative to that for an infinite earth is seen to be a simple combination of the effects due to each interface separately.

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