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Electromagnetic Pulse Transmission Through a Thin Sheet of Saturable Ferromagnetic Material of Infinite Surface Area

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***Abstract*—A numerical solution is presented for determining the shielding properties of a thin sheet of saturable ferromagnetic material of infinite surface area. Several examples are given to indicate the behavior of ferromagnetic shields in an intense electromagnetic environment. These examples illustrate that only a few numerical results are required to determine the electric field transmitted through the material for a given incident pulse of any amplitude.**

INTRODUCTION

THE SHIELDING action of metallic plates and shells to impinging low-frequency transients has been previously considered by Harrison [1]. This type of analysis indicates that ferromagnetic materials are advantageously employed where shielding against low-frequency fields is needed and much thicker aluminum or copper shields would be required to obtain the same shielding effectiveness.

When the amplitude of the external interference is very large, like the electromagnetic disturbance created by a nuclear explosion or a lightning flash, the effectiveness of a ferromagnetic shield may be reduced because a portion of the material is driven into saturation.

This paper is concerned with the action of ferromagnetic shields in intense electromagnetic fields. The model considered is the propagation of an intense transient plane wave through a thin sheet of saturable ferromagnetic material of infinite surface area.

A classical finite difference technique is used to solve the appropriate nonlinear diffusion equation for the field distribution inside the ferromagnetic sheet. This technique has previously been applied to a problem involving ferromagnetic materials by Poritsky and Butler [2]; the application to the shielding problem is new and the numerical results obtained are informative.

FORMULATION

To simplify the problem somewhat it shall be assumed that $f(t)$, the incident electric field transient (Fig. 1), is a causal function, i.e., $f(t) = 0$ if $t \leq 0$, and that the directions of the b and h vectors coincide while the magnitudes are related by a single-valued function (Fig. 2). The material then exhibits saturation, but hysteresis has been neglected.

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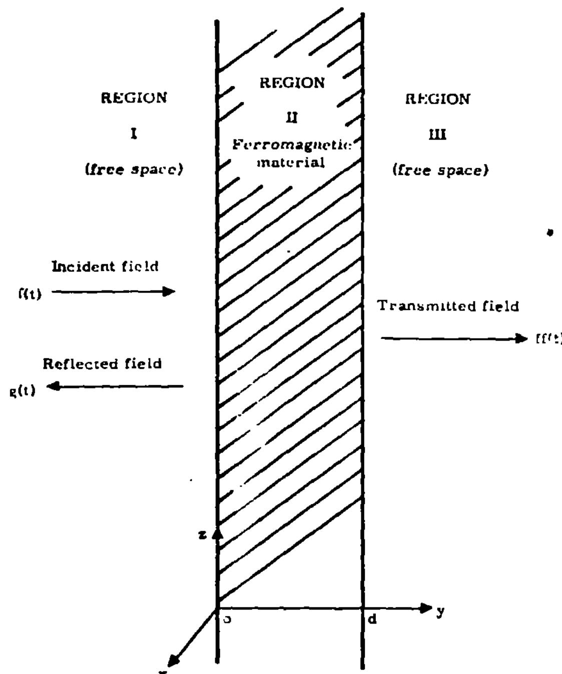

Fig. 1. Infinite sheet of ferromagnetic material.

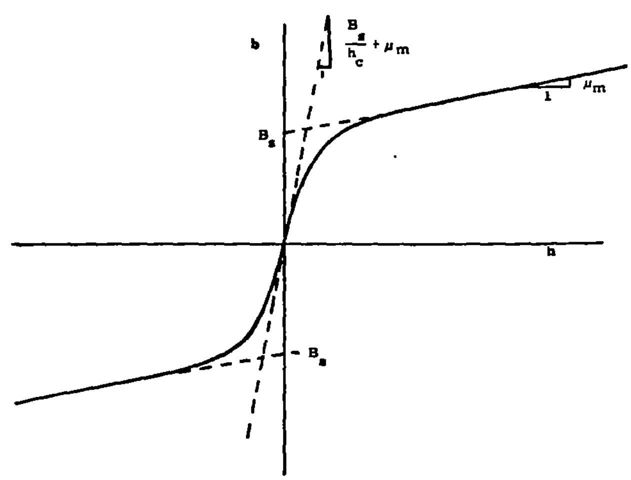

Fig. 2. Magnetic characteristics of ferromagnetic material.
$\mu(h) = \partial b/\partial h = \mu_m + B_S \exp(-|h|/h_c)/h_c$.

In all three regions of space (Fig. 2) the field components must satisfy Maxwell's equations:

$$\nabla \times e = -\frac{\partial b}{\partial t} \quad (1)$$

$$\nabla \times h = \sigma e + \epsilon_0 \frac{\partial e}{\partial t}. \quad (2)$$

If the incident electric field is directed along the z axis, the only nonzero components of the total field are e_z, h_x, and b_x. The curl equations reduce to

$$\frac{\partial e}{\partial y} = -\frac{\partial b}{\partial t} \quad (3)$$

$$\frac{\partial h}{\partial y} = -\sigma e - \epsilon_0 \frac{\partial e}{\partial t}. \quad (4)$$

For simplicity, the subscripts on the nonzero components have been dropped. In regions I and III, $\sigma = 0$; the field components are of the form

region I:

$$e(t,y) = f(t - y/c) + g(t + y/c),$$

$$h(t,y) = \frac{1}{\zeta_0}[f(t - y/c) - g(t + y/c)]; \quad (5)$$

region III:

$$e(t,y) = ff(t - (y - d)/c),$$

$$h(t,y) = \frac{1}{\zeta_0} ff(t - (y - d)/c). \quad (6)$$

Here $\zeta_0 = \sqrt{\mu_0/\epsilon_0} = 120\pi$ ohms, and c is the velocity of light in free space. The function f is identified as the incident electric field, g is the unknown reflected electric field, and ff is the unknown electric field transmitted through the sheet.

Inside the material, if the displacement current is neglected, (3) and (4) may be combined into one second-order equation:

$$\frac{\partial^2 h(t,y)}{\partial y^2} = \sigma\mu(h) \frac{\partial h(t,y)}{\partial t}, \quad 0 \le y \le d. \quad (7)$$

Here $\mu(h) \equiv db/dh$.

The boundary conditions that e and h must be continuous at the boundary yield from (5) and (6)

$$e(t,0) + \zeta_0 h(t,0) = 2f(t) \quad (8)$$

$$e(t,d) - \zeta_0 h(t,d) = 0; \quad (9)$$

or, applying (4),

$$\zeta_0 h(t,0) - \frac{1}{\sigma}\frac{\partial h(t,0)}{\partial y} = 2f(t) \quad (10)$$

$$\zeta_0 h(t,d) + \frac{1}{\sigma}\frac{\partial h(t,d)}{\partial y} = 0. \quad (11)$$

The electric field transmitted through the sheet, which is of primary interest in the shielding problem, is given by $ff(t) = \zeta_0 h(t,d)$. The problem reduces to finding an appropriate solution of the nonlinear diffusion equation (7) that satisfies the mixed boundary conditions (10) and (11).

Numerical Solution

To obtain a numerical solution of (7) with boundary conditions (10) and (11), a rectangular mesh of points is introduced into the y–t plane:

$$y_j = (j - 1)\,\Delta y, \quad j = 1, \cdots, J \quad (12)$$

$$t_k = (k - 1)\,\Delta t, \quad k = 1, 2, \cdots. \quad (13)$$

Here J is the number of nodes introduced between $y = 0$ and $y = d$, the boundaries of the sheet; $\Delta y = d/(J - 1)$; and Δt is the selected time increment. The derivatives in (7), (10), and (11) are replaced by the difference approximations

$$\frac{h_j^{k+1} - h_j^k}{\Delta t} = \frac{1}{\sigma\mu(h_j^{k+1/2})\,\Delta y^2}[h_{j+1}^{k+1} - 2h_j^{k+1} + h_{j-1}^{k+1}],$$
$$j = 2, 3, \cdots, (J - 1) \quad (14)$$

$$2f(t_{k+1}) = \zeta_0 h_1^{k+1} + \frac{1}{2\sigma\Delta y}[3h_1^{k+1} - 4h_2^{k+1} + h_3^{k+1}] \quad (15)$$

$$0 = \zeta_0 h_J^{k+1} + \frac{1}{2\sigma\Delta y}[h_{J-2}^{k+1} - 4h_{J-1}^{k+1} + 3h_J^{k+1}]. \quad (16)$$

Here $h_j^k \equiv h(t_k,y_j)$. An implicit differencing scheme (backward time differencing) was employed because the difference equations so obtained are stable for all values of Δt and Δy [3]. The value of μ used in the calculation was the value obtained at $h_j^{k+1/2} \equiv (h_j^k + h_j^{k+1})/2$.

Equations (14)–(16) constitute a system of J nonlinear equations in the unknowns h_j^{k+1}, $j = 1, 2, \cdots, J$. Since $f(t)$ was assumed to be causal, the magnetic field is initially zero throughout the material, $H_j^1 = 0$. The solution is effected by proceeding sequentially through the t_k solving the system of J nonlinear equations for h_j^{k+1} from the previously determined values of h_j^k.

When the equations are written in matrix form, every row of the coefficient matrix has only three elements centered on a dominate diagonal term. Under these conditions, the Gauss–Seidel iteration procedure may be used effectively to obtain an approximate solution to the system of equations, since the iteration needed to account for the nonlinear permeability may be incorporated in the normal iteration of the solution.

Attenuation of Half-Sine Pulses

To illustrate how saturation affects the shielding action of a ferromagnetic material, the transient electric field transmitted through a thin sheet of annealed steel was considered. The magnetic parameters μ_m, B_s, and h_c (Fig. 2) were chosen to be 1.67×10^{-4} H/m, 1.53 Wb/m² (15.3

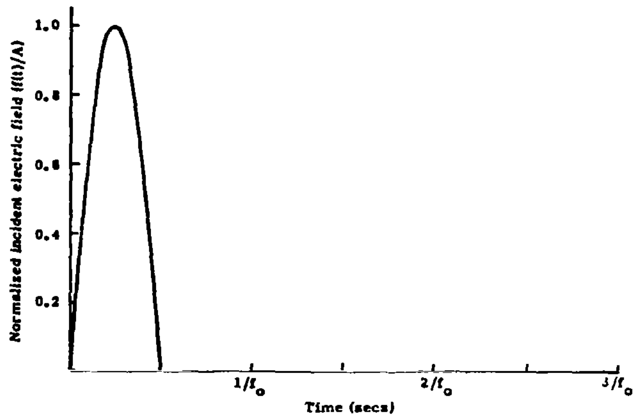

Fig. 3. Normalized incident electric field pulse,

$$f(t) = \begin{cases} A \sin 2\pi f_0 t, & 0 \le t \le 1/2f_0 \\ 0, & \text{otherwise.} \end{cases}$$

kG), and 120 A/m (1.51 Oe), respectively, to approximate the magnetization curve given by Stratton [4]. The conductivity σ was selected to be 10^7 mho/m, and the thickness of the sheet d was 0.005 inch (1.26×10^{-4} meters). The waveform of the incident electric field was a half-sine pulse (Fig. 3). Twenty-one spatial nodes were used in the numerical calculation, and the time increment Δt was selected to yield 400 sample points per half-cycle of the frequency f_0. Three different values of f_0 were selected, and for each value of f_0 the amplitude A of the incident electric field was varied over a wide range.

For the smallest amplitude considered, 10^4 V/m, the process is essentially linear; i.e., the transmitted electric field waveform, normalized by the amplitude A, is almost identical for all pulses with peak intensity less than 10^4 V/m. The result given agrees with predictions obtained with the theory given by Harrison for a linear medium [1].

For each value of f_0 considered, the amplitude A was increased until the entire thickness of the material was saturated $[h_j^k \ge 2h_c, j = 2, 3, \cdots, (J-1)]$ during some portion of the pulse. During this time interval, the behavior of the material is again almost linear, and the result given may be applied to more intense fields. The primary effect of increasing the amplitude is to widen the interval in which the material is totally saturated.

Discussion of Results

For the lowest value of f_0 considered (100 Hz) the spectral content of the incident field is primarily in the low-frequency range where, even for the linear material, the field suffers little attenuation in transversing the thin sheet of material. The shielding obtained results primarily from the reflection process. The transmitted field in this case is easily estimated. If the attenuation is small, the electric field is nearly independent of position within the material, and the electric field is continuous at $y = d$; therefore,

$$e(t,y) = \mathfrak{f}(t), \quad 0 \le y \le d. \tag{17}$$

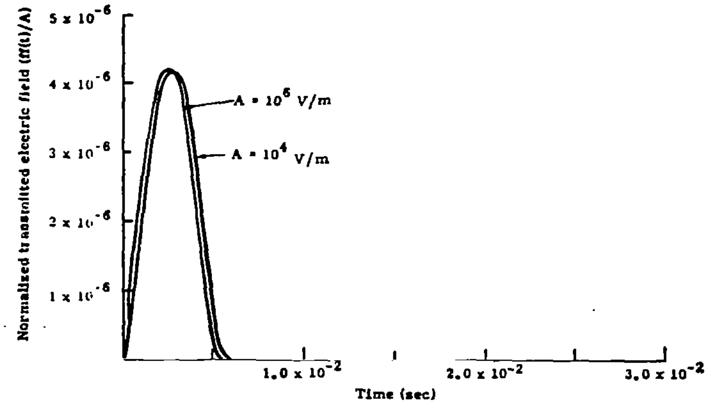

Fig. 4. Normalized electric field transmitted through sheet, f_0 = 100 Hz.

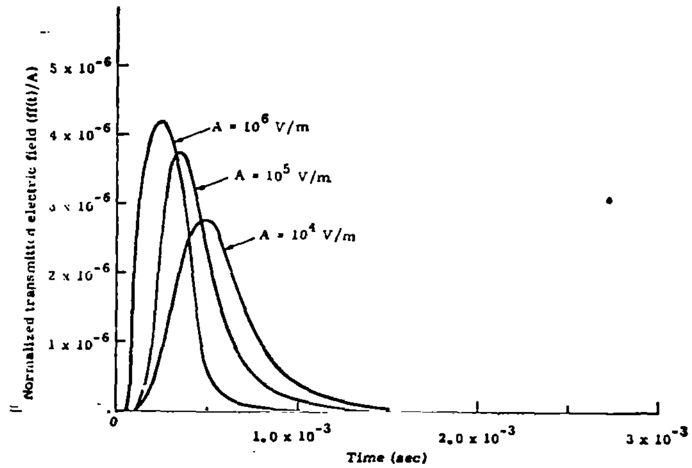

Fig. 5. Normalized electric field transmitted through sheet, f_0 = 1 kHz.

From (4), neglecting displacement current,

$$h(t,y) = h(t,0) - y\sigma \mathfrak{f}(t), \quad 0 \le y \le d. \tag{18}$$

Evaluating (18) at $y = d$ and recalling that $\mathfrak{f}(t) = \zeta_0 h(t,d)$ yields

$$h(t,0) = \frac{1}{\zeta_0}(1 + \sigma\zeta_0 d)\mathfrak{f}(t) \tag{19}$$

and, by combining (8), (17), and (19),

$$\mathfrak{f}(t) = \frac{2f(t)}{2 + \sigma\zeta_0 d}. \tag{20}$$

The amplitude predicted by (20) agrees well with the result given in Fig. 4. The important point here is that (20) is independent of μ; and, therefore, it is to be expected that the transmitted field will not be affected by saturation of the material. The amplitude dependence illustrated in Fig. 4 shows that the error made in employing (17) to deduce (20) is small.

For the second value of f_0 considered (1000 Hz) the spectral density of the pulse extends into the frequency range where the sheet offers significant attenuation at low levels. Some smoothing of the low-amplitude pulse is apparent (Fig. 5). As the amplitude is increased, the attenuation is

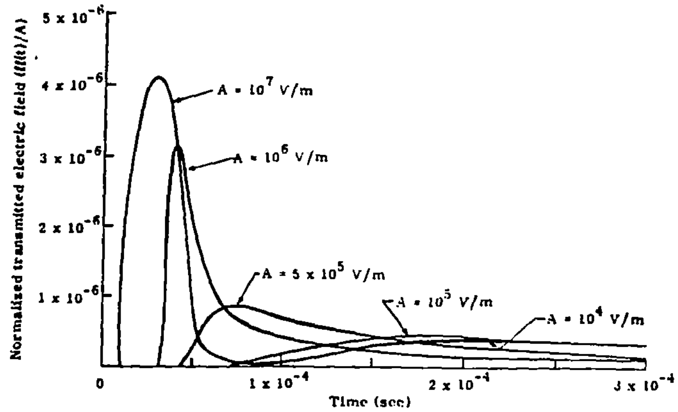

Fig. 6. Normalized electric field transmitted through sheet, $f_0 = 10$ kHz.

reduced and the high-frequency content of the transmitted field is increased. When the amplitude is large enough that the material is saturated all the way through the sheet, the attenuation is very small, and the peak field returns to the value predicted by (20).

For the final value of f_0 considered (10 kHz), a significant amount of attenuation and smoothing of the low-amplitude pulse is apparent (Fig. 6). For this pulse, a larger amplitude of the incident field is required to saturate the material throughout the sheet. The normalized amplitude of the peak, which occurs during the saturated interval, is lower than that predicted by (20). This results because the saturated material with $\mu = \mu_m = 1.67 \times 10^{-4}$ H/m offers some attenuation at the frequencies contained in this pulse. For pulses of shorter duration, this attenuation would be even more significant.

Conclusion

An analysis technique has been presented that will allow the shielding effectiveness of a nonlinear ferromagnetic material to be determined. For any given incident field pulse shape it is possible to describe the behavior of the material with a few calculations ranging between the low-level response and the response at very high level where the shield is totally saturated.

Although primarily intended for transient response studies, the procedure given could also be applied to steady-state phenomena by continuing the calculation until the field transmitted through the sheet becomes periodic.

The sample calculations illustrate that short pulses are heavily attenuated by passing through a thin sheet of annealed steel. When the pulse amplitude is increased until the material is saturated throughout the sheet, large reductions in the shielding are observed. Fortunately, the level needed to saturate the sheet is inversely proportional to the pulsewidth so that thin sheets of ferromagnetic material may be used to protect systems against fast transients.

Most shielding-grade ferromagnetic materials have a lower saturation flux density than that used in the ex-

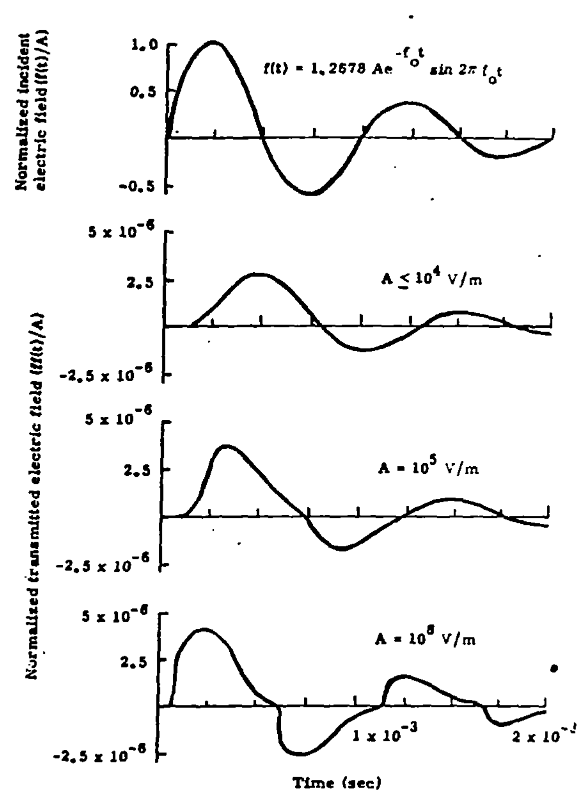

Fig. 7. Normalized transmitted electric field for a damped sine wave incident field, $f_0 = 1$ kHz.

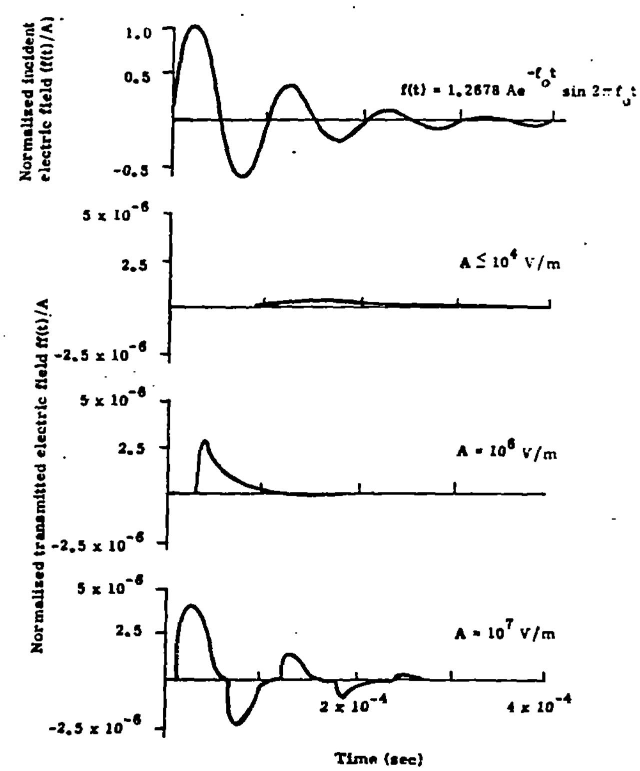

Fig. 8. Normalized transmitted electric field for a damped sine wave incident field, $f_0 = 10$ kHz.

amples. Since the amplitude of the field needed to saturate a sheet of given thickness is directly proportional to the saturation flux density, the effects illustrated can be expected to occur at a proportionately lower level.

Appendix

Because damped sine waves are often used in experiments where intense electromagnetic environments are required, the response of a ferromagnetic shield to this type of incident field is particularly interesting.

In this Appendix, the response of a thin sheet of annealed steel to a damped sine wave incident field,

$$f(t) = 1.2678\, Ae^{-f_0 t} \sin 2\pi f_0 t$$

is considered. For low values of f_0 the attenuation of the field through the sheet is small; consequently, the transmitted electric field given by (20) is a good approximation of the field predicted by the numerical solution. Numerical solutions were obtained for two higher values of f_0, 1 kHz and 10 kHz, and for several values of A. Here, as before, A is the peak value of incident field in volts per meter. The solutions are shown in Figs. 7 and 8.

For $f_0 = 1$ kHz and $A = 10^4$ V/m (Fig. 7) the transmitted electric field is similar in shape to the incident electric field. Larger values of A result in a shorter delay time through the material and in more distortion of the waveform. For $A = 10^6$ V/m, the material saturates throughout the sheet during each of several half cycles. It is interesting to note that on each successive half-cycle the delay time is a little longer since the incident field is smaller.

For $f_0 = 10$ kHz (Fig. 8), a different situation results: for low levels, the transmitted electric field does not resemble the incident field transient. This results because the sheet offers a large amount of attenuation at frequencies near f_0, and very little attenuation at low frequencies. Consequently, a filtered form of the incident field pulse is observed on the other side of the sheet. Again, as the amplitude of the incident electric field pulse is increased, the delay time is reduced and the high-frequency content of the transmitted pulse increases. However, it is not until the material is saturated all the way through the sheet during some portion of a half-cycle that the transmitted field begins to resemble the incident pulse.

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