Printed in the U.S.A.

IN 37

The Response of a Terminated Two-Wire Line Buried in the Earth and Excited by a Plane-Wave RF Field Generated in Free Space

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Abstract—A terminated two-wire transmission line is buried at constant depth near the earth–air interface with one conductor directly below the other. A plane-wave electromagnetic field, generated in free space, impinges upon the boundary where it undergoes partial reflection and transmission. The field transmitted into the earth excites the transmission line. The polarization of the electric field is chosen such that the field is directed parallel to the line conductors. The interaction of the line with the dispersive medium and the line losses are considered. The objective of the study is to determine the current in specified load impedances in terms of the amplitude of the incident electric field evaluated at the surface of the earth.

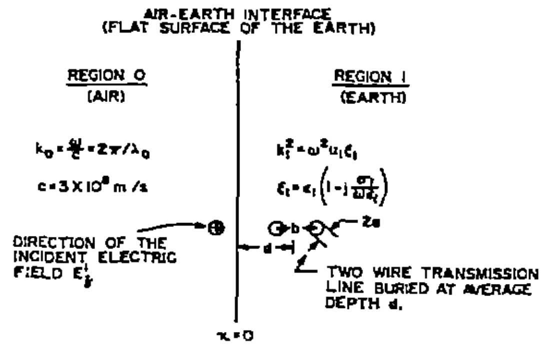

Fig. 1. Two-wire transmission line buried in a shallow trench and positioned for optimum response to the incident electric field, polarized as indicated.

Introduction

THE PURPOSE of the present investigation is to determine the response of an impedance-loaded two-wire transmission line that is laid in a shallow trench in the earth, with one conductor directly below the other, and driven by that portion of the incident plane-wave electromagnetic field originating in free space that is transmitted into the earth. The electric field midway between the line conductors is assumed to be directed parallel to the wire axes. In the theoretical development the earth is treated as a semi-infinite homogeneous dissipative medium, and it is assumed that the ditch is filled with soil following installation of the line. It is desired to develop expressions for the load currents in the line terminations in terms of the amplitude of the incident electric field evaluated at the surface of the earth.

The arrangement of the transmission line with respect to the interface, and the polarization of the incident field, are depicted graphically in Fig. 1. The usual right-hand Cartesian coordinate system is employed. The direction of propagation of the incident electric field is along the positive x axis; it is directed along the negative z axis. The incident magnetic field is directed along the positive y axis. The interface lies in the yz plane, and the transmission-line conductors are parallel to the z axis. An imaginary line orthogonal to the earth–air boundary contains the centers of both wires.

The procedure used to solve the problem consists in 1) integrating the differential value of the transmitted electric field over the length of the wires to obtain the load currents in terms of the amplitude of the transmitted field in the dispersive medium midway between the line conductors, and 2) solving the elementary boundary value problem to express the field at this point in terms of the incident field. It is to be remembered that at the boundary the sum of the incident and reflected fields equals the transmitted field.

Line losses are taken into account in the theory, as well as the interaction of the conductors with the dissipative medium. As long as the distance to the boundary is several times the line spacing there is no interaction of the transmission-line mode bidirectional currents with the interface. Only the transmission-line currents flow through the terminating impedances; the unidirectional or antenna current on the wires does not. If the antenna current in the line were sought, multiple reflections between the wires and the earth–air boundary would have to be taken into account. There are two additional points the writers feel require mentioning: 1) the transmitted field is a plane wave (with coincident surfaces of constant amplitude and phase) because the incident field is directed tangential to the earth's surface; 2) the line terminations play no role in the response of the line since there is no component of electric field tangential to these loads.

Elementary Transmission-Line Theory

Consider a two-wire transmission line of length s terminated in impedance Z_s, as shown in Fig. 2. The conductors are of radius a and are spaced a distance b apart. The line is lossy and located in an infinite dispersive medium designated region 1. The input current I_0 is delivered by generator V_0. The load current and load voltage are I_s and V_s, respectively.

Manuscript received April 7, 1969. This work was supported by the U. S. Atomic Energy Commission.
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braic manipulation that

$$I(h) = \frac{V_z}{D}\{Z_c \cosh \gamma(h+z) + Z_{-h} \sinh \gamma(h+z)\} \tag{18}$$

$$I(-h) = \frac{V_z}{D}\{Z_c \cosh \gamma(h-z) + Z_h \sinh \gamma(h-z)\} \tag{19}$$

where

$$D = Z_c(Z_h + Z_{-h}) \cosh 2\gamma h + (Z_h Z_{-h} + Z_c^2) \sinh 2\gamma h. \tag{20}$$

The Excitation of the Transmission Line

Refer to Fig. 1. Let the reference for phase be midway between the conductors at depth d below the earth–air boundary. The transmitted field at that point is designated E_d^t. Evidently the fields acting tangentially to the right-hand and left-hand conductors are, respectively,

$$E_r = E_d^t e^{-jk_1b/2} \tag{21}$$

and

$$E_l = E_d^t e^{jk_1b/2} \tag{22}$$

To resolve these fields into symmetrical and antisymmetrical components, one sets

$$E_r = E^s + E^a \tag{23}$$

$$E_l = E^s - E^a \tag{24}$$

It follows that

$$E^s = \frac{E_r + E_l}{2} = E_d^t \cos\left(\frac{k_1b}{2}\right) \tag{25}$$

$$E^a = \frac{E_r - E_l}{2} = jE_d^t \sin\left(\frac{k_1b}{2}\right). \tag{26}$$

The even field E^s is responsible for the antenna currents in the transmission line, and E^a, the odd field, sets up the transmission-line currents in the structure. Since E^a, the differential electric field acts at all points along the conductors, the cumulative effect may be obtained by integration. One sets

$$\frac{V_z}{2} = -E^a dz = -jE_d^t \sin\left(\frac{k_1b}{2}\right) dz. \tag{27}$$

Substituting (27) into (18) and (19) gives

$$I_T(h) = -j\frac{2}{D} E_d^t \sin\left(\frac{k_1b}{2}\right)\left[Z_c \int_{-h}^{h} \cosh \gamma(h+z)\, dz + Z_{-h} \int_{-h}^{h} \sinh \gamma(h+z)\, dz\right] \tag{28}$$

$$I_T(-h) = -j\frac{2}{D} E_d^t \sin\left(\frac{k_1b}{2}\right)\left[Z_c \int_{-h}^{h} \cosh \gamma(h-z)\, dz + Z_h \int_{-h}^{h} \sinh \gamma(h-z)\, dz\right]. \tag{29}$$

Integrating yields

$$I_T(h) = -j\frac{2}{\gamma D} E_d^t \sin\left(\frac{k_1b}{2}\right)[Z_c \sinh 2\gamma h + Z_{-h}(\cosh 2\gamma h - 1)] \tag{30}$$

$$I_T(-h) = -j\frac{2}{\gamma D} E_d^t \sin\left(\frac{k_1b}{2}\right)[Z_c \sinh 2\gamma h + Z_h(\cos 2\gamma h - 1)]. \tag{31}$$

Here $I_T(h)$ and $I_T(-h)$ are the total currents in the load impedances Z_h and Z_{-h}, respectively, resulting from the field acting at all points along the line.

If $Z_{-h} = 0$, (30) with (20) becomes[1]

$$I_T(h) = -j\frac{2}{\gamma} E_d^t \sin\left(\frac{k_1b}{2}\right)\left[\frac{\sinh 2\gamma h}{Z_h \cosh 2\gamma h + Z_c \sinh 2\gamma h}\right] \tag{32}$$

and if $Z_{-h} = \infty$, one obtains

$$I_T(h) = -j\frac{2}{\gamma} E_d^t \sin\left(\frac{k_1b}{2}\right)\left[\frac{\cosh 2\gamma h - 1}{Z_c \cosh 2\gamma h + Z_h \sinh 2\gamma h}\right]. \tag{33}$$

The Electromagnetic Field Transmitted into a Semi-Infinite Dissipative Medium

Let a plane-wave electromagnetic field arriving from free space be incident upon the air–earth interface located at $x = 0$. The field is polarized parallel to the z axis, and is propagated in the positive x direction as illustrated by Fig. 1. The lossless medium (air) is designated region 0, and the dispersive medium (earth) is denoted region 1.

Now in any source-free medium the wave equation

$$\nabla^2 E + k^2 E = 0 \tag{34}$$

applies. In the present application (34) reduces to

$$\frac{\partial^2 E_z}{\partial x^2} + k^2 E_z = 0. \tag{35}$$

Accordingly, one may write

$$(E_z^i)_{\text{air}} = E_0^i e^{-jk_0x} \tag{36}$$

$$(E_z^r)_{\text{air}} = E_0^r e^{jk_0x} \tag{37}$$

$$(E_z^t)_{\text{earth}} = E_0^t e^{-jk_1x} \tag{38}$$

Here the superscripts i, r, and t mean incident, reflected, and transmitted, respectively. $k_0 = 2\pi/\lambda_0$ is the free-space wavenumber.

[1] If upon investigation it is found that $2\alpha h \geq 5$, as is likely to occur for physically long transmission lines embedded in a moderately conducting earth, and the inequality $|k_1b| \ll 1$ is satisfied, (32) and (33) reduce to the same expression, namely,

$$I_T(h) = -jE_d^t \frac{k_1b}{\gamma}\left(\frac{1}{Z_c + Z_h}\right).$$

α is defined by (5). With (46),

$$I_T(h) = -j2E_0^i \frac{k_0k_1b}{\gamma(k_0 + k_1)} e^{-jk_1d}\left(\frac{1}{Z_c + Z_h}\right).$$

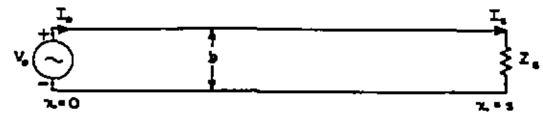

Fig. 2. Transmission line driven by one generator.

Since

$$I_0 = I_s \cosh \gamma s + \frac{V_s}{Z_c} \sinh \gamma s \tag{1}$$

and

$$V_s = I_s Z_s \tag{2}$$

it follows that

$$I_s = \frac{I_0 Z_c}{Z_c \cosh \gamma s + Z_s \sinh \gamma s}. \tag{3}$$

Also it can be shown easily that the driving-point impedance of the line is

$$Z_0 = Z_c \left\{ \frac{Z_s + Z_c \tanh \gamma s}{Z_c + Z_s \tanh \gamma s} \right\}. \tag{4}$$

In the above (see [4])

$$\gamma = \alpha + j\beta = \sqrt{ZY} \tag{5}$$

$$Z_c = \sqrt{\frac{Z}{Y}} \tag{6}$$

$$Z = z^i + j\omega l^e \tag{7}$$

$$Y = j\frac{k_1^2}{\omega l^e} = j\frac{\omega \pi \xi_1}{\ln (b/a)} \tag{8}$$

$$z^i = \frac{1}{\pi a}\sqrt{\frac{\omega \mu_c}{2\sigma_c}}\,(1 + j), \quad a\sqrt{\omega \sigma_c \mu_c} \geq 10 \tag{9}$$

$$l^e = \frac{\mu_1}{\pi} \ln \left(\frac{b}{a}\right) \tag{10}$$

$$k_1^2 = \omega^2 \mu_1 \xi_1 \tag{11}$$

$$\xi_1 = \epsilon_1 \left(1 - j\frac{\sigma_1}{\omega \epsilon_1}\right). \tag{12}$$

μ_1, ϵ_1, and σ_1 are the absolute permeability, dielectric constant, and conductivity of the dissipative medium, respectively. μ_c and σ_c refer to the transmission-line conductors.

Application to a Terminated Two-Conductor Transmission Line Driven by Two Generators

Equations (3) and (4), if properly interpreted, may be employed to find the load currents $I(h)$ and $I(-h)$ of Fig. 3. This figure illustrates a transmission line of length $2h$ driven at point z, measured from the midpoint of the line, by two generators of voltage $V_z/2$ of the polarity shown, and terminated in impedances Z_h and Z_{-h}. These impedances need not be of equal value. For example, the line might be short-circuited or open-circuited at one end, and be terminated in a finite value of impedance at the other end. From Fig. 4 it is clear that the current I_g, which corresponds to I_0 in (1), is given by the relation

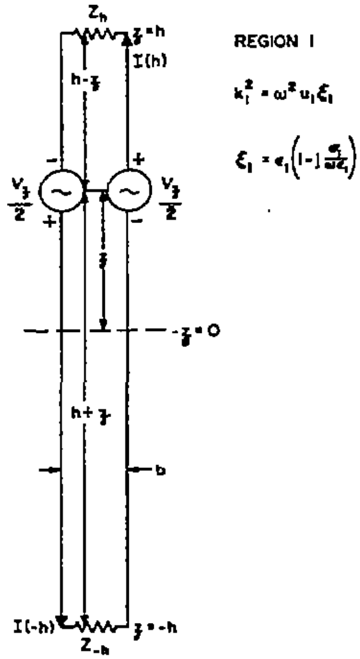

Fig. 3. Lossy transmission line embedded in a dissipative medium. The line is driven by two series generators.

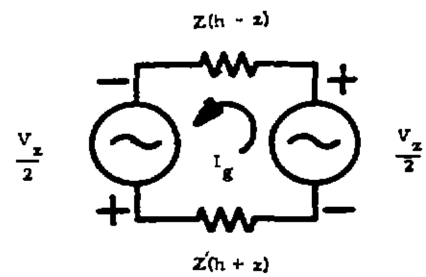

Fig. 4. Generator region of Fig. 3. $Z(h - z)$ and $Z(h + z)$ are the impedances looking toward the loads Z_h and Z_{-h}, respectively.

$$I_g = \frac{V_z}{Z(h - z) + Z(h + z)}. \tag{13}$$

Evidently, from (4),

$$Z(h - z) = Z_c \left\{ \frac{Z_h + Z_c \tanh \gamma(h - z)}{Z_c + Z_h \tanh \gamma(h - z)} \right\} \tag{14}$$

$$Z(h + z) = Z_c \left\{ \frac{Z_{-h} + Z_c \tanh \gamma(h + z)}{Z_c + Z_{-h} \tanh \gamma(h + z)} \right\}. \tag{15}$$

Also, from (3),

$$I(h) = \frac{I_g Z_c}{Z_c \cosh \gamma(h - z) + Z_h \sinh \gamma(h - z)} \tag{16}$$

$$I(-h) = \frac{I_g Z_c}{Z_c \cosh \gamma(h + z) + Z_{-h} \sinh \gamma(h + z)}. \tag{17}$$

Substituting (14) and (15) into (13), and (13) into (16) and (17), it is found after a reasonable amount of alge-

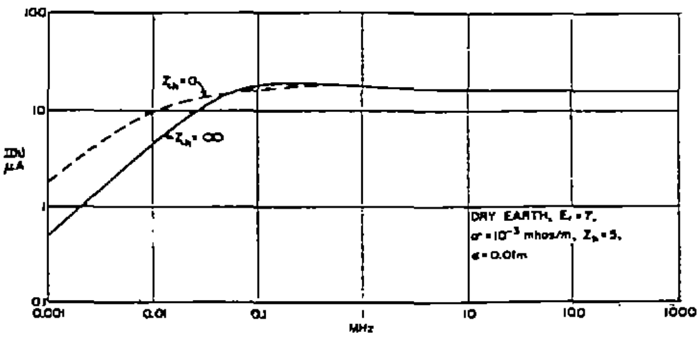

Fig. 5. Dry earth. $I(h)$ in μA versus frequency in MHz for $Z_h = 0$ and ∞, $d = 0.01$ m.

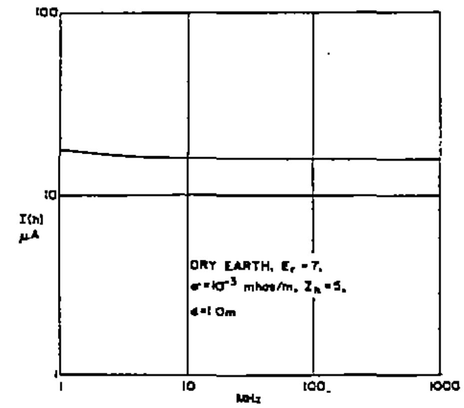

Fig. 6. Dry earth. Like Fig. 5, except for a reduced frequency range and $d = 1.0$ m.

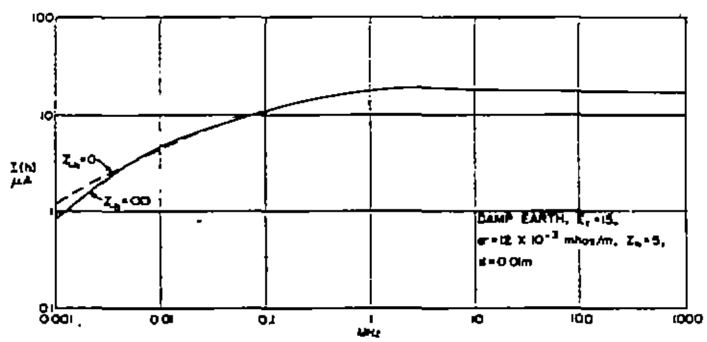

Fig. 7. Damp earth. Like Fig. 5.

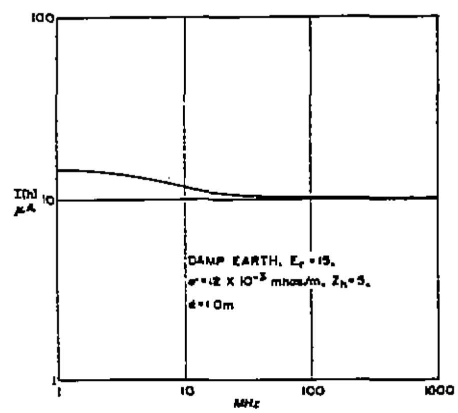

Fig. 8. Damp earth. Like Fig. 6.

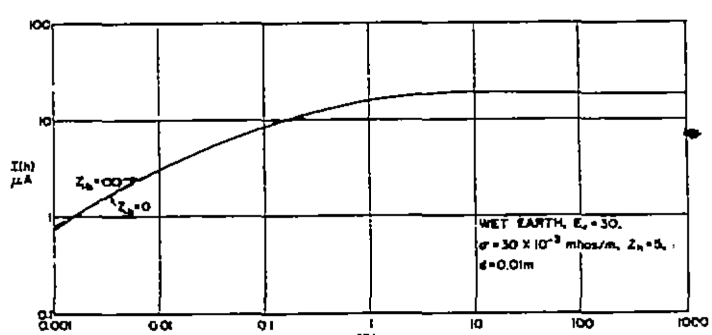

Fig. 9. Wet earth. Like Fig. 5.

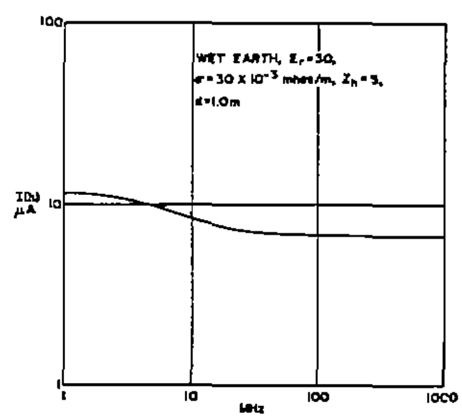

Fig. 10. Wet earth. Like Fig. 6.

The boundary conditions require continuity of the electric and magnetic fields at $x = 0$. Hence

$$E_0^i + E_0^r = E_0^t \tag{39}$$

$$k_0E_0^i - k_0E_0^r = k_1E_0^t. \tag{40}$$

In writing (40), use has been made of the relation

$$\nabla \times E = -j\omega B = -j\omega\mu H \tag{41}$$

so that in the present application,

$$-\nabla_y \times E = \frac{\partial E_z}{\partial x} = j\omega\mu H_y. \tag{42}$$

It follows that

$$E_0^t = \frac{2k_0E^i}{k_0 + k_1}. \tag{43}$$

Accordingly,

$$(E_z^t)_{\text{earth}} = \left(\frac{2k_0E_0^i}{k_0 + k_1}\right) e^{-jk_1x}. \tag{44}$$

Specifically,

$$E_d^t = \left(\frac{2k_0E_0^i}{k_0 + k_1}\right) e^{-jk_1d}. \tag{45}$$

Equation (45) gives the transmitted field at the midpoint of the line (a distance d below the earth–air boundary) in terms of the incident field E_0^i at $x = 0$.

Numerical Results

A two-wire transmission line for which $2h = 100$ m, $b = 2.5$ mm, $a = 0.5$ mm, $\sigma_e = 5.8 \times 10^7$ mho/m (copper), $Z_{-h} = 0$ or ∞ and $Z_h = 5$ ohms was selected for the numerical study. The line is buried in the earth an average depth d and is assumed to be oriented with respect to the incident field and earth–air boundary as illustrated in Fig. 1. The objective is to determine the response of the line for an incident field strength of 1 V/m, i.e., the current in the terminating impedance Z_h when the other end of the line is short-circuited or open-circuited ($Z_{-h} = 0$ or ∞, respectively).

Fig. 5 presents graphically the current $I(h)$ (in μA) in the load impedance $Z_h = 5$ ohms for $Z_{-h} = 0$ or ∞ when the line is buried $d = 0.01$ m in dry earth over the frequency range 10^2 to 10^9 Hz.

Note that the response of the line for $Z_{-h} = 0$ is somewhat greater than for $Z_{-h} = \infty$ when $f < 10^5$ Hz. Similar data are presented in Fig. 6 except that the frequency range is 10^4 to 10^9 Hz and $d = 1.0$ m. Observe that the results are the same whether $Z_{-h} = 0$ or ∞. Corresponding data are presented in Figs. 7 and 8 for damp earth, and in Figs. 9 and 10 for wet earth. Computations were also made for the case $Z_h = 1$ ohm and $Z_{-h} = 0$ or ∞, but for the sake of brevity these results are omitted. It can be stated that $I(h)$ cannot exceed by more than a factor of 2 the results reported here when $f < 10^5$ Hz. At higher frequencies the curves all merge whether or not $Z_{-h} = 0$ or ∞. Thus proof is presented that for a buried line it does not make any difference whether the line is open- or short-circuited provided $f > 10^5$ Hz and $1 \leq Z_h \leq 5$ ohms.

Conclusions

Within the restrictions stated in the text of this paper an impeccable solution to the problem of the response of a two-wire transmission line buried in a homogeneous earth to a plane-wave electromagnetic field generated in free space has been achieved. The principal restrictions imposed in the theory are that the electric field arriving at the conductors is directed parallel to their axes; one conductor of the line lies directly below the other, and the dielectric insulation surrounding the wires has negligible effect on the electrical properties of the line. Line losses as well as coupling to the dispersive medium are taken into account. There is no significant interaction of the line with the earth–air boundary if the conductor spacing is small compared to the distance of the wires to the interface. Also, for the polarization of the incident field chosen there is no interaction of the field with the line terminations.

Evidently, if the cord is twisted, i.e., one wire not directly below the other, the response to the incident radio frequency field will be reduced.

It has been shown that when $2\alpha h \geq 5$ and $|k_1b| \ll 1$, the formulas for the load currents are identical and assume a singularly simple form. Accordingly, from the radio-frequency hazard point of view (spurious radio signal emissions or lightning) it is immaterial whether the transmission line is open- or short-circuited prior to connection of the detonation battery by the road blasting crew.

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