Interaction Notes
Note 48
January 1969

Penetration of Electromagnetic Fields Through Small Apertures into Closed Shields

Y. P. Liu
Mississippi State University
State College, Mississippi

ABSTRACT

An approximate treatment is presented for the penetration of electromagnetic fields through a small aperture into an otherwise closed shield. The formulation is a modification of the treatment of cavity coupling through small apertures. The shield is considered to be a thin walled rectangular cavity and aperture is considered to be a circular hole in one side. General model expansions for the field components inside the cavity are derived and expressed in terms of the field components that would be present if the aperture were completely shorted. A method is presented and discussed for obtaining the latter field components.

TABLE OF CONTENTS

ACKNOWLEDGMENTS

I would like to take this opportunity to express my sincere appreciation to the people who have aided me in this work. I am particularly grateful to Dr. J. O. Hebert, Jr., my course work advisor, and Dr. C. D. Taylor, my research advisor, for their thoughtful direction in the preparation of this thesis. I also would like to thank Dr. T. T. Crow for his assistance and Mr. Dong-Hoa Lam for his help with the computer programming.

To Sandia Corporation, Albuquerque, New Mexico, the author expresses his appreciation for their financial support.

Y. P. Liu

State College, Mississippi

January 1969

CHAPTER I

INTRODUCTION

The ideal electromagnetic shield is a completely closed highly conducting shell. But invariably small apertures are present in most practical situations. It is obvious that a degradation of shielding properties must accompany the presence of small apertures. To obtain this degradation requires the solution of a horrendous electromagnetic boundary value problem. And it appears at the present that an exact solution for any physical configuration is virtually impossible. Thus some approximating methodology is in order. It is the purpose of this thesis to present an attack on this problem by using a modification of the treatment of cavity coupling through small apertures.

The problem formulated is that of determining the electromagnetic field produced inside a perfectly conducting cavity by an external field when an aperture is present in the cavity wall. For convenience, the cavity geometry is taken to be rectangular, and the dimensions of the aperture are considered small as compared to the wavelength of the incident radiation. General model expansions for the field components inside the cavity are derived and the expansion coefficients are expressed in terms of aperture fields for an arbitrary shaped aperture. The theory of diffraction by small circular apertures is then used to obtain the aperture fields. However these fields are expressed in terms of the field components that would present if the aperture were completely shorted. A method is presented and discussed for obtaining the latter field components.

CHAPTER II

ELECTROMAGNETIC FIELD PENETRATION THROUGH SMALL APERTURE

The various formulations dealing with the problem of electromagnetic field penetrating through small apertures have been reviewed by Bouwkamp.[1] Under the restriction that the aperture dimensions be small compared with the wavelength, it is found that the Rayleigh-Bethe approach may be used. This formulation is based on the use of fictitious magnetic charges and currents in the aperture which has the advantage of automatically satisfying the boundary conditions on the conducting screen. Although the higher order terms in the obtained expressions for the aperture fields apparently are not correct, the zeroth order terms are consistent with other formulations.[2] The primary reason for using this formulation is that it may be readily adapted to the treatment of the coupling of cavities or the field penetration through small apertures where only the zeroth order terms in the aperture field expressions are used.[1]

First, consider the hole in an infinite, perfectly conducting screen to be placed at $z = 0$. Let H_o^i, E_o^i be the field on the upper side of the screen if there is no hole.* Then the actual field for

---

* Following the procedure of Reference 6.

this problem can be represented as

$$H = H_o^{\,i} + H_1 \qquad \text{for } z < 0$$

$$H = H_2 \qquad \text{for } z > 0$$

and similarly for the electric field.

The boundary conditions for the fields at $z = 0$:

$$E_{1tan} = E_{2tan} \qquad \text{in the hole} \qquad (1)$$

$$E_{1tan} = E_{2tan} = 0 \qquad \text{outside the hole} \qquad (2)$$

$$H_{2tan} - H_{1tan} = H_{0tan}^{\,i} \qquad \text{in the hole} \qquad (3)$$

For plane waves, it is obvious that the boundary conditions for the normal components are automatically fulfilled if those for the tangential components are satisfied.

The solutions for the field components will have certain symmetry properties because of the form of Maxwell's equations and the considered geometry.[3] These are:

$$E_{tan}\ (x, y, z) = \pm E_{tan}\ (x, y, -z)$$

$$H_{tan}\ (x, y, z) = \mp H_{tan}\ (x, y, -z)$$

$$E_z\ (x, y, z) = \mp E_z\ (x, y, -z)$$

$$H_z\ (x, y, z) = \pm H_z\ (x, y, -z)$$

Each odd solution, whose tangential electric field components vanish in the aperture as well as upon the screen, describes a field configuration with the plane $z = 0$ completely occupied by a perfect conductor. So the fields E_1, E_2, H_1, H_2 attributed to the presence of an aperture then must belong to the class of even solutions, i.e.

$$E_{1tan}\ (x, y, -z) = E_{2tan}\ (x, y, z) \qquad (4)$$

$$H_{1tan}(x, y, -z) = -H_{2tan}(x, y, z) \quad (5)$$

$$E_{1z}(x, y, -z) = -E_{2z}(x, y, z) \quad (6)$$

$$H_{1z}(x, y, -z) = H_{2z}(x, y, z) \quad (7)$$

These four equations are consistent with Maxwell's equations.

Inserting (5) into (3), it is found that H must satisfy the following boundary condition:

$$H_{2tan} = \tfrac{1}{2} H^{i}_{0tan} \qquad \text{in the hole} \quad (8)$$

The normal component of the electric field is known to be continuous and the use of (7), we get a similar:

$$E_{2z} = \tfrac{1}{2} E^{i}_{0z} \qquad \text{in the hole} \quad (9)$$

Now the problem turns out to be the technical procedure to calculate the field E_2, H_2 which satisfies the boundary conditions mentioned above which hold for an aperture of any shape. For convenience, take the shape of the hole to be circular, with radius a.

In order to satisfy the aforementioned boundary condition on the magnetic field component assume a magnetic current density $\vec{J}^*$ and charge density ρ^*.[4] The $\vec{J}^*$ is not simply $\hat{n} \times \vec{E}$ as for the "no hole" problem.[+] The time dependence of all quantities is assumed to be of the form e^{+jwt}.

$$\nabla \cdot \vec{H} = \frac{1}{\mu} \rho^*$$

$$\nabla \times \vec{E} + jw\mu \vec{H} = -\vec{J}^*$$

[+]The interchange of Stratton's notation into field point $\vec{r}$ and source point $\vec{r}'$.

implies

$$\nabla \cdot \vec{J}^{*} + jw\rho^{*} = 0 \tag{10}$$

Instead of using magnetic volume currents and charges, it is sufficient to use magnetic surface current density $\vec{K}$ and surface charge density η. Then according to equation (10):

$$\nabla \cdot \vec{K} = -jw\eta$$

Scalar and vector potentials help to simplify most problems. Consider the vector potential $\vec{F}$ and scalar potential ψ.

$$\vec{E} = \nabla \times \vec{F} \tag{11}$$

$$\vec{H} = \epsilon \frac{\partial \vec{F}}{\partial t} - \nabla \psi \tag{12}$$

Using the Green's function $\varphi(|\vec{r} - \vec{r}'|) = \frac{e^{jk|\vec{r} - \vec{r}'|}}{|\vec{r} - \vec{r}'|}$ and the gauge condition $\mu \frac{\partial \psi}{\partial t} - \nabla \cdot \vec{F} = 0$, yields the following relationship:

$$\vec{F}(\vec{r}) = -\int \vec{K}(\vec{r}')\varphi(|\vec{r} - \vec{r}'|)\, d\vec{r}' \tag{13}$$

$$\psi(\vec{r}) = \int \eta(\vec{r}')\varphi(|\vec{r} - \vec{r}'|)\, d\vec{r}' \tag{14}$$

Substituting these results into (11) and (12), will yield the field equations.[5]

$$\vec{E}(\vec{r}) = \int \vec{K}(\vec{r}') \times \nabla\varphi \, ds$$

$$\vec{H}(\vec{r}) = \int [jk\vec{K}(\vec{r}')\varphi - \eta(\vec{r}')\nabla\varphi]\, ds$$

where the integral goes over the area of the hole. Because the hole is very small, the retardation, which is of the second order of ka, may be neglected. Then (12) and (14) reduce respectively to:

$$\vec{H} \sim -\nabla\psi$$

$$\psi(\vec{r}) = \int \eta(\vec{r}') \frac{d\vec{r}'}{|\vec{r} - \vec{r}'|}$$

In order to satisfy the boundary condition on the normal component of the electric field, the magnetic current distribution $\vec{K}$ is used.

According to the foregoing:

$$E_{2z} = \tfrac{1}{2} E_{0z} = (\nabla \times \vec{F})_z$$

and a suitable form of $\vec{F}$ is[6]

$$\vec{F} = \tfrac{1}{4} \vec{E}_0{}^i \times \vec{r} \quad .$$

Neglecting retardation,(13) yields

$$\int \vec{K}(\vec{r}\,') \frac{d\vec{r}\,'}{|\vec{r}-\vec{r}\,'|} = -\vec{F}(\vec{r}) = -\tfrac{1}{4} \vec{E}_o{}^i \times \vec{r}$$

which implies

$$\vec{K}(\vec{r}\,') \simeq \frac{\vec{r} \times \vec{E}_o{}^i}{2\pi^2 (a^2 - r'^2)^{1/2}}$$

provided $\vec{E}_o{}^i$ is constant over the aperture region.

Note that $\nabla \cdot \vec{K} \simeq 0$.

So that the obtained expression for the magnetic current does not contribute appreciably to the magnetic charge density.

Now consider the magnetic field

$$\vec{H}_{tan} = \tfrac{1}{2} \vec{H}^{\,i}_{otan} \simeq -\nabla \psi .$$

The foregoing suggests

$$\psi = -\tfrac{1}{2} \vec{H}_o{}^i \cdot \vec{r} \quad . \tag{15}$$

Since a constant inside field is produced by a uniform distribution of dipoles on an ellipsoid and the surface density of dipoles is proportional to the ordinate of the ellipsoid,[7] i.e.

$$\xi = (a^2 - r'^2)^{1/2}$$

the appropriate surface charge density is then,

$$\eta \simeq C_0 \vec{H}_0^{\,i} \cdot \nabla \xi \simeq -C_0 \frac{\vec{H}_0^{\,i} \cdot \vec{r}\,'}{(a^2 - r'^2)^{1/2}} \quad . \tag{16}$$

By direct calculation, we found $C_0 = \frac{1}{\pi^2}$.[6] Then the magnetic charge density is

$$\eta \simeq - \frac{1}{\pi^2 (a^2 - r'^2)^{1/2}} \vec{H}_0^{\,i} \cdot \vec{r}\,' \quad .$$

The foregoing expression is used by Bethe to obtain a first order contribution to the magnetic current density. However, an erroneous result is obtained. Bouwkamp correctly obtains this first order contribution and it is of the order of ka and hence may be neglected in comparison to the zeroth order term at low frequency.

The tangential component of electric field is given by

$$\begin{aligned} \vec{E}_{tan} &= 2\pi \hat{n} \times \vec{K} \\ &= \frac{\vec{r}\,'}{\pi (a^2 - r'^2)^{1/2}} E_{0z}^{\,i} \quad . \end{aligned}$$

Expressing the electrical field in the aperature in cylindrical coordinates:

$$\begin{aligned} E_x &= \frac{r' \cos\theta}{\pi (a^2 - r'^2)^{1/2}} E_{0z}^{\,i} \\ E_y &= \frac{r' \sin\theta}{\pi (a^2 - r'^2)^{1/2}} E_{0z}^{\,i} \\ E_z &= \tfrac{1}{2} \, E_{0z}^{\,i} \quad . \end{aligned}$$

The normal component of the magnetic field is given by:

$$H_n = 2\pi \eta \quad .$$

Then the magnetic field components are

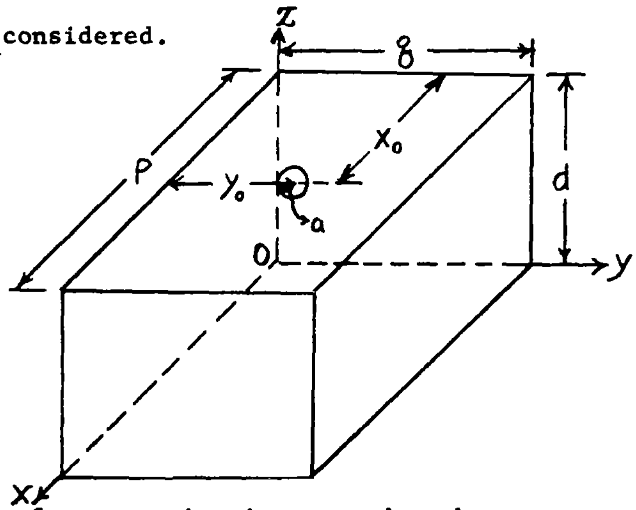
$$\begin{aligned} H_x &= \tfrac{1}{2} \, H_{0x}^{\,i} \\ H_y &= \tfrac{1}{2} \, H_{0y}^{\,i} \\ H_z &= - \frac{1}{\pi (a^2 - r'^2)^{1/2}} \vec{H}_0^{\,i} \cdot \vec{r}\,' \, . \end{aligned}$$

Note that the origin of the coordinate system coincides with the center of the hole.

CHAPTER III

CAVITY THEORY

The theory of wave guides and cavity resonators is well formulated in almost every field theory book.[8] For convenience the cavity shown below is considered.



The medium of propagation is assumed to be vacuum and the walls are assumed to be perfectly conducting. The electric field in the cavity then can be written as

$$\vec{E} = (E_{0x}\hat{i} + E_{0y}\hat{j} + E_{0z}\hat{k})\, e^{j(wt - k_g Z)}$$

where E_{ox}, E_{oy}, E_{oz} are functions of x and y only with the z dependence in the exponential term, and k_g is the effective propagation constant. The magnetic field has the similar form

$$\vec{H} = (H_{0x}\hat{i} + H_{0y}\hat{j} + H_{0z}\hat{k})\, e^{j(wt - k_g Z)}$$

From Maxwell's equations, the following formulas are easily obtained:

$$E_{ox} = -\frac{jw\mu}{K^2-K_g^2}\left(\frac{K_g}{w\mu}\frac{\partial E_{oz}}{\partial x} + \frac{\partial H_{oz}}{\partial y}\right)$$

$$E_{oy} = \frac{jw\mu}{K^2-K_g^2}\left(-\frac{K_g}{w\mu}\frac{\partial E_{oz}}{\partial y} + \frac{\partial H_{oz}}{\partial x}\right)$$

$$H_{ox} = \frac{jw\epsilon}{K^2-K_g^2}\left(\frac{\partial E_{oz}}{\partial y} - \frac{K_g}{K^2-K_g^2}\frac{\partial H_{oz}}{\partial x}\right) \tag{17}$$

$$H_{oy} = -\frac{jw\epsilon}{K^2-K_g^2}\left(\frac{\partial E_{oz}}{\partial x} + \frac{K_g}{K^2-K_g^2}\frac{\partial H_{oz}}{\partial y}\right).$$

According to Maxwell's equations

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + (K^2 - K_g^2)\psi = 0$$

where ψ is E_{oz} or H_{oz}. For the TM mode ($H_{oz} = 0$)

$$\frac{\partial^2 E_{oz}}{\partial x^2} + \frac{\partial^2 E_{oz}}{\partial y^2} + (K')^2 E_{oz} = 0$$

where

$$(K')^2 = K^2 - K_g^2.$$

A solution of the foregoing is

$$E_{oz} = (A \sin K_x x + B \cos K_x x)(C \sin K_y y + D \cos K_y y)$$

where

$$K_x^2 + K_y^2 = (K')^2$$

From the boundary conditions on the cavity walls, $K_x = n\pi/p$ and $K_y = m\pi/q$

$$E_{oz} = \sum_{nm} G_{nm} \sin\frac{n\pi x}{p} \sin\frac{m\pi y}{q}$$

The same procedure may be followed for the TE mode ($E_{oz} = 0$)

$$\frac{\partial^2 H_{oz}}{\partial x^2} + \frac{\partial^2 H_{oz}}{\partial y^2} + (K')^2 H_{oz} = 0$$

is the equation for H_{oz}. Using the appropriate boundary condition on the cavity walls yields

$$H_{oz} = \sum_{nm} F_{nm} \cos\frac{n\pi x}{p} \cos\frac{m\pi y}{q}$$

So the general expression for the field within the cavity is a linear combination of both modes, including the explicit z dependence.

That is:

$$E_{oz} = \sum_{nm} (G_{nm} e^{-jK_g z} + G'_{nm} e^{jK_g z}) \sin\frac{n\pi x}{p} \sin\frac{m\pi y}{q}$$

$$H_{oz} = \sum_{nm} (F_{nm} e^{-jK_g z} + F'_{nm} e^{jK_g z}) \cos\frac{n\pi x}{p} \cos\frac{m\pi y}{q}$$

From (17) , if G_{nm}, G_{nm}', and F_{nm} and F_{nm}' can be determined from boundary conditions, the fields in the cavity will be specified completely. The other field components are:

$$E_{ox} = \sum_{nm} \cos\frac{n\pi x}{p} \sin\frac{m\pi y}{q} (\alpha_{nm} e^{-jK_g z} + \alpha'_{nm} e^{jK_g z})$$

$$H_{ox} = \sum_{nm}\Big[\frac{jw\epsilon}{(K')^2} \frac{m\pi}{q} \sin\frac{n\pi x}{p} \cos\frac{m\pi y}{q} (G_{nm} e^{-jK_g z} + G'_{nm} e^{jK_g z}) + \frac{jK_g}{(K')^2} \frac{n\pi}{p} \sin\frac{n\pi x}{p} \cos\frac{m\pi y}{q} (F_{nm} e^{-jK_g z} + F'_{nm} e^{jK_g z})\Big]$$

$$E_{oy} = \sum_{nm} \sin\frac{n\pi x}{p} \cos\frac{m\pi y}{q} (\beta_{nm} e^{-jK_g z} + \beta'_{nm} e^{jK_g z})$$

$$H_{oy} = \sum_{nm}\Big[-\frac{jw\epsilon}{(K')^2} \frac{n\pi}{p} \cos\frac{n\pi x}{p} \sin\frac{m\pi y}{q} (G_{nm} e^{-jK_g z} + G'_{nm} e^{jK_g z}) + \frac{jK_g}{(K')^2} \frac{m\pi}{q} \cos\frac{n\pi x}{p} \sin\frac{m\pi y}{q} (F_{nm} e^{-jK_g z} + F'_{nm} e^{jK_g z})\Big]$$

where

$$\alpha_{nm} = -\frac{jK_g}{(K')^2} \frac{n\pi}{p} G_{nm} + \frac{jw\mu}{(K')^2} \frac{m\pi}{q} F_{nm}$$

$$\alpha'_{nm} = -\frac{jK_g}{(K')^2} \frac{n\pi}{p} G'_{nm} + \frac{jw\mu}{(K')^2} \frac{m\pi}{q} F'_{nm}$$

$$\beta_{nm} = -\frac{jK_g}{(K')^2}\frac{m\pi}{g}G_{nm} - \frac{jw\mu}{(K')^2}\frac{n\pi}{P}F_{nm}$$

$$\beta'_{nm} = -\frac{jK_g}{(K')^2}\frac{m\pi}{g}G'_{nm} - \frac{jw\mu}{(K')^2}\frac{n\pi}{P}F'_{nm}$$

On the z = 0 surface:

$$E_{0x} = E_{0y} = 0 \ .$$

Therefore $\alpha_{nm} = -\alpha'_{nm}$, $\beta_{nm} = -\beta'_{nm}$ which implies

$$G_{nm} = -G'_{nm}$$

$$F_{nm} = -F'_{nm} \ .$$

By using the foregoing, the tangential components of the electric field become

$$E_{0x} = \sum_{nm} 2j\,\alpha_{nm}\cos\frac{n\pi x}{P}\sin\frac{m\pi y}{g}\sin K_g Z$$

$$E_{0y} = \sum_{nm} 2j\,\beta_{nm}\sin\frac{n\pi x}{P}\cos\frac{m\pi y}{g}\sin K_g Z \ .$$

By using the following orthogonality relation:

$$\int_0^g \sin\frac{\ell\pi}{g}y\,\sin\frac{m\pi}{g}y\,dy = g/2\ \delta_{\ell m}$$

$$\int_0^P \cos\frac{i\pi}{P}x\,\cos\frac{n\pi}{P}x\,dx = P/2\ \delta_{in}$$

$$\int_0^g \cos\frac{\ell\pi}{g}y\,\sin\frac{m\pi}{g}y\,dy = g/2\ \varepsilon_{\ell m}$$

$$\int_0^P \cos\frac{i\pi}{P}x\,\sin\frac{n\pi}{P}x\,dx = P/2\ \varepsilon_{in}$$

where

$\delta_{\ell m}$ and δ_{in} are Delta functions

$$\varepsilon_{in} = \frac{1}{\pi}\frac{i}{i^2-n^2}\left[1-(-1)^{i+n}\right]$$

$$\varepsilon_{ii} = 0 \ .$$

For the cavity problem, on the z = d surface it follows:

$$2j\alpha_{nm} = \frac{4}{pq\sin k_g d}\int_{x_o-a}^{x_o+a}\cos\frac{n\pi x}{p}\,dx\int_{y_o-\sqrt{a^2-(x-x_o)^2}}^{y_o+\sqrt{a^2-(x-x_o)^2}} E_{0x}\sin\frac{m\pi y}{q}\,dy$$

$$2j\beta_{nm} = \frac{4}{pq\sin k_g d}\int_{x_o-a}^{x_o+a}\sin\frac{n\pi x}{p}\,dx\int_{y_o-\sqrt{a^2-(x-x_o)^2}}^{y_o+\sqrt{a^2-(x-x_o)^2}} E_{0y}\cos\frac{m\pi y}{q}\,dy.$$

It is convenient to transform the cartesian coordinates into cylindrical coordinates in the expression for the aperture field. If the origin of the coordinate system is then shifted to the (x_o, y_o, 0) point, the calculation will be much more convenient. Then for m and n even:

$$2j\alpha_{nm} = \frac{4}{pq\sin k_g d}\int_0^a r\,dr\int_{-\pi}^{\pi}d\theta\, E_{0z}^{i}\,\frac{(r\cos\theta+x_o)\cos\frac{n\pi}{p}(r\cos\theta+x_o)}{\pi\,(a^2-r^2)^{1/2}}\cdot\sin\frac{m\pi}{q}(r\sin\theta+y_o)$$

$$2j\beta_{nm} = \frac{4}{pq\sin k_g d}\int_0^a r\,dr\int_{-\pi}^{\pi}d\theta\, E_{0z}^{i}\,\frac{(r\sin\theta+y_o)\sin\frac{n\pi}{p}(r\cos\theta+x_o)}{\pi\,(a^2-r^2)^{1/2}}\cdot\cos\frac{m\pi}{q}(r\sin\theta+y_o)$$

and for m and n odd integers:

$$2j\alpha_{nm} = \frac{-4}{pq\sin k_g d}\int_0^a r\,dr\int_{-\pi}^{\pi}d\theta\, E_{0z}^{i}\,\frac{(r\cos\theta+x_o)\sin\frac{n\pi}{p}(r\cos\theta+x_o)}{\pi\,(a^2-r^2)^{1/2}}\cdot\cos\frac{m\pi}{q}(r\sin\theta+y_o)$$

$$2j\beta_{nm} = \frac{-4}{pq\sin k_g d}\int_0^a r\,dr\int_{-\pi}^{\pi}d\theta\, E_{0z}^{i}\,\frac{(r\sin\theta+y_o)\cos\frac{n\pi}{p}(r\cos\theta+x_o)}{\pi\,(a^2-r^2)^{1/2}}\cdot\sin\frac{m\pi}{q}(r\sin\theta+y_o)$$

where (x_o, y_o, 0) are the coordinates of the center of the aperture.

Integrating with respect to r first and using the approximation $p,q \gg a$, than for m and n even:

$$2j\alpha_{nm} = \frac{E_{oz}^{i}}{\pi p q \sin K_g d} \int_{-\pi}^{\pi} d\theta \cos\frac{n\pi}{p}(a\cos\theta + X_0)\sin\frac{m\pi}{q}(a\sin\theta + Y_0)$$
$$\cdot\ (a^2\pi\cos\theta + 4X_0 a)$$

$$2j\beta_{nm} = \frac{E_{oz}^{i}}{\pi p q \sin K_g d} \int_{-\pi}^{\pi} d\theta \sin\frac{n\pi}{p}(a\cos\theta + X_0)\cos\frac{m\pi}{q}(a\sin\theta + Y_0)$$
$$\cdot\ (a^2\pi\sin\theta + 4Y_0 a)$$

and for m and n odd integers:

$$2j\alpha_{nm} = \frac{-E_{oz}^{i}}{\pi p q \sin K_g d} \int_{-\pi}^{\pi} d\theta \sin\frac{n\pi}{p}(a\cos\theta + X_0)\cos\frac{m\pi}{q}(a\sin\theta + Y_0)$$
$$\cdot\ (a^2\pi\cos\theta + 4X_0 a)$$

$$2j\beta_{nm} = \frac{-E_{oz}^{i}}{\pi p q \sin K_g d} \int_{-\pi}^{\pi} d\theta \cos\frac{n\pi}{p}(a\cos\theta + X_0)\sin\frac{m\pi}{q}(a\sin\theta + Y_0)$$
$$\cdot\ (a^2\pi\cos\theta + 4Y_0 a)$$

Because of the restriction that a be very small, we can use the following approximation*:

$$\sin\left(\frac{m\pi}{q}a\sin\theta\right) \simeq \frac{m\pi}{q}a\sin\theta$$
$$\sin\left(\frac{n\pi}{p}a\cos\theta\right) \simeq \frac{n\pi}{p}a\cos\theta$$
$$\cos\left(\frac{m\pi}{q}a\sin\theta\right) \simeq 1$$
$$\cos\left(\frac{n\pi}{p}a\cos\theta\right) \simeq 1$$

The previous relation yields:

---

* Of course the approximation becomes inaccurate for very large m and n. However the approximation is valid when $m<q/30a$ and $n<p/30a$. The error of approximation will be less than 0.00016. The knowledge of α_{nm} and β_{nm} of those m and n should sufficient.

For m and n even:

$$\alpha_{nm} \simeq -\frac{j E_{oz}^{i}}{P g \sin K_g d}\left(4 x_o a \cos\frac{n\pi}{P} x_o \sin\frac{m\pi}{g} y_o\right)$$

$$\beta_{nm} \simeq -\frac{j E_{oz}^{i}}{P g \sin K_g d}\left(4 y_o a \sin\frac{n\pi}{P} x_o \cos\frac{m\pi}{g} y_o\right)$$

and for m and n odd integers:

$$\alpha_{nm} \simeq \frac{j E_{oz}^{i}}{P g \sin K_g d}\left(4 x_o a \sin\frac{n\pi}{P} x_o \cos\frac{m\pi}{g} y_o\right)$$

$$\beta_{nm} \simeq \frac{j E_{oz}^{i}}{P g \sin K_g d}\left(4 y_o a \cos\frac{n\pi}{P} x_o \sin\frac{m\pi}{g} y_o\right).$$

As noted earlier in chapter II, Bethe's approximation considers only a single plate. In this problem, there is a reflected wave contribution to the fields in the aperture. A means for determining if this approximation is valid in this problem would be to compare the size of the E_z field induced in the cavity to the impressed field, E_{oz}^{i}. If the induced field is small in comparison with impressed field, then the above approximation should be good, since the aperture radius is small compared to the wavelength.

CHAPTER IV

SCATTERING CONSIDERATIONS

According to the forgoing formulation, in order to determine the field inside the cavity, it is necessary to find the impressed electric field. Obtaining the impressed field components is simplified somewhat by not having to consider the aperture present, provided the cavity walls are good conductors. Then

$$E_{oz}^{i} = \rho/\epsilon$$

where ρ is the surface charge density on the outside wall of the cavity at z = d.

For harmonic time dependence, the equation of continuity yields

$$\rho = \frac{j}{w}\left[\frac{\partial}{\partial x} J_x(x,y) + \frac{\partial}{\partial y} J_y(x,y)\right]$$

Therefore

$$E_{oz}^{i} = \frac{j}{w\epsilon}\left[\frac{\partial}{\partial x} J_x(x,y) + \frac{\partial}{\partial y} J_y(x,y)\right]$$

which expresses the impressed field in terms of the surface current density. In general, the surface current is a periodic function of position. Hence the interior fields are proportional to the surface current density that would exist at the position of the aperture.

A theoretical-numerical approach may be used for solving Maxwell's equations, directly applying the finite-difference methods suggested by K. S. Yee.[9] In theory, this numerical attack can be employed for the most general case. However, because of the limited memory capacity of present day computers, numerical solutions to a scattering problem can probably be obtained only for the two dimensional problem. In order to fit this restriction, it is assumed that p is very large and the incident electric field is polarized in the yz plane so that all field components, incident and scattered, have little variation in x coordinate, requiring the cavity to appear more like a waveguide.

From symmetry considerations, the only nonzero field compoents are H_{ox}, E_{oz} and E_{oy}. Then Maxwell's equations give:

$$\epsilon \frac{\partial E_{oy}}{\partial t} = \frac{\partial H_{ox}}{\partial Z}$$
$$\epsilon \frac{\partial E_{oz}}{\partial t} = -\frac{\partial H_{ox}}{\partial y}$$
$$-\mu \frac{\partial H_{ox}}{\partial t} = \frac{\partial E_{oz}}{\partial y} - \frac{\partial E_{oy}}{\partial Z} .$$

The system of equations is particularly suited to solution by difference technique. The method proposed by Lax is used to set up the difference equations.[10] It automatically "centers" the difference formulas thereby reducing the translation (or round-off) error.

The resulting difference equations are:

$$\frac{\epsilon}{\Delta t}\left[E_{oy}^{N+1/2}(g,k+1/2)-E_{oy}^{N-1/2}(g,k+1/2)\right]=\frac{1}{\Delta Z}\left[H_{ox}^{N}(g,k+1)-H_{ox}^{N}(g,k)\right]$$
$$\frac{\epsilon}{\Delta t}\left[E_{oz}^{N+1/2}(g+1/2,k)-E_{oz}^{N-1/2}(g+1/2,k)\right]=\frac{-1}{\Delta y}\left[H_{ox}^{N}(g+1,k)-H_{ox}^{N}(g,k)\right]$$
$$\frac{-\mu}{\Delta t}\left[H_{ox}^{N+1}(g,k)-H_{ox}^{N}(g,k)\right]=\frac{1}{\Delta y}\left[E_{oz}^{N+1/2}(g+1/2,k)-E_{oz}^{N+1/2}(g-1/2,k)\right]$$
$$-\frac{1}{\Delta Z}\left[E_{cy}^{N+1/2}(g,k+1/2)-E_{oy}^{N+1/2}(g,k-1/2)\right]$$

Letting

$$\Delta\tau = c\,\Delta t = \frac{\Delta t}{\sqrt{\mu_0 \epsilon_0}}$$
$$\mathcal{Z} = \sqrt{\frac{\mu_0}{\epsilon_0}} = 376.7$$

then:

$$E_{oy}^{N+1/2}(g,k+1/2) = E_{oy}^{N-1/2}(g,k+1/2) + \frac{\zeta\Delta\tau}{\Delta Z}\left[H_{ox}^{N}(g,k+1) - H_{ox}^{N}(g,k)\right]$$

$$E_{oz}^{N+1/2}(g+1/2,k) = E_{oz}^{N-1/2}(g+1/2,k) - \frac{\zeta\Delta\tau}{\Delta Z}\left[H_{ox}^{N}(g+1,k) - H_{ox}^{N}(g,k)\right]$$

$$H_{ox}^{N+1}(g,k) = H_{ox}^{N}(g,k) - \frac{\Delta\tau}{\zeta\Delta y}\left[E_{oz}^{N+1/2}(g+1/2,k) - E_{oz}^{N+1/2}(g-\tfrac{1}{2},k)\right] + \frac{\Delta\tau}{\zeta\Delta Z}\left[E_{oy}^{N+1/2}(g,k+1/2) - E_{oy}^{N+1/2}(g,k-1/2)\right]$$

The value for $H_{ox}^{0}(g,h)$, $E_{oy}^{1/2}(g, h+1/2)$, $E_{oz}^{1/2}(g+1/2, h)$ are obtained from the incident wave. For further discussion in the mechanics of solving the difference equations, the author defers to Yee.[9]

The quantity that is needed for the aperture problem is the value of the normal component of the electric field at the surface containing the aperture, in particular $E_{oz}^{i}(x_o, y_o, d, t)$. It is pointed out that the difference formulation is carried out in the time domain whereas the previous formulation is developed for the frequency domain. As the input for the frequency domain the Fourier transform of $E_{oz}^{i}(x_o, y_o, d, t)$ must be used. Then the calculated values for the interior fields of the cavity are interpreted as the Fourier transforms of the interior fields for an incident electromagnetic pulse (the particular pulse that is used to obtain $E_{oz}^{i}(x_o, y_o, d, t)$).

CHAPTER V

CONCLUSION

By choosing a rectangular cavity with a small aperture in one side it has been possible to use small aperture diffraction theory to obtain the field penetration from the cavity exterior into the interior of the cavity. Also the same formulation can simply be extended to include the treatment of penetration of fields through small apertures into wave guides. Bethe predicted that similar field penetration through small apertures in curved surfaces would occur.[11] This leads one to conject that the results obtained for the rectangular cavity considered in this thesis would also apply, at least qualitatively, for cavities or shields of arbitrary configuration.

With the advent of higher capacity high speed digital computers the basic formulation presented in this thesis could be used in the treatment of more complex shield or cavity configurations. Also it is observed that a knowledge of the field configuration within the cavity would allow the calculation of the pick-up by small antennas inside the shield.

REFERENCES

1. C. F. Bouwkamp, "Diffraction Theory," Reports on Progress in Physics, Vol. XVII, p. 35 (1954).

2. Ibid., p.77.
H. Levine and J. Schwinger, "On the Theory of Diffraction by an Aperture in an Infinite Plane Screen," Phys. Rev. Vol. 74, p. 958 (1948) and Vol. 75 p. 1423 (1949).

3. H. Levine and J. Schwinger, "On the Theory of Electromagnetic Wave Diffraction by an Aperture in an Infinite Plane Conducting Screen," Comm. Pure and Appl. Math. 3, p. 360 (1950).

4. J. A. Stratton, "Direct Integration of the Field Equation," Electromagnetic Theory, p. 464 (New York: McGraw-Hill 1941).

5. Ibid., p. 466.

6. H. A. Bethe, "Theory of Diffraction by Small Holes," Phys. Rev. Vol. 66, p. 168 (1944).

7. Ibid., p. 167.

8. J. C. Slater and N. H. Frank, Electromagnetism, p. 118 (New York: McGraw-Hill 1947).
J. D. Jackson, "Wave Guides and Resonant Cavities," Classical Electrodynamics, p. 235 (New York: John Wiley and Sons 1967).

9. K. S. Yee, "Numerical Solution of Initial Boundary Problems Involving Maxwell's Equations in Isotropic Media," IEEE Trans. Ant. Prop. Vol. AP-14, No. 3, p.302 May 1966.

10. P. D. Lax, "Differential Equations, Difference Equations and Matrix Theory," Comm. Pure and Appl. Math. Vol II, p. 175, Nov. 1958.

11. H. A. Bethe, "Theory of Diffraction by Small Holes," Phys. Rev. Vol. 66, p. 175 (1944).