

NOTES ON PLANAR, CYLINDRICAL
ELECTROMAGNETIC WAVES AND PROPAGATION
ON CIRCULARLY-SYMMETRIC TRANSMISSION LINES

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ABSTRACT

It is the purpose of these notes to present a relatively detailed, self-consistent solution for the propagation of electromagnetic energy along circularly symmetric conductors. Numerical results are presented in addition to the theoretical summary. It is the intent of these notes to summarize in one convenient document the necessary procedures for calculating the propagation constants and characteristic impedance of buried insulated cables.

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CHAPTER 1
INTRODUCTION

1.1 GENERAL

Recently, considerable interest has arisen on the subject of induced currents in buried cables. As a consequence, a considerable amount of literature on the subject has been generated, and unfortunately a considerable amount of inconsistency has appeared. The difficulty arises primarily from two sources. First, two conventions exist concerning sinusoidal time variations --- $\exp(+i\omega t)$ and $\exp(-i\omega t)$; and second, the definition and use of propagation constants, together with the associated value of the square root relation, varies from reference to reference.

For these reasons, it is the purpose of these notes to operate a relatively detailed, self-consistent solution for the propagation of electromagnetic energy along circularly-symmetric conductors. In addition, sample numerical values are presented for the behavior of propagation in an infinite conductive medium. These values are compared with those obtained from conventional transmission line theory, and the direction of future investigation is indicated.

The material herein makes use of sinusoidal time variations of the form $\exp(+i\omega t)$, so that inductive reactance is positive and capacitive reactance is negative.

CHAPTER 2

PROPAGATION ALONG AN INFINITELY LONG, INSULATED, CYLINDRICAL CONDUCTOR BURIED IN AN INFINITE HOMOGENEOUS CONDUCTING MEDIUM

2.1 ANALYTICAL RESULTS

The material in this chapter considers solutions to Maxwell's equations (Appendices B, C and D) for propagation along an insulated conductor buried in an infinite, homogeneous, conducting medium. An analysis similar to this situation is considered in Reference 2, pp. 545-554. Here, however, all modes are considered, not just those which are transverse magnetic. In addition sinusoidal variations of the form $\exp(+i\omega t)$ are considered. Finally, the determinantal equation for propagation constant is numerically evaluated for a specific cable configuration.

Figure 2-1 is an illustration of the geometry under consideration for the infinitely-long, insulated conductor. The electromagnetic fields which can exist in regions 1, 2 and 3 are (see Appendix D and Equations D-34 through D-39)

Region 1: ($r < r_i$)

$$E_r^1 = \sum_{\nu=-\infty}^{\infty} \left[-\frac{\omega\lambda_1 h}{\gamma_1^2} J'_\nu(\lambda_1 r) a_\nu^1 - \frac{i\nu}{r} J_\nu(\lambda_1 r) b_\nu^1 \right] K_\nu, \quad (2-1)$$

$$E_\theta^1 = \sum_{\nu=-\infty}^{\infty} \left[-\frac{i\omega h \nu}{\gamma_1^2 r} J_\nu(\lambda_1 r) a_\nu^1 + \lambda_1 J'_\nu(\lambda_1 r) b_\nu^1 \right] K_\nu, \quad (2-2)$$

$$E_z^1 = \sum_{\nu=-\infty}^{\infty} \left[-\frac{i\omega\lambda_1^2}{\gamma_1^2} J_\nu(\lambda_1 r) a_\nu^1 \right] K_\nu, \quad (2-3)$$

$$B_r^1 = \sum_{\nu=-\infty}^{\infty} \left[\frac{i\nu}{r} J_\nu(\lambda_1 r) a_\nu^1 - \frac{h\lambda_1}{\omega} J'_\nu(\lambda_1 r) b_\nu^1 \right] K_\nu, \quad (2-4)$$

$$B_{\theta}^1 = \sum_{\nu=-\infty}^{\infty} \left[-\lambda_1 J_{\nu}'(\lambda_1 r) a_{\nu}^1 + \frac{\nu h}{i\omega} J_{\nu}(\lambda_1 r) b_{\nu}^1 \right] K_{\nu}, \quad (2-5)$$

$$B_z^1 = \sum_{\nu=-\infty}^{\infty} \left[\frac{\lambda_1^2}{i\omega} J_{\nu}(\lambda_1 r) b_{\nu}^1 \right] K_{\nu}; \quad (2-6)$$

Region 2: ($r_i < r < r_o$)

$$E_r^2 = \sum_{\nu=-\infty}^{\infty} \left\{ -\frac{\omega \lambda_2 h}{\gamma_2} \left[J_{\nu}'(\lambda_2 r) a_{\nu}^2 + Y_{\nu}'(\lambda_2 r) \overline{a_{\nu}^2} \right] - \frac{i\nu}{r} \left[J_{\nu}(\lambda_2 r) b_{\nu}^2 + Y_{\nu}(\lambda_2 r) \overline{b_{\nu}^2} \right] \right\} K_{\nu}, \quad (2-7)$$

$$E_{\theta}^2 = \sum_{\nu=-\infty}^{\infty} \left\{ -\frac{i\omega h \nu}{\gamma_2 r} \left[J_{\nu}(\lambda_2 r) a_{\nu}^2 + Y_{\nu}(\lambda_2 r) \overline{a_{\nu}^2} \right] + \lambda_2 \left[J_{\nu}'(\lambda_2 r) b_{\nu}^2 + Y_{\nu}'(\lambda_2 r) \overline{b_{\nu}^2} \right] \right\} K_{\nu}, \quad (2-8)$$

$$E_z^2 = \sum_{\nu=-\infty}^{\infty} \left\{ -\frac{i\omega \lambda_2^2}{\gamma_2} \left[J_{\nu}(\lambda_2 r) a_{\nu}^2 + Y_{\nu}(\lambda_2 r) \overline{a_{\nu}^2} \right] \right\} K_{\nu}, \quad (2-9)$$

$$B_r^2 = \sum_{\nu=-\infty}^{\infty} \left\{ \frac{i\nu}{r} \left[J_{\nu}(\lambda_2 r) a_{\nu}^2 + Y_{\nu}(\lambda_2 r) \overline{a_{\nu}^2} \right] - \frac{h\lambda_2}{\omega} \left[J_{\nu}'(\lambda_2 r) b_{\nu}^2 + Y_{\nu}'(\lambda_2 r) \overline{b_{\nu}^2} \right] \right\} K_{\nu}, \quad (2-10)$$

$$B_{\theta}^2 = \sum_{\nu=-\infty}^{\infty} \left\{ -\lambda_2 \left[J_{\nu}'(\lambda_2 r) a_{\nu}^2 + Y_{\nu}'(\lambda_2 r) \overline{a_{\nu}^2} \right] + \frac{\nu h}{i\omega} \left[J_{\nu}(\lambda_2 r) b_{\nu}^2 + Y_{\nu}(\lambda_2 r) \overline{b_{\nu}^2} \right] \right\} K_{\nu}, \quad (2-11)$$

$$B_z^2 = \sum_{\nu=-\infty}^{\infty} \left\{ \frac{\lambda_2^2}{i\omega} \left[J_{\nu}(\lambda_2 r) b_{\nu}^2 + Y_{\nu}(\lambda_2 r) \overline{b_{\nu}^2} \right] \right\} K_{\nu}; \quad (2-12)$$

Region 3: ($r > r_0$)

$$E_r^3 = \sum_{\nu=-\infty}^{\infty} \left[-\frac{\omega\lambda_3 h}{\gamma_3^2} H_\nu^{(2)'}(\lambda_3 r) a_\nu^3 - \frac{i\nu}{r} H_\nu^{(2)}(\lambda_3 r) b_\nu^3 \right] K_\nu, \quad (2-13)$$

$$E_\theta^3 = \sum_{\nu=-\infty}^{\infty} \left[-\frac{i\omega h \nu}{\gamma_3^2 r} H_\nu^{(2)'}(\lambda_3 r) a_\nu^3 + \lambda_3 H_\nu^{(2)'}(\lambda_3 r) b_\nu^3 \right] K_\nu, \quad (2-14)$$

$$E_z^3 = \sum_{\nu=-\infty}^{\infty} \left[-\frac{i\omega\lambda_3^2}{\gamma_3^2} H_\nu^{(2)}(\lambda_3 r) a_\nu^3 \right] K_\nu, \quad (2-15)$$

$$B_r^3 = \sum_{\nu=-\infty}^{\infty} \left[\frac{i\nu}{r} H_\nu^{(2)}(\lambda_3 r) a_\nu^3 - \frac{h\lambda_3}{\omega} H_\nu^{(2)'}(\lambda_3 r) b_\nu^3 \right] K_\nu, \quad (2-16)$$

$$B_\theta^3 = \sum_{\nu=-\infty}^{\infty} \left[-\lambda_3 H_\nu^{(2)'}(\lambda_3 r) a_\nu^3 + \frac{\nu h}{i\omega} H_\nu^{(2)}(\lambda_3 r) b_\nu^3 \right] K_\nu, \quad (2-17)$$

$$B_z^3 = \sum_{\nu=-\infty}^{\infty} \left[\frac{\lambda_3^2}{i\omega} H_\nu^{(2)}(\lambda_3 r) b_\nu^3 \right] K_\nu; \quad (2-18)$$

where: $K_\nu = \epsilon^{i\nu\theta} \epsilon^{-ihz}$,

a_ν and b_ν are associated with transverse magnetic and transverse electric waves, respectively.

The constant coefficients in Equations 2-1 through 2-18, together with the propagation constant (h), are determined from the electromagnetic boundary conditions; that is, the tangential components of both electric and magnetic fields must be continuous across the boundaries at both $r = r_1$ and $r = r_0$. If the tangential components of the series of Equations 2-1 through 2-18 are equated term by term at these boundaries, then eight, linear, homogeneous equations results for each ν . Since there

TABLE 2-1

BOUNDARY CONDITION EQUATIONS FOR THE GENERAL CASE

$$-\frac{i\omega h\nu}{\gamma_1 r} J_\nu(\lambda_1 r_1) a_\nu^1 + \lambda_1 J'_\nu(\lambda_1 r_1) b_\nu^1 = -\frac{i\omega h\nu}{\gamma_2 r_1} \left[J_\nu(\lambda_2 r_1) a_\nu^2 + Y_\nu(\lambda_2 r_1) \overline{a_\nu^2} \right] + \lambda_2 \left[J'_\nu(\lambda_2 r_1) b_\nu^2 + Y'_\nu(\lambda_2 r_1) \overline{b_\nu^2} \right] \quad (\text{T2-1.1})$$

$$-\frac{i\omega \lambda_1^2}{\gamma_1} J_\nu(\lambda_1 r_1) a_\nu^1 = -\frac{i\omega \lambda_2^2}{\gamma_2} \left[J_\nu(\lambda_2 r_1) a_\nu^2 + Y_\nu(\lambda_2 r_1) \overline{a_\nu^2} \right] \quad (\text{T2-1.2})$$

$$-\lambda_1 J'_\nu(\lambda_1 r_1) a_\nu^1 + \frac{\nu h}{i\omega} J_\nu(\lambda_1 r_1) b_\nu^1 = -\lambda_2 \left[J'_\nu(\lambda_2 r_1) a_\nu^2 + Y'_\nu(\lambda_2 r_1) \overline{a_\nu^2} \right] + \frac{\nu h}{i\omega} \left[J_\nu(\lambda_2 r_1) b_\nu^2 + Y_\nu(\lambda_2 r_1) \overline{b_\nu^2} \right] \quad (\text{T2-1.3})$$

$$\frac{\lambda_1^2}{i\omega} J_\nu(\lambda_1 r_1) b_\nu^1 = \frac{\lambda_2^2}{i\omega} \left[J_\nu(\lambda_2 r_1) b_\nu^2 + Y_\nu(\lambda_2 r_1) \overline{b_\nu^2} \right] \quad (\text{T2-1.4})$$

$$-\frac{i\omega h\nu}{\gamma_2 r_0} \left[J_\nu(\lambda_2 r_0) a_\nu^2 + Y_\nu(\lambda_2 r_0) \overline{a_\nu^2} \right] + \lambda_2 \left[J'_\nu(\lambda_2 r_0) b_\nu^2 + Y'_\nu(\lambda_2 r_0) \overline{b_\nu^2} \right] = -\frac{i\omega h\nu}{\gamma_3 r_0} H_\nu^{(2)}(\lambda_3 r_0) a_\nu^3 + \lambda_3 H_\nu^{(2)}(\lambda_3 r_0) b_\nu^3 \quad (\text{T2-1.5})$$

$$-\frac{i\omega \lambda_2^2}{\gamma_2} \left[J_\nu(\lambda_2 r_0) a_\nu^2 + Y_\nu(\lambda_2 r_0) \overline{a_\nu^2} \right] = -\frac{i\omega \lambda_3^2}{\gamma_3} H_\nu^{(2)}(\lambda_3 r_0) a_\nu^3 \quad (\text{T2-1.6})$$

$$-\lambda_2 \left[J'_\nu(\lambda_2 r_0) a_\nu^2 + Y'_\nu(\lambda_2 r_0) \overline{a_\nu^2} \right] + \frac{\nu h}{i\omega} \left[J_\nu(\lambda_2 r_0) b_\nu^2 + Y_\nu(\lambda_2 r_0) \overline{b_\nu^2} \right] = -\lambda_3 H_\nu^{(2)}(\lambda_3 r_0) a_\nu^3 + \frac{\nu h}{i\omega} H_\nu^{(2)}(\lambda_3 r_0) b_\nu^3 \quad (\text{T2-1.7})$$

$$\frac{\lambda_2^2}{i\omega} \left[J_\nu(\lambda_2 r_0) b_\nu^2 + Y_\nu(\lambda_2 r_0) \overline{b_\nu^2} \right] = \frac{\lambda_3^2}{i\omega} H_\nu^{(2)}(\lambda_3 r_0) b_\nu^3 \quad (\text{T2-1.8})$$

TABLE 2-2

BOUNDARY CONDITION EQUATIONS FOR THE SYMMETRIC CASE ($\nu=0$)

$$\lambda_1 J'_0(\lambda_1 r_1) b_0^1 = \lambda_2 \left[J'_0(\lambda_2 r_1) b_0^2 + Y'_0(\lambda_2 r_1) \overline{b_0^2} \right] \quad (T2-2.1)$$

$$-\frac{i\omega\lambda_1^2}{\gamma_1} J_0(\lambda_1 r_1) a_0^1 = -\frac{i\omega\lambda_2^2}{\gamma_2} \left[J_0(\lambda_2 r_1) a_0^2 + Y_0(\lambda_2 r_1) \overline{a_0^2} \right] \quad (T2-2.2)$$

$$-\lambda_1 J'_0(\lambda_1 r_1) a_0^1 = -\lambda_2 \left[J'_0(\lambda_2 r_1) a_0^2 + Y'_0(\lambda_2 r_1) \overline{a_0^2} \right] \quad (T2-2.3)$$

$$\frac{\lambda_1^2}{i\omega} J_0(\lambda_1 r_1) b_0^1 = \frac{\lambda_2^2}{i\omega} \left[J_0(\lambda_2 r_1) b_0^2 + Y_0(\lambda_2 r_1) \overline{b_0^2} \right] \quad (T2-2.4)$$

$$\lambda_2 \left[J'_0(\lambda_2 r_0) b_0^2 + Y'_0(\lambda_2 r_0) \overline{b_0^2} \right] = \lambda_3 H_0^{(2)'}(\lambda_3 r_0) b_0^3 \quad (T2-2.5)$$

$$-\frac{i\omega\lambda_2^2}{\gamma_2} \left[J_0(\lambda_2 r_0) a_0^2 + Y_0(\lambda_2 r_0) \overline{a_0^2} \right] = -\frac{i\omega\lambda_3^2}{\gamma_3} H_0^{(2)}(\lambda_3 r_0) a_0^3 \quad (T2-2.6)$$

$$-\lambda_2 \left[J'_0(\lambda_2 r_0) a_0^2 + Y'_0(\lambda_2 r_0) \overline{a_0^2} \right] = -\lambda_3 H_0^{(2)'}(\lambda_3 r_0) a_0^3 \quad (T2-2.7)$$

$$\frac{\lambda_2^2}{i\omega} \left[J_0(\lambda_2 r_0) b_0^2 + Y_0(\lambda_2 r_0) \overline{b_0^2} \right] = \frac{\lambda_3^2}{i\omega} H_0^{(2)}(\lambda_3 r_0) b_0^3 \quad (T2-2.8)$$

Note: Equations T2-2.1, T2-2.4, T2-2.5 and T2-2.8 are decoupled from Equations T2-2.2, T2-2.3, T2-2.6 and T2-2.7. The former are associated with transverse electric waves, while the latter are associated with transverse magnetic waves.

Matrix entries for Table 2-2 equations showing decoupling of equations

a_0^1	b_0^1	a_0^2	$\overline{a_0^2}$	b_0^2	$\overline{b_0^2}$	a_0^3	b_0^3	Equation
0	⊗	0	0	⊗	⊗	0	0	T2-2.1
✓	0	✓	✓	0	0	0	0	T2-2.2
✓		✓	✓	0	0	0	0	T2-2.3
0	⊗	0	0	⊗	⊗	0	0	T2-2.4
0	0	0	0	⊗	⊗	0	⊗	T2-2.5
0	0	✓	✓	0	0	✓	0	T2-2.6
0	0	✓	✓	0	0	✓	0	T2-2.7
0	0	0	0	○	○	0	○	T2-2.8

are eight unknown coefficients, generally speaking both transverse magnetic and transverse electric fields are necessary to satisfy the boundary conditions. An exception to this latter occurs when the fields are angularly symmetric with $\nu = 0$. Table 2-1 shows the equations which must be satisfied in the general case, while Table 2-2 shows the resulting equations when $\nu = 0$. It can be seen from Table 2-2 that the original set of eight coupled equations has degenerated into two sets of four uncoupled equations, one set involving transverse electric propagation and the other set involving transverse magnetic propagation. Therefore, either isolated transverse electric or transverse magnetic propagation can only exist for angularly symmetric propagation modes.

The boundary condition equations shown in Tables 2-1 and 2-2 are homogeneous; thus, in order for a solution to exist, the determinant of their coefficients must vanish. Of course in the case of Table 2-2, two fourth order determinants establishes conditional equations for the propagation constant (h).

For the angularly symmetric ($\nu = 0$) case, the equations which h must satisfy are:

Transverse Magnetic Modes:

$$\frac{a_o^2}{a_o^2} = \frac{\left[\frac{Y'_o(\lambda_2 r_i)}{J'_o(\lambda_1 r_i)} - \frac{\lambda_1^2 \lambda_2}{\lambda_2^2 \lambda_1} \frac{Y_o(\lambda_2 r_i)}{J_o(\lambda_1 r_i)} \right]}{\left[\frac{\gamma_1^2 \lambda_2}{\lambda_1 \gamma_2} \frac{J_o(\lambda_2 r_i)}{J_o(\lambda_1 r_i)} - \frac{J'_o(\lambda_2 r_i)}{J'_o(\lambda_1 r_i)} \right]} = \frac{\left[\frac{Y'_o(\lambda_2 r_o)}{H_o^{(2)'(\lambda_3 r_o)} - \frac{\gamma_3^2 \lambda_2}{\gamma_3 \lambda_3} \frac{Y_o(\lambda_2 r_o)}{H_o^{(2)'(\lambda_3 r_o)}} \right]}{\left[\frac{\gamma_3^2 \lambda_2}{\gamma_2 \lambda_3} \frac{J_o(\lambda_2 r_o)}{H_o^{(2)'(\lambda_3 r_o)} - \frac{J'_o(\lambda_2 r_o)}{H_o^{(2)'(\lambda_3 r_o)}} \right]} ; \quad (2-19)$$

Transverse Electric Modes:

$$\frac{b_o^2}{b_o^2} = \frac{\left[\frac{\lambda_2}{\lambda_1} \frac{Y_o(\lambda_2 r_i)}{J_o(\lambda_1 r_i)} - \frac{Y'_o(\lambda_2 r_i)}{J'_o(\lambda_1 r_i)} \right]}{\left[\frac{J'_o(\lambda_2 r_i)}{J'_o(\lambda_1 r_i)} - \frac{\lambda_2}{\lambda_1} \frac{J_o(\lambda_2 r_i)}{J_o(\lambda_1 r_i)} \right]} = \frac{\left[\frac{\lambda_2}{\lambda_3} \frac{Y_o(\lambda_2 r_o)}{H_o^{(2)'(\lambda_3 r_o)} - \frac{Y'_o(\lambda_2 r_o)}{H_o^{(2)'(\lambda_3 r_o)}} \right]}{\left[\frac{J'_o(\lambda_2 r_o)}{H_o^{(2)'(\lambda_3 r_o)} - \frac{\lambda_2}{\lambda_3} \frac{J_o(\lambda_2 r_o)}{H_o^{(2)'(\lambda_3 r_o)}} \right]} ; \quad (2-20)$$

Many important applications of the preceding theory are applied to situations in which the center conductor is an excellent (metallic) conductor. In these cases, the propagation mode away from the source is primarily transverse magnetic (see Reference 2) because the attenuation constant for transverse electric propagation is quite large. For this reason, it is common practice to consider all propagation in these geometries as transverse magnetic.

2.2 Numerical Results

The roots of Equation 2-19⁽¹⁾ were numerically evaluated as a function of frequency by means of a Newton-Rapheson procedure. For numerical purposes, the following parameters were used: (refer to Figure 2-1)

$$r_i = 1.79 \times 10^{-2} \text{ meters,}$$

$$r_o = 3.588 \times 10^{-2} \text{ meters,}$$

medium 1 - copper conductor,

medium 2 - insulator with $\sigma_2=0$, $\epsilon_2=4\epsilon_o$,

medium 3 - conductor with $\epsilon_3=4\epsilon_o$, $\sigma=10^{-2}$, 10^{-3}
and 10^{-4} mhos/meter.

The results of the evaluation are shown in various forms in Figures 2-2 through 2-8. Figures 2-2 through 2-7 show the real and imaginary parts of the roots versus frequency for conductivities of 10^{-4} , 10^{-3} and 10^{-2} mhos/meter in medium 3. The numerical evaluations were terminated at 10^4 - 10^5 Hz because of numerical problems in root evaluation. It is intended to evaluate the roots at higher frequencies. Figure 2-8 is a root locus plot of the roots for two medium 3 conductivities, 10^{-2} and 10^{-4} mhos/meter. It can be seen from

(1) The actual equation used for solution was taken from Reference 2, equation 11, p. 547.

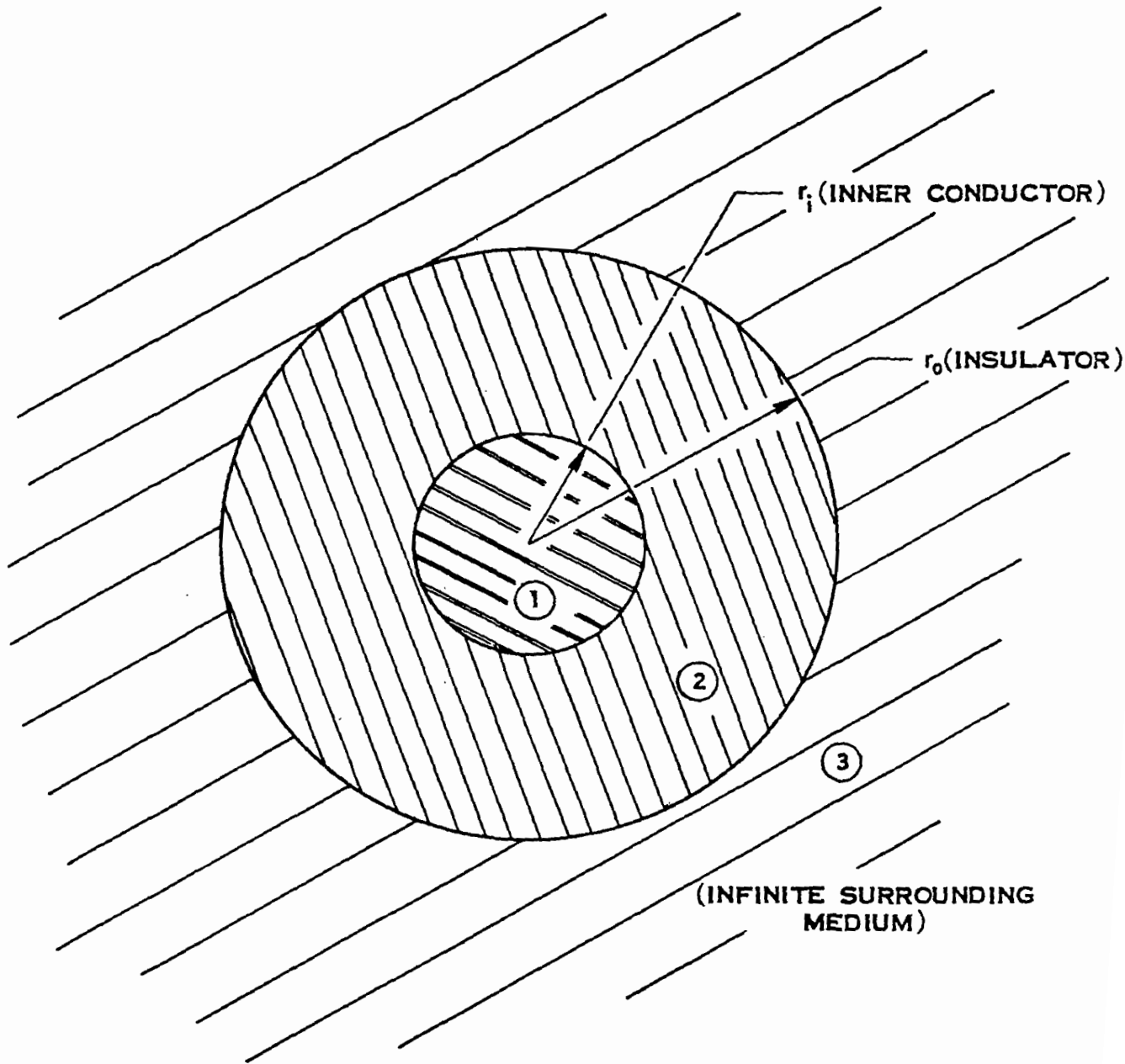


FIGURE 2-1

GEOMETRY OF INSULATED CABLE BURIED IN AN INFINITE MEDIUM

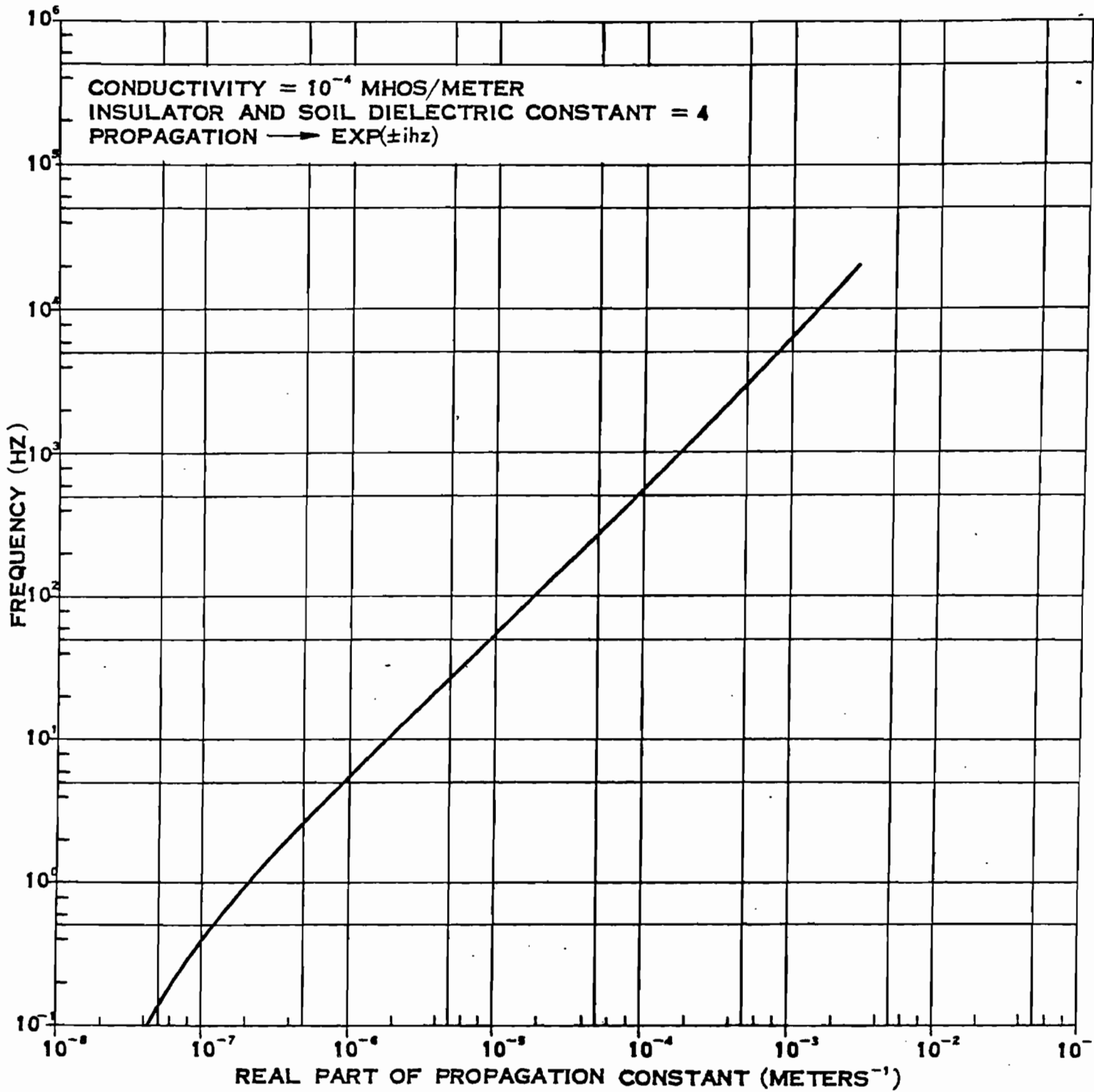


FIGURE 2-2

REAL PART OF PROPAGATION CONSTANT FOR INSULATED CONDUCTOR
BURIED IN INFINITE, CONDUCTIVE SOIL

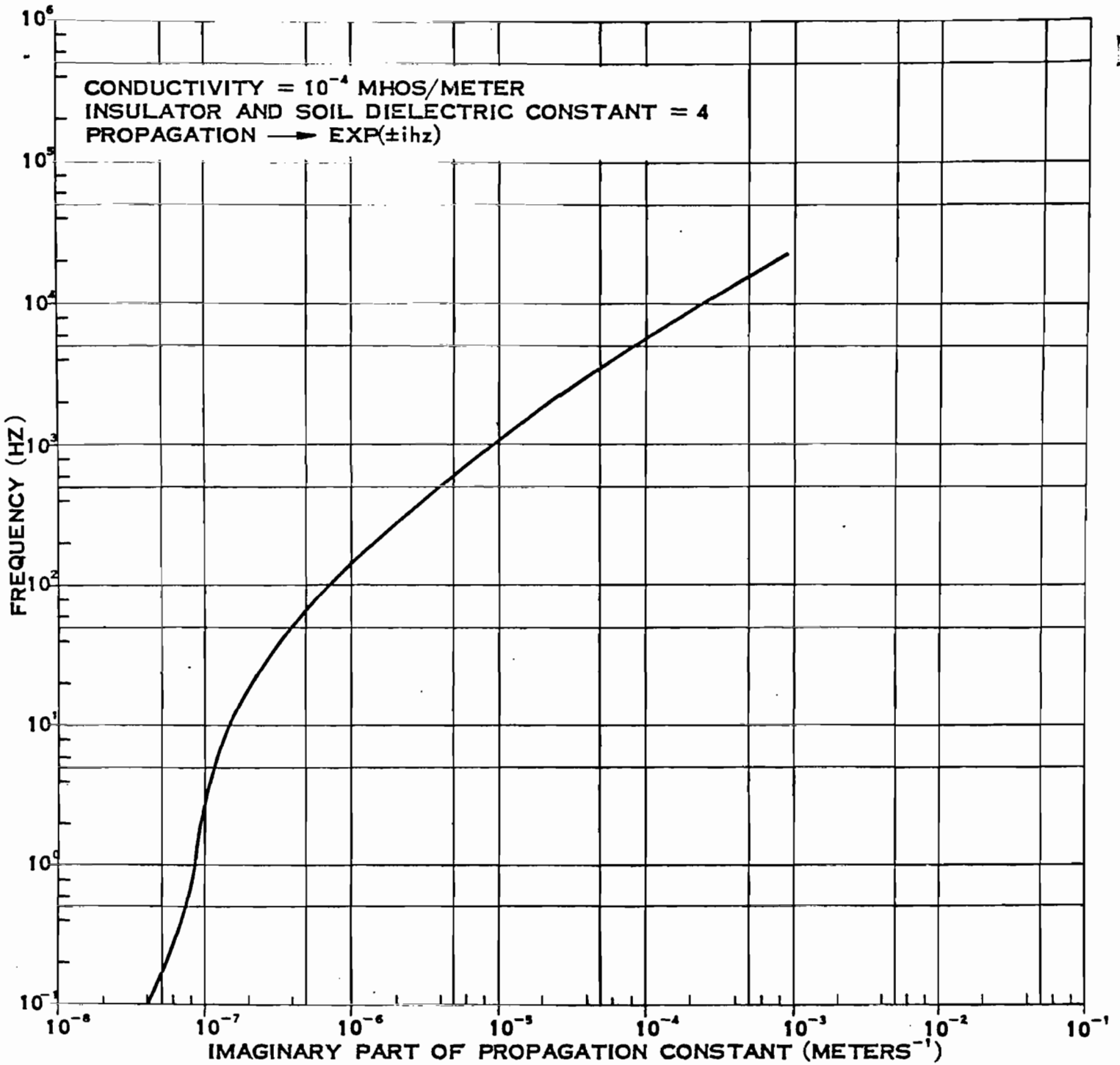


FIGURE 2-3

IMAGINARY PART OF PROPAGATION CONSTANT FOR INSULATED CONDUCTOR
BURIED IN INFINITE, CONDUCTIVE SOIL

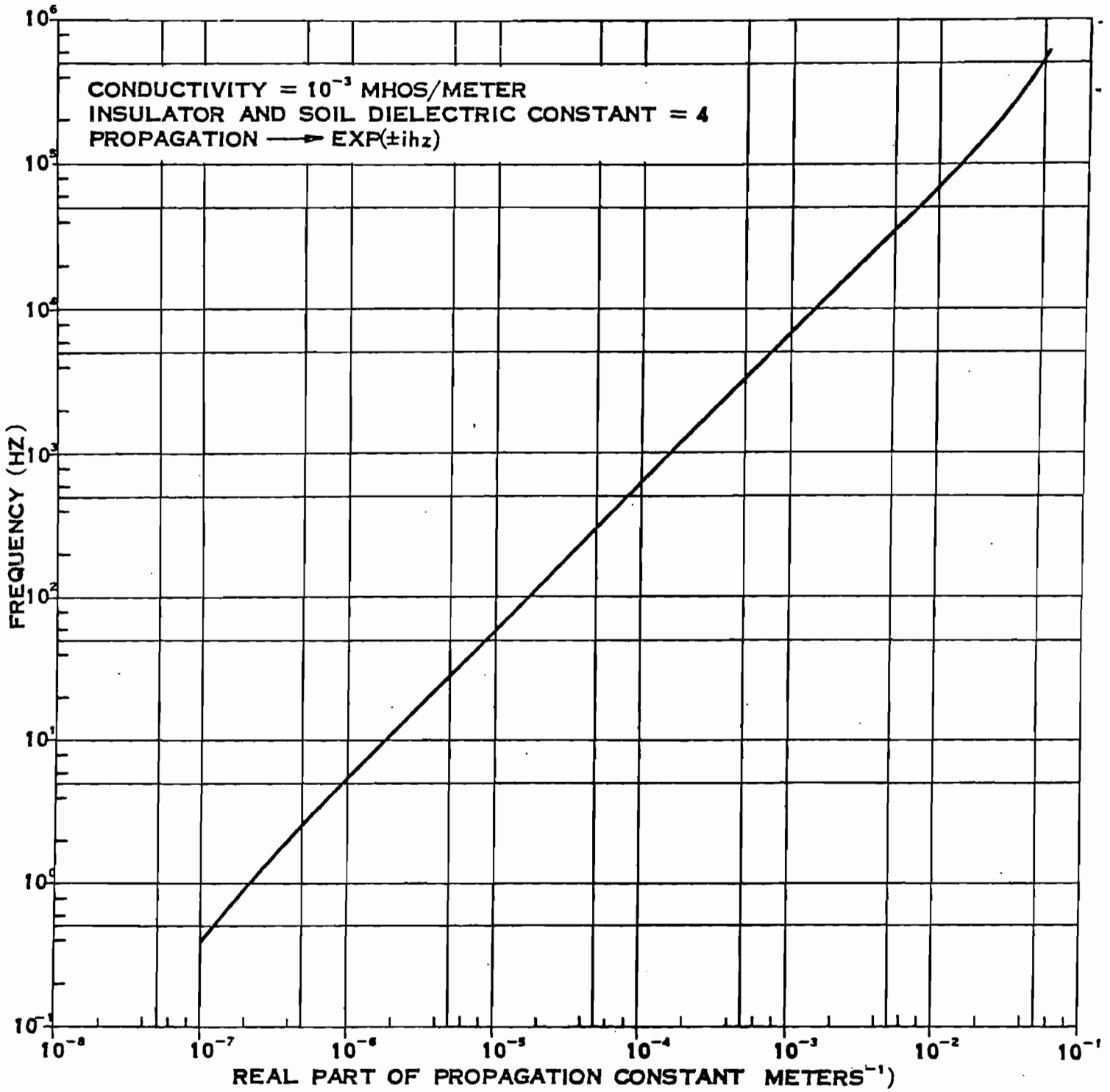


FIGURE 2-4

REAL PART OF PROPAGATION CONSTANT FOR INSULATED CONDUCTOR
BURIED INFINITE, CONDUCTIVE SOIL

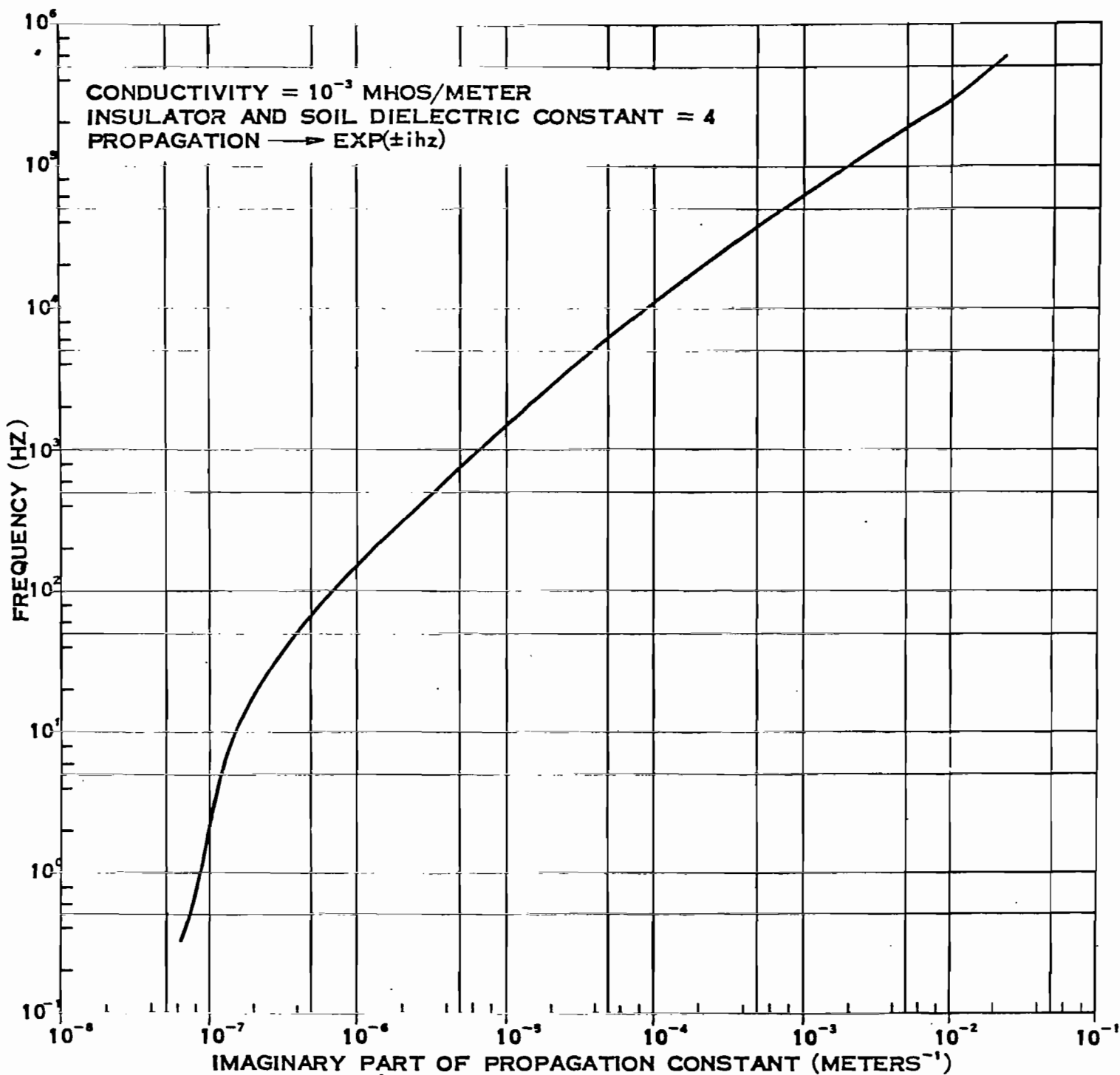


FIGURE 2-5

IMAGINARY PART OF PROPAGATION CONSTANT FOR INSULATED CONDUCTOR
BURIED IN INFINITE, CONDUCTIVE SOIL

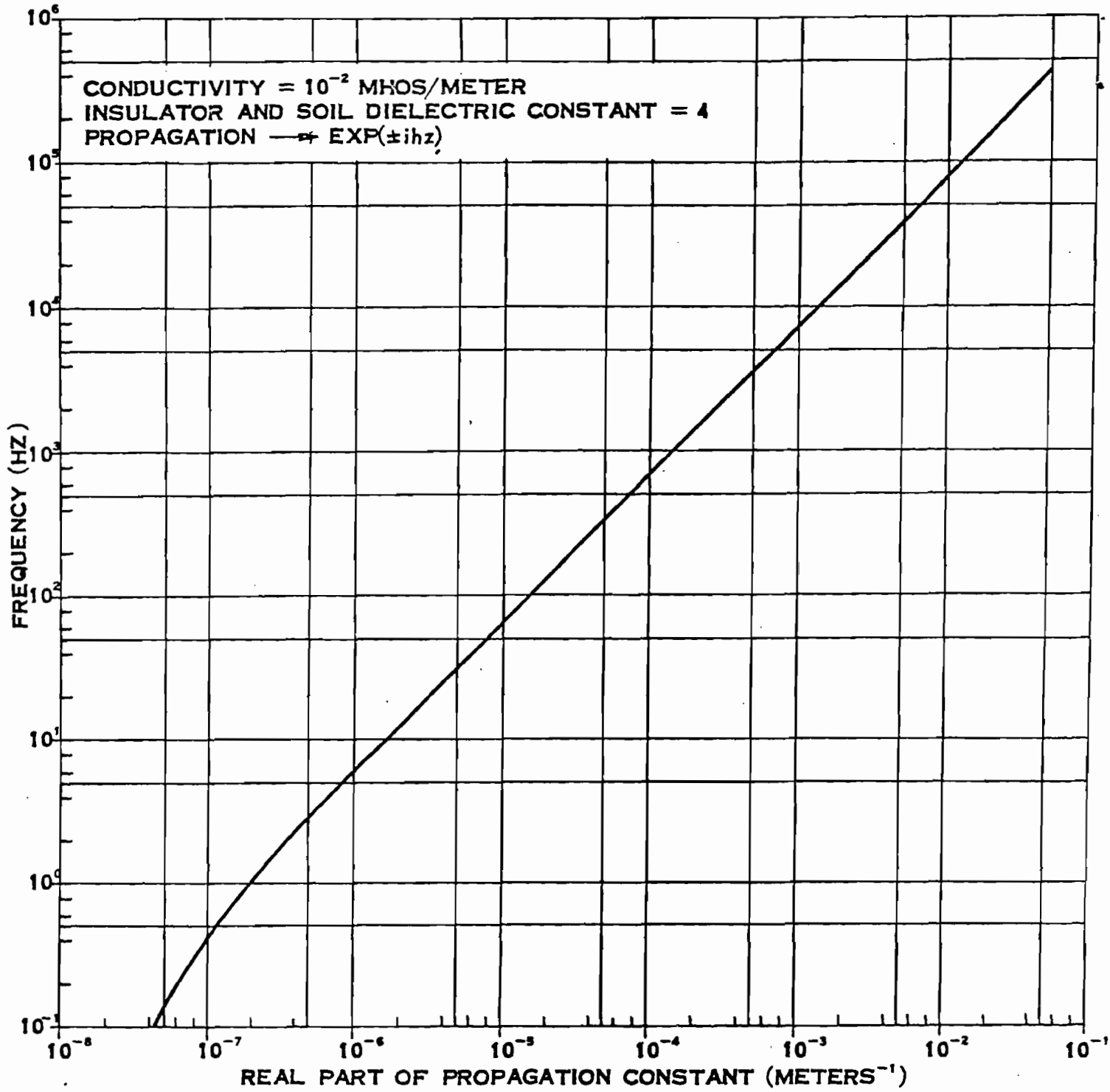


FIGURE 2-6

REAL PART OF PROPAGATION CONSTANT FOR INSULATED CONDUCTOR
BURIED IN INFINITE, CONDUCTIVE SOIL

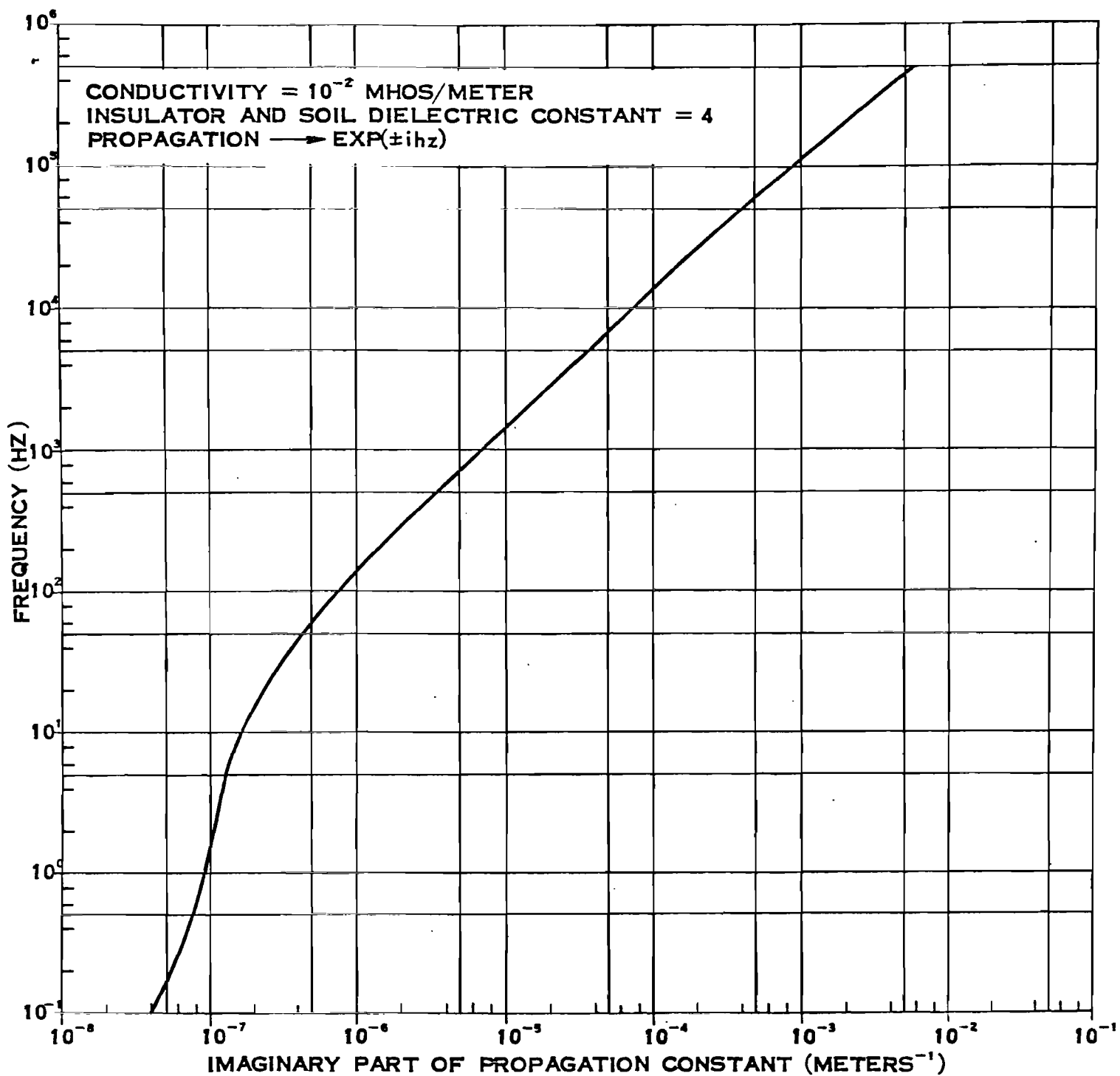


FIGURE 2-7

IMAGINARY PART OF PROPAGATION CONSTANT FOR INSULATED CONDUCTOR
BURIED IN INFINITE, CONDUCTIVE SOIL

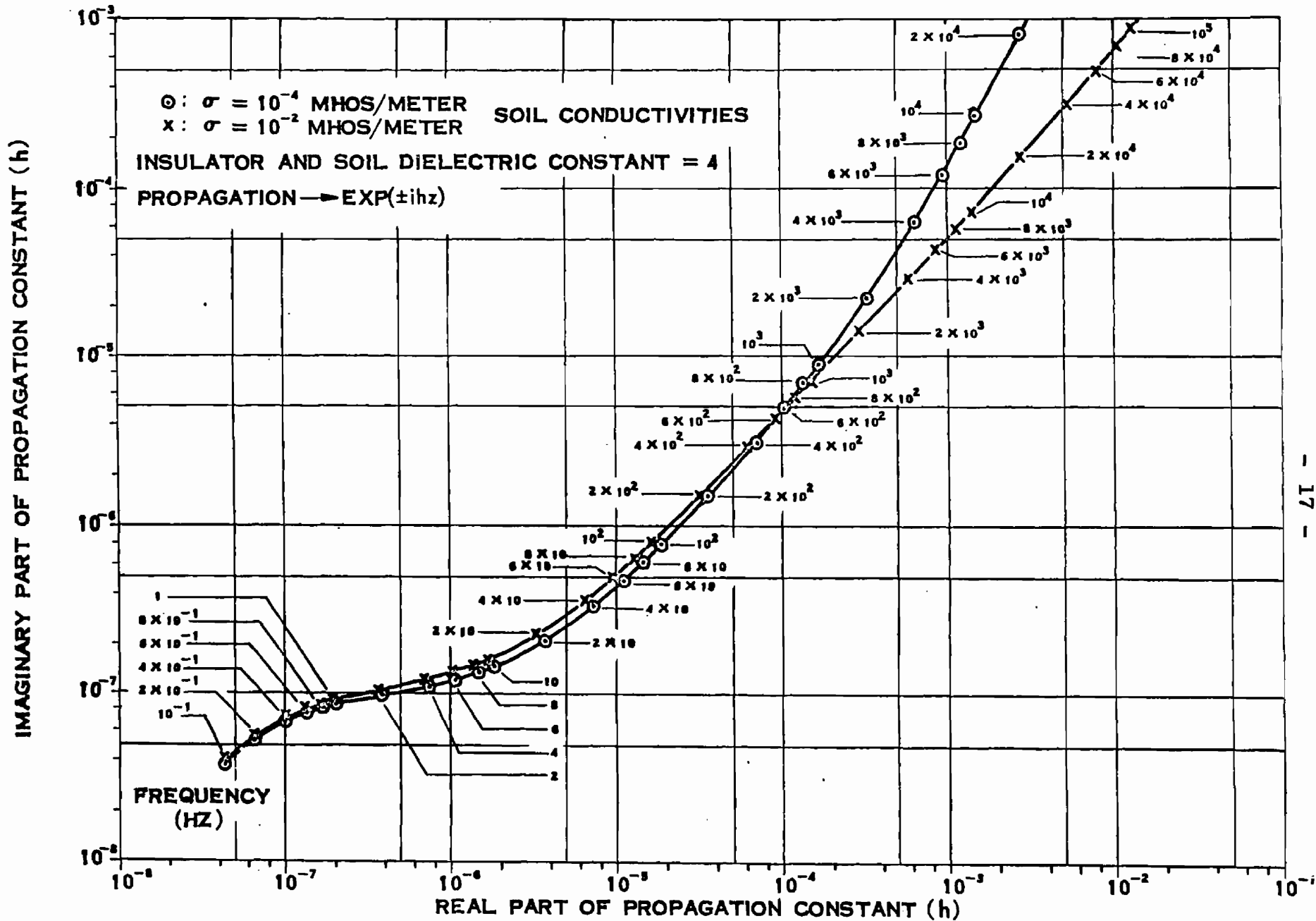


FIGURE 2-8

INFINITE MEDIUM PROPAGATION CONSTANT FOR INSULATED CONDUCTOR BURIED IN INFINITE, CONDUCTIVE SOIL

this illustration that the roots are relatively insensitive to conductivity below 10^3 Hz, for this particular geometry and conductivity range. This fact is perhaps better shown in Figure 2-9, which is a plot of the attenuation part (imaginary part) for the roots at three frequencies. For sufficiently high medium 3 conductivity ($> 10^3$ mhos/meter), the attenuation on the line decreases, which is consistent with the fact that the attenuation should be quite low, if medium 3 were metallic.

The preceding discussion implies that there exists only one root (for fixed frequency) of Equation 2-19. Actually this transcendental equation has an infinity of roots. It is believed that the root shown in the preceding figures is the principle root; i.e., the root with the lowest attenuation constant. Figures 2-10 through 2-12 are presented to support this contention. These graphs are plots of the magnitude of Equation 2-19 in the vicinity of a root, and tend to show that another root is not present within a similar distance of the origin. Although not shown on Figures 2-10 through 2-12, these figures are symmetric about a ray through the origin and the root and a ray through the origin and orthogonal to the root ray. That is, the root and its negative should be present. Finally, numerical attempts to find additional roots always resulted in roots whose real parts were greater than 100.

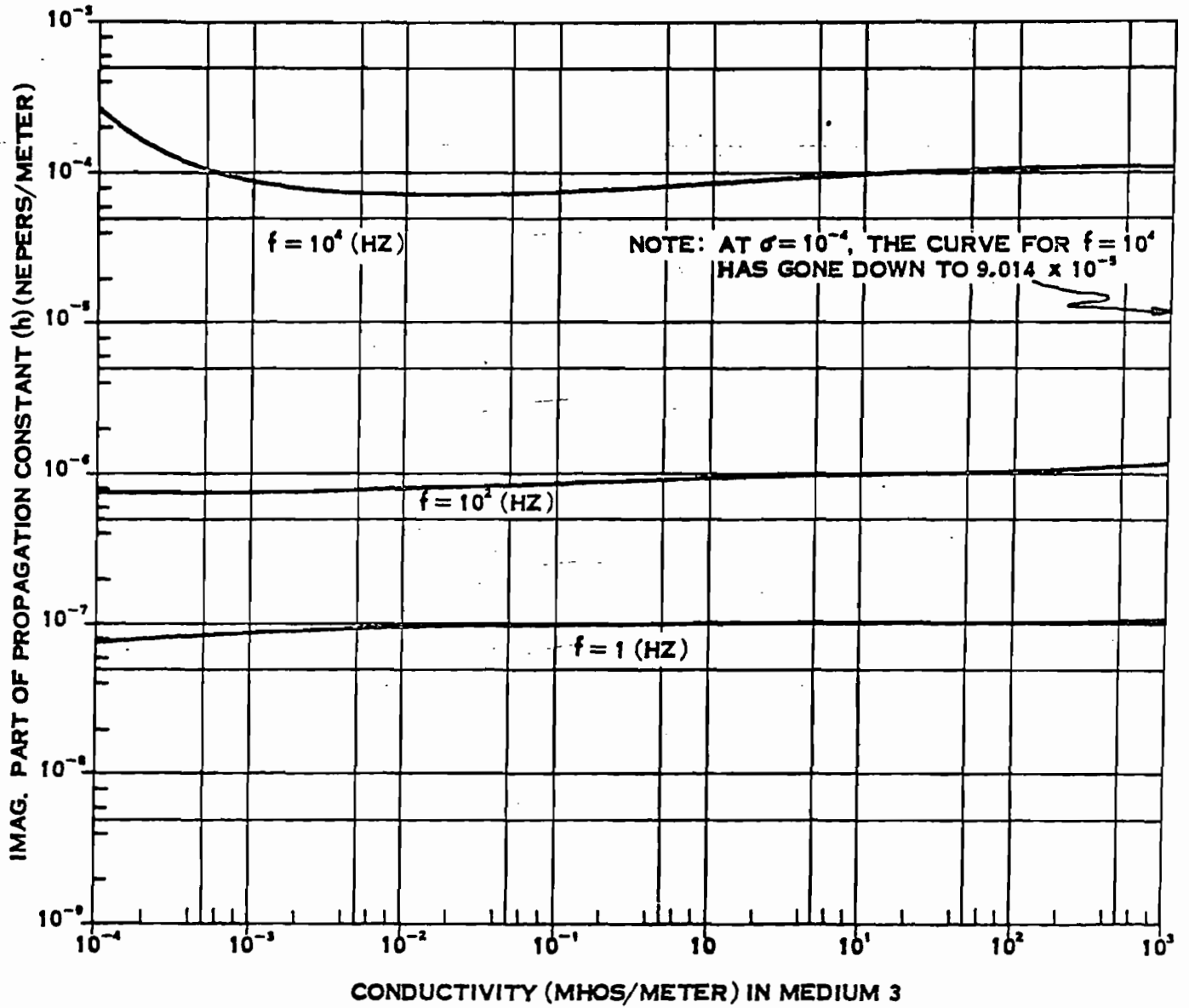


FIGURE 2-9

ATTENUATION PORTION OF INSULATED CABLE PROPAGATION CONSTANT
VERSUS SOIL CONDUCTIVITY
DIELECTRIC CONSTANT OF INSULATOR AND SOIL = 4

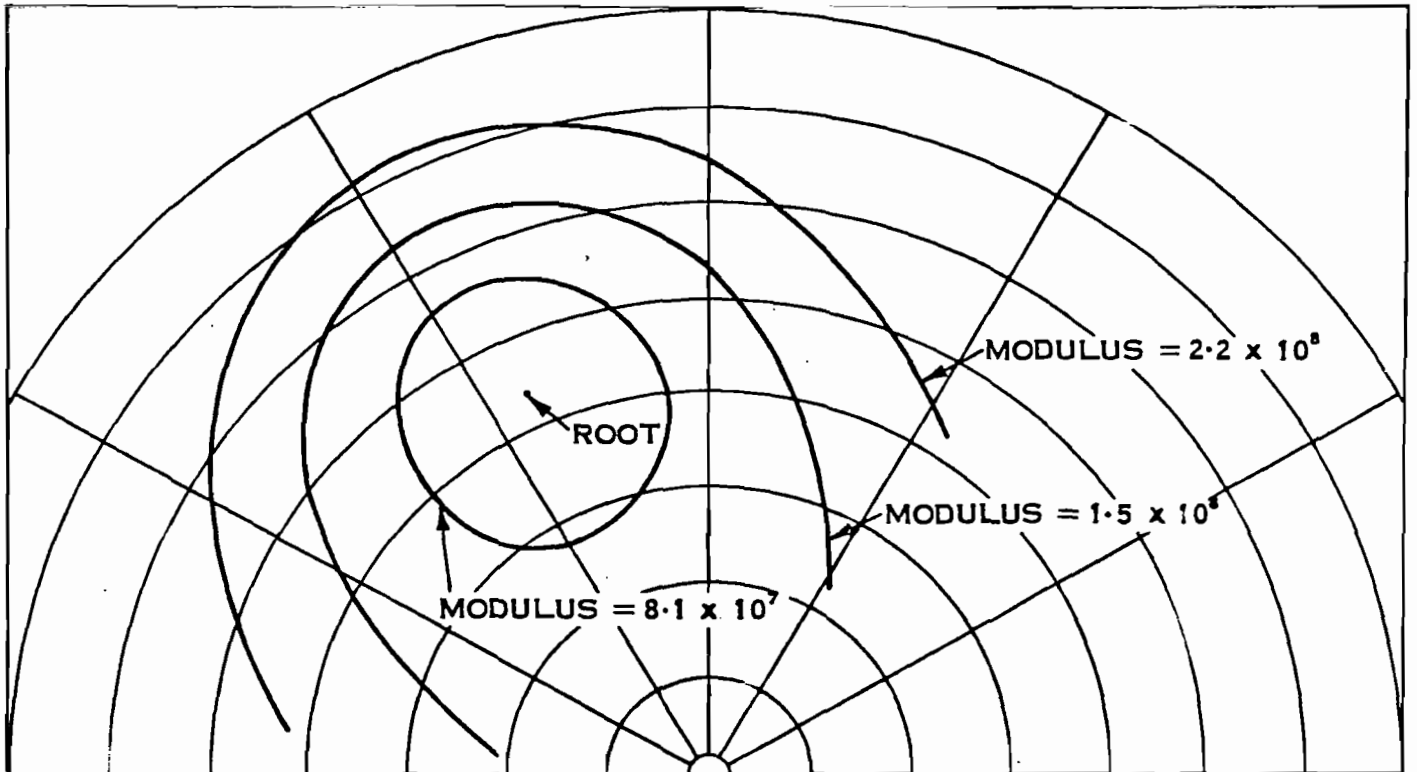


FIGURE 2-10

DETERMINANTAL EQUATION EQUI-MODULUS
CONTOURS IN THE VICINITY OF THE PRINCIPLE ROOT

SOIL CONDUCTIVITY = 10^{-2} MHOS/METER
INSULATOR AND SOIL DIELECTRIC CONSTANT = 4
FREQUENCY = 1 (HZ)

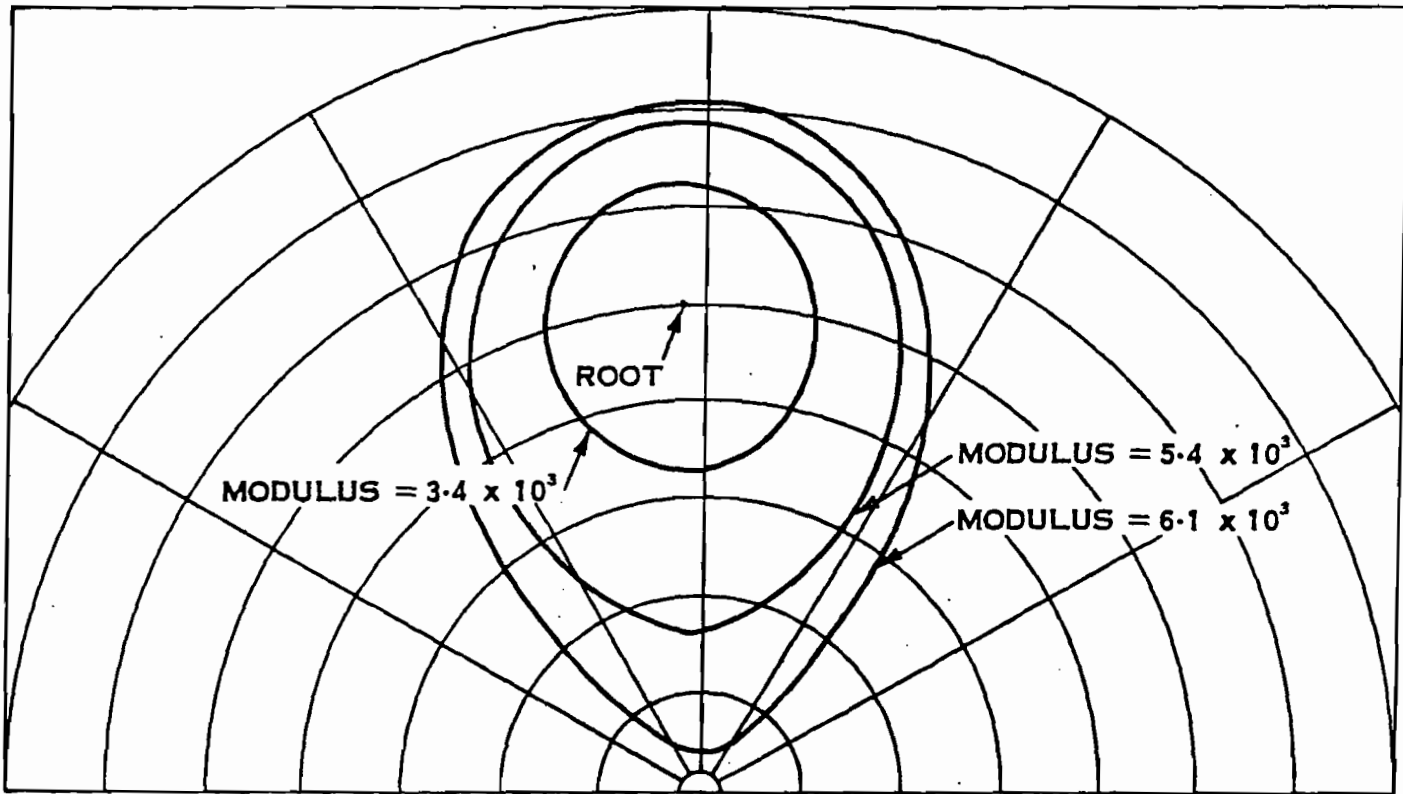


FIGURE 2-11

DETERMINANTAL EQUATION EQUI-MODULUS
CONTOURS IN THE VICINITY OF THE PRINCIPLE ROOT

SOIL CONDUCTIVITY = 10^{-2} MHOS/METER
INSULATOR AND SOIL DIELECTRIC CONSTANT = 4
FREQUENCY = 10^3 (HZ)

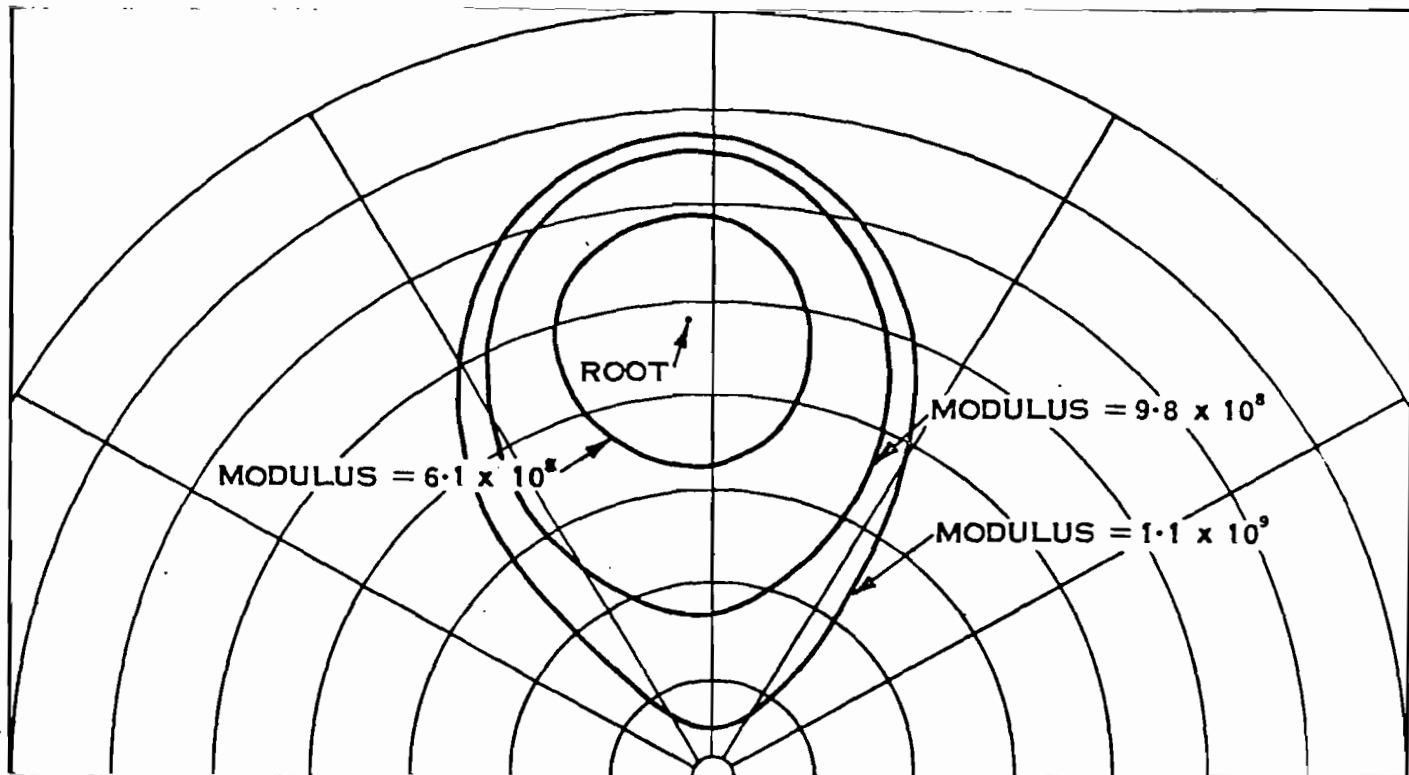


FIGURE 2-12

DETERMINANTAL EQUATION EQUI-MODULUS
CONTOURS IN THE VICINITY OF THE PRINCIPLE ROOT

SOIL CONDUCTIVITY = 10^{-2} MHOS/METER
INSULATOR AND SOIL DIELECTRIC CONSTANT = 4
FREQUENCY = 10^4 (HZ)

CHAPTER 3

TRANSMISSION LINE ANALOG TO TRANSVERSE MAGNETIC PROPAGATION

3.1 Analytic Results

A very common problem is that of determining the current induced on a center conductor due to the presence of an electric field. That is to say, a situation similar to that considered in Chapter 2. If this problem is to be solved by the methods of Chapter 2, however, a relatively complicated boundary value results. In order to more easily obtain estimates of induced currents, the center conductor geometry is often analoged with a conventional transmission line (for example, References 2, 3, and 5). As will be shown in the following section, such an analog is very often not too valid; however, prior to the numerical comparisons, it is expedient to review the transmission line theory.

The transmission line theory treats the conductor geometry as a lumped constant circuit distributed in space, as shown in Figure 3-1. By means of Kirchhoff's voltage and current laws applied to this small section of line, it is easily determined that the voltage and current along the line must satisfy

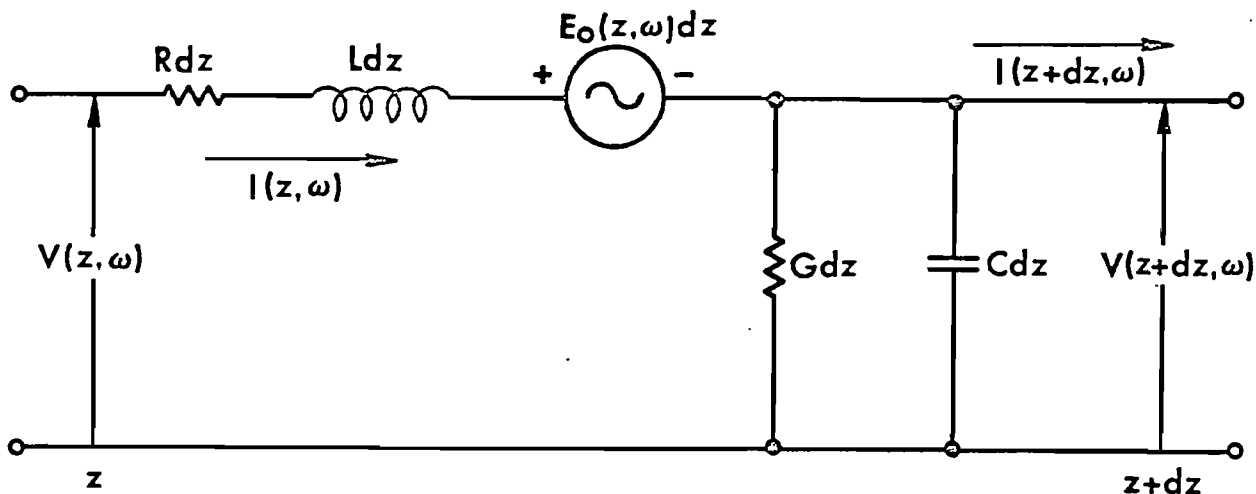
$$\frac{\partial V(z, \omega)}{\partial z} + Z(\omega)I(z, \omega) = -E_0(z, \omega), \quad (3-1)$$

$$\frac{\partial I(z, \omega)}{\partial z} + Y(\omega)V(z, \omega) = 0; \quad (3-2)$$

where $Z(\omega) = R(\omega) + i\omega L(\omega)$,

$$Y(\omega) = G(\omega) + i\omega C(\omega).$$

Complete solutions to Equations 3-1 and 3-2 can be determined from linear combinations of travelling waves of the form



R , L , G and C are the resistance, inductance, conductance and capacitance per unit length, respectively. In general, they are frequency dependent.

$E_0(z, \omega)$ is the net electric field per unit length serving to drive the line. If the impressed electric field vanishes at infinity, then $E_0(z, \omega)$ is equal to the impressed field at the wire.

FIGURE 3-1

SMALL SECTION OF TRANSMISSION LINE

$$V\epsilon^{\pm\gamma Z}, \quad (3-3)$$

$$I\epsilon^{\pm\gamma Z}; \quad (3-4)$$

where: $\gamma = \sqrt{ZY}$.

In order to make use of the preceding material, it is necessary to determine suitable values for Z and Y. In order to do this, the assumption is made, in the following material, that the geometry is that shown in Figure 2-1; that is, a center conductor surrounded by an insulator, with the combination surrounded by an infinite conducting medium. The impedance of the equivalent transmission line is then assumed to be the sum of the center conductor impedance, the insulator impedance and the surrounding medium impedance, i.e.,

$$Z = Z_w + Z_i + Z_s; \quad (3-5)$$

where Z_w is the center conductor impedance,
 Z_i is the insulator impedance,
 Z_s is the surrounding medium impedance,

Transmission line analogs are discussed in References 3, 5, 8, 9, 10 and the references cited therein.

The impedances Z_s and Z_w are derived in Appendices F and G, respectively, and are

$$Z_s = \frac{-\lambda_2 H_0^{(2)}(\lambda_2 r_i)}{2\pi r_i (\sigma_2 + i\omega\epsilon_2) H_1^{(2)}(\lambda_2 r_i)}, \quad (3-6)$$

$$Z_w = \frac{\lambda_1}{2\pi r_i (\sigma_1 + i\omega\epsilon_1)} \frac{J_0(\lambda_1 r_i)}{J_1(\lambda_1 r_i)}; \quad (3-7)$$

where: $\lambda_2^2 = \gamma_2^2 - h^2$,
 $\lambda_1^2 = \gamma_1^2 - h^2$,

$$\gamma_1^2 = -i\omega\mu_0(\sigma_1+i\omega\epsilon_1),$$

$$\gamma_2^2 = -i\omega\mu_0(\sigma_2+i\omega\epsilon_2).$$

In order to evaluate Equations 3-6 and 3-7 it is necessary to know h ; that is, the solution to the determinantal equation for symmetrical transverse magnetic propagation (Equation 2-19). It would be desirable not to have to solve this equation. If the conductivities of the wire are sufficiently high, then since h must remain finite

$$\lambda_1 = \sqrt{\gamma_1^2 - h^2} \rightarrow \gamma_1,$$

$$\lambda_2 = \sqrt{\gamma_2^2 - h^2} \rightarrow \gamma_2,$$

$$\gamma_1 \rightarrow \sqrt{-i\omega\mu_0\sigma_1},$$

$$\gamma_2 \rightarrow \sqrt{-i\omega\mu_0\sigma_2}.$$

With the assumption of high conductivity, then equations 3-6 and 3-7 become

$$Z_s \approx -\frac{\sqrt{-i\omega\mu_0\sigma_2} H_0^{(2)}(\sqrt{-i\omega\mu_0\sigma_2} r_i)}{2\pi r_i \sigma_2 H_1^{(2)}(\sqrt{-i\omega\mu_0\sigma_2} r_i)}, \quad (3-8)$$

$$Z_w \approx \frac{\sqrt{-i\omega\mu_0\sigma_1}}{2\pi r_i \sigma_1} \frac{J_0(\sqrt{-i\omega\mu_0\sigma_1} r_i)}{J_1(\sqrt{-i\omega\mu_0\sigma_1} r_i)}. \quad (3-9)$$

The insulator impedance could be determined from Equation E-16 (Appendix E). However, for purposes of calculational simplicity, it is often taken as the inductive reactance of the insulator. The inductance of the insulator is

$$L_i = \frac{\phi_i}{I} ; \quad (3-10)$$

where ϕ_i is the flux in the insulator

I is the current producing the flux

If the current in the center conductor is I , then the flux density in the insulator is

$$B_\theta = \mu_o \frac{I}{2\pi r} .$$

Thus the flux in the insulator is (in one unit length)

$$\phi_i = \int_{r_i}^{r_o} B_\theta dr = \int_{r_i}^{r_o} \frac{\mu_o}{2\pi} \frac{I}{r} dr = \frac{\mu_o I}{2\pi} \ln \frac{r_o}{r_i} ,$$

so that the inductance from Equation 3-10 is

$$L_i = \frac{\mu_o}{2\pi} \ln \frac{r_o}{r_i} .$$

It follows that the impedance of the insulator is

$$Z_i = i\omega L_i = \frac{i\omega\mu_o}{2\pi} \ln \frac{r_o}{r_i} . \quad (3-11)$$

The admittance Y can be considered to be composed of two parts: the capacitive reactance of the insulator and the admittance of the surrounding medium. The capacitive admittance of the insulator is (references 2, 3, 5, 8, 9 and 10)

$$Y_i = i \frac{2\pi\omega\epsilon_2}{\ln \frac{r_o}{r_i}}$$

In calculating the admittance of the surrounding medium, it is assumed that

$$\gamma_2^2 = Y_S Z_S ,$$

so that

$$Y_s = \frac{\gamma_2^2}{Z_s} \quad (3-12)$$

The total admittance (insulator plus surrounding conductor) is then

$$Y = \frac{Y_i Y_s}{Y_i + Y_s} \quad (3-13)$$

3.2 Numerical Results

The transmission line equivalent parameters; i.e., γ , Y and Z have been numerically evaluated using Equations 3-5, 3-8, 3-9, 3-11, 3-12 and 3-13. The results for the propagation constant ($\gamma = \sqrt{YZ}$) are shown graphically in Figure 3-2. Also shown on this figure, for comparison, are the actual values of h from Chapter 2. The evaluations were made using the same geometry and electrical parameters as those used in Chapter 2. Note that the agreement is rather good, at least for the known values of h . Also shown on Figure 3-2 is the propagation constant calculated with $Y = Y_i$. Here the agreement with calculated h is poor above approximately 10^3 Hz.

The seemingly good agreement of γ with h is not without problems, however. The corresponding impedances are not physically real; that is, there exist regions of frequency for which the real part of Z is negative. This unfortunate circumstance is undoubtedly due to the approximation for large σ ; i.e.,

$$\lambda \rightarrow \gamma,$$

used in obtaining Equations 3-8 and 3-9. If the actual h were known, then the corresponding equations to 3-8 and 3-9 (Equations 3-6 and 3-7) could be used to compute physically real values.

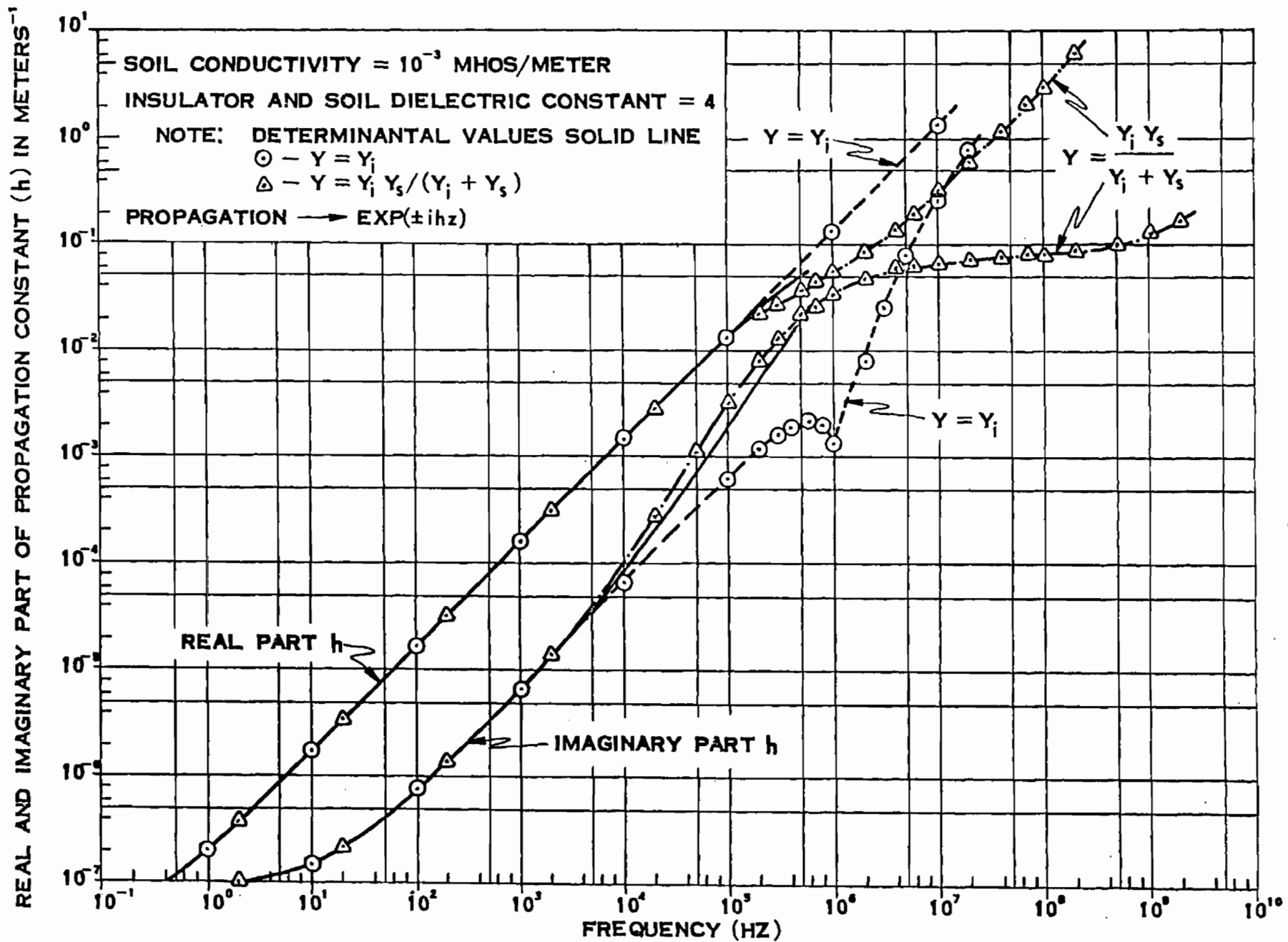


FIGURE 3-2
 EQUIVALENT TRANSMISSION LINE PARAMETERS VERSUS DETERMINANTAL EQUATION VALUES

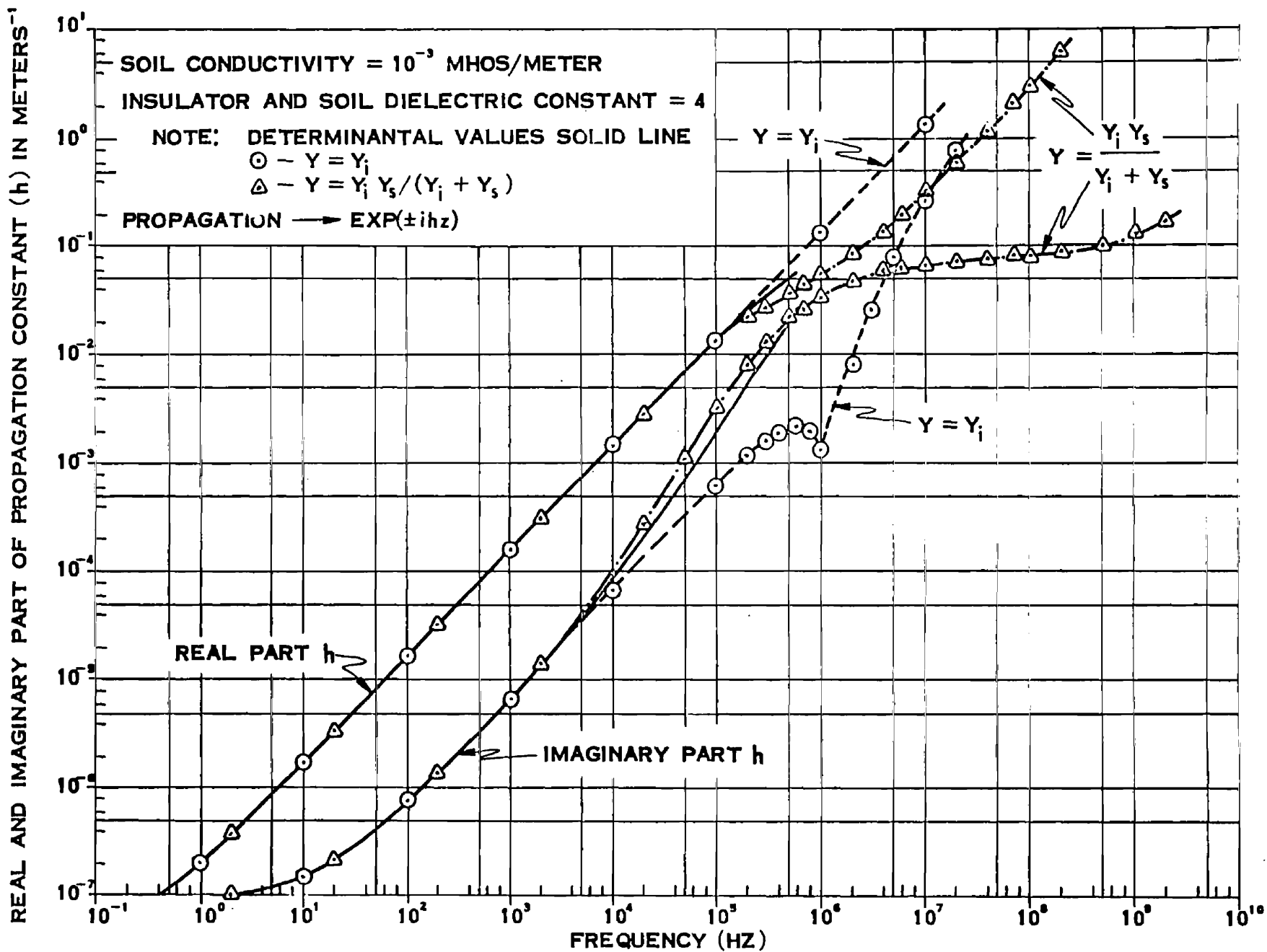


FIGURE 3-2

EQUIVALENT TRANSMISSION LINE PARAMETERS VERSUS DETERMINANTAL EQUATION VALUES

Tables 3-1 through 3-6 are listings of the calculated values for propagation constant, impedance and admittance for conductivity values in the surrounding medium of 10^{-2} , 10^{-3} and 10^{-4} mhos/meter. The first three of these tables are for $Y = (Y_i Y_s)/(Y_i + Y_s)$, while the second three tables are for $Y = Y_i$.

TABLE 3-1

$$Y = \frac{Y_L Y_S}{Y_L + Y_S}$$

SIGW SIGS KI KS
5.800E+07 1.000E-02 4.000E+00 4.000E+00

FREQ.	RE(ZW)	IM(ZW)	RE(ZS)	IM(ZS)	RE(ZI)	IM(ZI)	RE(Z)	IM(Z)	INDUCTANCE
1.0E-02	1.7128E-05	3.1400E-09	9.8646E-09	1.7488E-07	0.	8.7340E-09	1.7138E-05	1.8675E-07	2.9723E-06
2.0E-02	1.7128E-05	6.2800E-09	1.9729E-08	3.4105E-07	0.	1.7468E-08	1.7148E-05	3.6480E-07	2.9030E-06
3.0E-02	1.7128E-05	9.4200E-09	2.9594E-08	5.0394E-07	0.	2.6202E-08	1.7158E-05	5.3956E-07	2.8625E-06
4.0E-02	1.7128E-05	1.2560E-08	3.9458E-08	6.6469E-07	0.	3.4936E-08	1.7168E-05	7.1219E-07	2.8337E-06
5.0E-02	1.7128E-05	1.5700E-08	4.9323E-08	8.2306E-07	0.	4.3670E-08	1.7178E-05	8.8323E-07	2.8114E-06
6.0E-02	1.7128E-05	1.8840E-08	5.9188E-08	9.8176E-07	0.	5.2404E-08	1.7188E-05	1.0530E-06	2.7932E-06
7.0E-02	1.7128E-05	2.1980E-08	6.9052E-08	1.1386E-06	0.	6.1138E-08	1.7197E-05	1.2217E-06	2.7778E-06
8.0E-02	1.7128E-05	2.5120E-08	7.8917E-08	1.2946E-06	0.	6.9872E-08	1.7207E-05	1.3896E-06	2.7644E-06
9.0E-02	1.7128E-05	2.8260E-08	8.8781E-08	1.4497E-06	0.	7.8606E-08	1.7217E-05	1.5566E-06	2.7527E-06
1.0E-01	1.7128E-05	3.1400E-08	9.8646E-08	1.6042E-06	0.	8.7340E-08	1.7227E-05	1.7229E-06	2.7421E-06
2.0E-01	1.7128E-05	6.2800E-08	1.9729E-07	3.1213E-06	0.	1.7468E-07	1.7326E-05	3.3508E-06	2.6729E-06
3.0E-01	1.7129E-05	9.4200E-08	2.9594E-07	4.6056E-06	0.	2.6202E-07	1.7424E-05	4.9618E-06	2.6323E-06
4.0E-01	1.7129E-05	1.2560E-07	3.9458E-07	6.0685E-06	0.	3.4936E-07	1.7523E-05	6.5435E-06	2.6036E-06
5.0E-01	1.7129E-05	1.5700E-07	4.9323E-07	7.5156E-06	0.	4.3670E-07	1.7622E-05	8.1093E-06	2.5813E-06
6.0E-01	1.7129E-05	1.8839E-07	5.9188E-07	8.9500E-06	0.	5.2404E-07	1.7721E-05	9.6625E-06	2.5630E-06
7.0E-01	1.7129E-05	2.1979E-07	6.9052E-07	1.0374E-05	0.	6.1138E-07	1.7820E-05	1.1205E-05	2.5476E-06
8.0E-01	1.7130E-05	2.5119E-07	7.8917E-07	1.1789E-05	0.	6.9872E-07	1.7919E-05	1.2739E-05	2.5343E-06
9.0E-01	1.7130E-05	2.8258E-07	8.8781E-07	1.3196E-05	0.	7.8606E-07	1.8018E-05	1.4265E-05	2.5225E-06
1.0E+00	1.7130E-05	3.1398E-07	9.8646E-07	1.4596E-05	0.	8.7340E-07	1.8117E-05	1.5783E-05	2.5120E-06
2.0E+00	1.7136E-05	6.2786E-07	1.9729E-06	2.8321E-05	0.	1.7468E-06	1.9109E-05	3.0696E-05	2.4427E-06
3.0E+00	1.7146E-05	9.4153E-07	2.9594E-06	4.1718E-05	0.	2.6202E-06	2.0105E-05	4.5280E-05	2.4022E-06
4.0E+00	1.7159E-05	1.2549E-06	3.9458E-06	5.4901E-05	0.	3.4936E-06	2.1105E-05	5.9650E-05	2.3734E-06
5.0E+00	1.7176E-05	1.5678E-06	4.9323E-06	6.7926E-05	0.	4.3670E-06	2.2109E-05	7.3861E-05	2.3511E-06
6.0E+00	1.7197E-05	1.8802E-06	5.9188E-06	8.0824E-05	0.	5.2404E-06	2.3116E-05	8.7945E-05	2.3328E-06
7.0E+00	1.7222E-05	2.1920E-06	6.9052E-06	9.3617E-05	0.	6.1138E-06	2.4127E-05	1.0192E-04	2.3174E-06
8.0E+00	1.7251E-05	2.5030E-06	7.8917E-06	1.0632E-04	0.	6.9872E-06	2.5142E-05	1.1501E-04	2.3040E-06
9.0E+00	1.7283E-05	2.8133E-06	8.8781E-06	1.1894E-04	0.	7.8606E-06	2.6161E-05	1.2962E-04	2.2922E-06
1.0E+01	1.7319E-05	3.1226E-06	9.8646E-06	1.3150E-04	0.	8.7340E-06	2.7183E-05	1.4336E-04	2.2816E-06
2.0E+01	1.7869E-05	6.1446E-06	1.9729E-05	2.5429E-04	0.	1.7468E-05	3.7599E-05	2.7790E-04	2.2115E-06
3.0E+01	1.8727E-05	8.9832E-06	2.9594E-05	3.7380E-04	0.	2.6202E-05	4.8321E-05	4.0898E-04	2.1697E-06
4.0E+01	1.9818E-05	1.1585E-05	3.9458E-05	4.9117E-04	0.	3.4936E-05	5.9276E-05	5.3769E-04	2.1394E-06
5.0E+01	2.1064E-05	1.3929E-05	4.9323E-05	6.0696E-04	0.	4.3670E-05	7.0386E-05	6.6456E-04	2.1193E-06
6.0E+01	2.2392E-05	1.6019E-05	5.9187E-05	7.2148E-04	0.	5.2404E-05	8.1579E-05	7.8990E-04	2.0953E-06
7.0E+01	2.3748E-05	1.7877E-05	6.9052E-05	8.3495E-04	0.	6.1138E-05	9.2799E-05	9.1396E-04	2.0760E-06
8.0E+01	2.5090E-05	1.9534E-05	7.8916E-05	9.4752E-04	0.	6.9872E-05	1.0401E-04	1.0369E-03	2.0629E-06
9.0E+01	2.6395E-05	2.1024E-05	8.8780E-05	1.0593E-03	0.	7.8606E-05	1.1518E-04	1.1509E-03	2.0494E-06
1.0E+02	2.7648E-05	2.2376E-05	9.8645E-05	1.1704E-03	0.	8.7340E-05	1.2629E-04	1.2801E-03	2.0373E-06
2.0E+02	3.7501E-05	3.2291E-05	1.9729E-04	2.2537E-03	0.	1.7468E-04	2.3479E-04	2.4607E-03	1.9581E-06
3.0E+02	4.4785E-05	3.9736E-05	2.9593E-04	3.3042E-03	0.	2.6202E-04	3.4071E-04	3.6059E-03	1.9130E-06
4.0E+02	5.0950E-05	4.6023E-05	3.9456E-04	4.3333E-03	0.	3.4936E-04	4.4551E-04	4.7207E-03	1.8815E-06
5.0E+02	5.8398E-05	5.1544E-05	4.9320E-04	5.3466E-03	0.	4.3670E-04	5.4960E-04	5.8340E-03	1.8573E-06
6.0E+02	6.1327E-05	5.6525E-05	5.9183E-04	6.3472E-03	0.	5.2404E-04	6.5316E-04	6.9277E-03	1.8376E-06
7.0E+02	6.5860E-05	6.1099E-05	6.9046E-04	7.3373E-03	0.	6.1138E-04	7.5632E-04	8.0098E-03	1.8211E-06
8.0E+02	7.0084E-05	6.5357E-05	7.8909E-04	8.3184E-03	0.	6.9872E-04	8.5918E-04	9.0825E-03	1.8069E-06
9.0E+02	7.4056E-05	6.9357E-05	8.8772E-04	9.2916E-03	0.	7.8606E-04	9.6177E-04	1.0147E-02	1.7944E-06
1.0E+03	7.3337E-05	7.3557E-05	9.8634E-04	1.0258E-02	0.	8.7340E-04	1.0597E-03	1.1205E-02	1.7833E-06
2.0E+03	1.0371E-04	1.0371E-04	1.9725E-03	1.9645E-02	0.	1.7468E-03	2.0762E-03	2.1496E-02	1.7106E-06
3.0E+03	1.2702E-04	1.2702E-04	2.9584E-03	2.8704E-02	0.	2.6202E-03	3.0854E-03	3.1451E-02	1.6685E-06
4.0E+03	1.4667E-04	1.4667E-04	3.9441E-03	3.7549E-02	0.	3.4936E-03	4.0908E-03	4.1189E-02	1.6389E-06
5.0E+03	1.6399E-04	1.6399E-04	4.9296E-03	4.6236E-02	0.	4.3670E-03	5.0936E-03	5.0767E-02	1.6160E-06
6.0E+03	1.7964E-04	1.7964E-04	5.9150E-03	5.4796E-02	0.	5.2404E-03	6.0946E-03	6.0216E-02	1.5973E-06
7.0E+03	1.9403E-04	1.9403E-04	6.9001E-03	6.3251E-02	0.	6.1138E-03	7.0941E-03	6.9559E-02	1.5815E-06
8.0E+03	2.0743E-04	2.0743E-04	7.8851E-03	7.1616E-02	0.	6.9872E-03	8.0925E-03	7.8811E-02	1.5679E-06
9.0E+03	2.2001E-04	2.2001E-04	8.8699E-03	7.9903E-02	0.	7.8606E-03	9.0899E-03	8.7983E-02	1.5559E-06

SIGW 5.800E+07 SIOS 1.000E-02 KI 4.000E+00 KS 4.000E+00

FREQ.	RE(ZW)	IM(ZW)	RE(ZS)	IM(ZS)	RE(ZI)	IM(ZI)	RE(Z)	IM(Z)	INDUCTANCE
1.0E+04	2.3191E-04	2.3191E-04	9.8545E-03	8.8119E-02	0.	8.7340E-03	1.0086E-02	9.7085E-02	1.5452E-06
2.0E+04	3.2797E-04	3.2797E-04	1.9691E-02	1.6753E-01	0.	1.7468E-02	2.0019E-02	1.8533E-01	1.4748E-06
3.0E+04	4.0168E-04	4.0168E-04	2.9510E-02	2.4367E-01	0.	2.6202E-02	2.9912E-02	2.7027E-01	1.4338E-06
4.0E+04	4.6382E-04	4.6382E-04	3.9313E-02	3.1767E-01	0.	3.4936E-02	3.9776E-02	3.5307E-01	1.4048E-06
5.0E+04	5.1857E-04	5.1857E-04	4.9099E-02	3.9008E-01	0.	4.3670E-02	4.9618E-02	4.3427E-01	1.3823E-06
6.0E+04	5.6807E-04	5.6807E-04	5.8870E-02	4.6124E-01	0.	5.2404E-02	5.9438E-02	5.1421E-01	1.3640E-06
7.0E+04	6.1358E-04	6.1358E-04	6.8626E-02	5.3134E-01	0.	6.1138E-02	6.9239E-02	5.9309E-01	1.3485E-06
8.0E+04	6.5595E-04	6.5595E-04	7.8366E-02	6.0055E-01	0.	6.9872E-02	7.9022E-02	6.7108E-01	1.3351E-06
9.0E+04	6.9574E-04	6.9574E-04	8.8091E-02	6.6897E-01	0.	7.8606E-02	8.8787E-02	7.4827E-01	1.3232E-06
1.0E+05	7.3337E-04	7.3337E-04	9.7802E-02	7.3670E-01	0.	8.7340E-02	9.8535E-02	8.2477E-01	1.3127E-06
2.0E+05	1.0371E-03	1.0371E-03	1.9412E-01	1.3866E+00	0.	1.7468E-01	1.9516E-01	1.5623E+00	1.2432E-06
3.0E+05	1.2702E-03	1.2702E-03	2.8907E-01	2.0038E+00	0.	2.6202E-01	2.9034E-01	2.2671E+00	1.2027E-06
4.0E+05	1.4667E-03	1.4667E-03	3.8272E-01	2.5999E+00	0.	3.4936E-01	3.8418E-01	2.9508E+00	1.1741E-06
5.0E+05	1.6399E-03	1.6399E-03	4.7510E-01	3.1805E+00	0.	4.3670E-01	4.7674E-01	3.6188E+00	1.1519E-06
6.0E+05	1.7964E-03	1.7964E-03	5.6625E-01	3.7486E+00	0.	5.2404E-01	5.6804E-01	4.2744E+00	1.1338E-06
7.0E+05	1.9403E-03	1.9403E-03	6.5620E-01	4.3064E+00	0.	6.1138E-01	6.5814E-01	4.9197E+00	1.1186E-06
8.0E+05	2.0743E-03	2.0743E-03	7.4496E-01	4.8555E+00	0.	6.9872E-01	7.4704E-01	5.5562E+00	1.1054E-06
9.0E+05	2.2001E-03	2.2001E-03	8.3257E-01	5.3969E+00	0.	7.8606E-01	8.3477E-01	6.1852E+00	1.0938E-06
1.0E+06	2.3191E-03	2.3191E-03	9.1903E-01	5.9316E+00	0.	8.7340E-01	9.2135E-01	6.8073E+00	1.0834E-06
2.0E+06	3.2797E-03	3.2797E-03	1.7237E+00	1.1017E+01	0.	1.7468E+00	1.7270E+00	1.2767E+01	1.0160E-06
3.0E+06	4.0168E-03	4.0168E-03	2.4259E+00	1.5801E+01	0.	2.6202E+00	2.4299E+00	1.8425E+01	9.7747E-07
4.0E+06	4.6382E-03	4.6382E-03	3.0321E+00	2.0398E+01	0.	3.4936E+00	3.0368E+00	2.3896E+01	9.5080E-07
5.0E+06	5.1857E-03	5.1857E-03	3.5473E+00	2.4866E+01	0.	4.3670E+00	3.5524E+00	2.9238E+01	9.3066E-07
6.0E+06	5.6807E-03	5.6807E-03	3.9751E+00	2.9237E+01	0.	5.2404E+00	3.9808E+00	3.4483E+01	9.1470E-07
7.0E+06	6.1358E-03	6.1358E-03	4.3191E+00	3.3535E+01	0.	6.1138E+00	4.3252E+00	3.9655E+01	9.0161E-07
8.0E+06	6.5595E-03	6.5595E-03	4.5820E+00	3.7775E+01	0.	6.9872E+00	4.5886E+00	4.4768E+01	8.9064E-07
9.0E+06	6.9574E-03	6.9574E-03	4.7667E+00	4.1969E+01	0.	7.8606E+00	4.7737E+00	4.9837E+01	8.8131E-07
1.0E+07	7.3337E-03	7.3337E-03	4.8756E+00	4.6128E+01	0.	8.7340E+00	4.8830E+00	5.4870E+01	8.7328E-07
2.0E+07	1.0371E-02	1.0371E-02	2.2654E+00	8.7012E+01	0.	1.7468E+01	2.2757E+00	1.0449E+02	8.3151E-07
3.0E+07	1.2702E-02	1.2702E-02	-5.8343E+00	1.2862E+02	0.	2.6202E+01	-5.8216E+00	1.5483E+02	8.2140E-07
4.0E+07	1.4667E-02	1.4667E-02	-1.8021E+01	1.7216E+02	0.	3.4936E+01	-1.8006E+01	2.0711E+02	8.2407E-07
5.0E+07	1.6399E-02	1.6399E-02	-3.3276E+01	2.1800E+02	0.	4.3670E+01	-3.3259E+01	2.6169E+02	8.3298E-07
6.0E+07	1.7964E-02	1.7964E-02	-5.0880E+01	2.6617E+02	0.	5.2404E+01	-5.0862E+01	3.1859E+02	8.4509E-07
7.0E+07	1.9403E-02	1.9403E-02	-7.0329E+01	3.1657E+02	0.	6.1138E+01	-7.0309E+01	3.7772E+02	8.5881E-07
8.0E+07	2.0743E-02	2.0743E-02	-9.1261E+01	3.6906E+02	0.	6.9872E+01	-9.1240E+01	4.3896E+02	8.7327E-07
9.0E+07	2.2001E-02	2.2001E-02	-1.1341E+02	4.2352E+02	0.	7.8606E+01	-1.1339E+02	5.0215E+02	8.8799E-07
1.0E+08	2.3191E-02	2.3191E-02	-1.3659E+02	4.7980E+02	0.	8.7340E+01	-1.3657E+02	5.6716E+02	9.0267E-07
2.0E+08	3.2797E-02	3.2797E-02	-4.0277E+02	1.1220E+03	0.	1.7468E+02	-4.0274E+02	1.2967E+03	1.0319E-06
3.0E+08	4.0168E-02	4.0168E-02	-7.0159E+02	1.8684E+03	0.	2.6202E+02	-7.0155E+02	2.1305E+03	1.1303E-06
4.0E+08	4.6382E-02	4.6382E-02	-1.0160E+03	2.6879E+03	0.	3.4936E+02	-1.0159E+03	3.0373E+03	1.2085E-06
5.0E+08	5.1857E-02	5.1857E-02	-1.3390E+03	3.5638E+03	0.	4.3670E+02	-1.3389E+03	4.0006E+03	1.2734E-06
6.0E+08	5.6807E-02	5.6807E-02	-1.6671E+03	4.4856E+03	0.	5.2404E+02	-1.6670E+03	5.0097E+03	1.3289E-06
7.0E+08	6.1358E-02	6.1358E-02	-1.9982E+03	5.4460E+03	0.	6.1138E+02	-1.9981E+03	6.0575E+03	1.3773E-06
8.0E+08	6.5595E-02	6.5595E-02	-2.3309E+03	6.4398E+03	0.	6.9872E+02	-2.3308E+03	7.1386E+03	1.4202E-06
9.0E+08	6.9574E-02	6.9574E-02	-2.6643E+03	7.4628E+03	0.	7.8606E+02	-2.6642E+03	8.2489E+03	1.4587E-06
1.0E+09	7.3337E-02	7.3337E-02	-2.9978E+03	8.5118E+03	0.	8.7340E+02	-2.9977E+03	9.3853E+03	1.4937E-06
2.0E+09	1.0371E-01	1.0371E-01	-6.2780E+03	2.0014E+04	0.	1.7468E+03	-6.2779E+03	1.761E+04	1.7317E-06
3.0E+09	1.2702E-01	1.2702E-01	-9.4077E+03	3.2687E+04	0.	2.6202E+03	-9.4076E+03	3.5307E+04	1.8731E-06
4.0E+09	1.4667E-01	1.4667E-01	-1.2386E+04	4.6074E+04	0.	3.4936E+03	-1.2386E+04	4.9568E+04	1.9723E-06
5.0E+09	1.6399E-01	1.6399E-01	-1.5231E+04	5.9963E+04	0.	4.3670E+03	-1.5231E+04	6.4331E+04	2.0477E-06
6.0E+09	1.7964E-01	1.7964E-01	-1.7958E+04	7.4231E+04	0.	5.2404E+03	-1.7958E+04	7.9472E+04	2.1080E-06
7.0E+09	1.9403E-01	1.9403E-01	-2.0582E+04	8.8798E+04	0.	6.1138E+03	-2.0582E+04	9.4912E+04	2.1580E-06
8.0E+09	2.0743E-01	2.0743E-01	-2.3113E+04	1.0361E+05	0.	6.9872E+03	-2.3113E+04	1.1060E+05	2.2003E-06
9.0E+09	2.2001E-01	2.2001E-01	-2.5563E+04	1.1863E+05	0.	7.8606E+03	-2.5563E+04	1.2649E+05	2.2368E-06

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SIGW SIGS KI KS
 5.800E+07 1.000E-02 4.000E+00 4.000E+00

FREQ.	RE(ZW)	IM(ZW)	RE(ZS)	IM(ZS)	RE(ZI)	IM(ZI)	RE(Z)	IM(Z)	INDUCTANCE
1.0E+10	2.3191E-01	2.3191E-01	-2.7938E+04	1.3382E+05	0.	8.7340E+03	-2.7938E+04	1.4256E+05	2.2688E-06
2.0E+10	3.2797E-01	3.2797E-01	-4.8727E+04	2.9192E+05	0.	1.7468E+04	-4.8727E+04	3.0939E+05	2.4621E-06
3.0E+10	4.0168E-01	4.0168E-01	-6.6055E+04	4.5630E+05	0.	2.6202E+04	-6.6055E+04	4.8250E+05	2.5597E-06
4.0E+10	4.6382E-01	4.6382E-01	-8.1261E+04	6.2402E+05	0.	3.4936E+04	-8.1261E+04	6.5895E+05	2.6219E-06
5.0E+10	5.1857E-01	5.1857E-01	-9.4999E+04	7.9390E+05	0.	4.3670E+04	-9.4998E+04	8.3757E+05	2.6661E-06
6.0E+10	5.6807E-01	5.6807E-01	-1.0764E+05	9.6533E+05	0.	5.2404E+04	-1.0764E+05	1.0177E+06	2.6996E-06
7.0E+10	6.1358E-01	6.1358E-01	-1.1935E+05	1.1379E+06	0.	6.1138E+04	-1.1935E+05	1.1990E+06	2.7261E-06
8.0E+10	6.5595E-01	6.5595E-01	-1.3049E+05	1.3114E+06	0.	6.9872E+04	-1.3049E+05	1.3813E+06	2.7480E-06
9.0E+10	6.9574E-01	6.9574E-01	-1.4118E+05	1.4857E+06	0.	7.8606E+04	-1.4118E+05	1.5643E+06	2.7662E-06

SIGW SIGS KI KS
 5.800E+07 1.000E-02 4.000E+00 4.000E+00

FREQ.	RI(YI)	IM(YI)	RE(YI)	IM(YI)	RE(YI)	IM(YI)	CAPACITANCE	RE(GAMMA)	IM(GAMMA)	RE(Z(0))	IM(Z(0))
1.0E+10	0.	2.008E+01	1.339E-01	4.273E-01	1.259E-01	6.091E-01	9.694E-12	1.551E+00	3.006E+02	4.738E+02	9.539E+01
2.0E+10	0.	4.016E+01	1.977E-01	1.168E+00	1.867E-01	1.136E+00	9.043E-12	1.996E+00	6.006E+02	5.149E+02	8.289E+01
3.0E+10	0.	6.024E+01	2.478E-01	1.694E+00	2.344E-01	1.648E+00	8.744E-12	2.345E+00	9.004E+02	5.357E+02	7.475E+01
4.0E+10	0.	8.032E+01	2.904E-01	2.210E+00	2.750E-01	2.152E+00	8.563E-12	2.642E+00	1.200E+03	5.489E+02	6.891E+01
5.0E+10	0.	1.004E+02	3.282E-01	2.722E+00	3.111E-01	2.651E+00	8.439E-12	2.905E+00	1.500E+03	5.582E+02	6.441E+01
6.0E+10	0.	1.205E+02	3.626E-01	3.230E+00	3.439E-01	3.146E+00	8.348E-12	3.144E+00	1.800E+03	5.654E+02	6.030E+01
7.0E+10	0.	1.406E+02	3.942E-01	3.735E+00	3.740E-01	3.639E+00	8.274E-12	3.364E+00	2.099E+03	5.710E+02	5.770E+01
8.0E+10	0.	1.606E+02	4.240E-01	4.237E+00	4.025E-01	4.129E+00	8.215E-12	3.572E+00	2.399E+03	5.757E+02	5.525E+01
9.0E+10	0.	1.807E+02	4.526E-01	4.738E+00	4.298E-01	4.618E+00	8.166E-12	3.771E+00	2.699E+03	5.795E+02	5.312E+01

TABLE 3-2

$$Y = \frac{Y_1 Y_2}{Y_1 + Y_2}$$

SIGW 5.800E+07
 SIOS 1.000E-03
 KI 4.000E+00
 KS 4.000E+00

FREQ.	RE(ZW)	IM(ZW)	RE(ZS)	IM(ZS)	RE(ZI)	IM(ZI)	RE(Z)	IM(Z)	INDUCTANCE
1.0E-02	1.7128E-05	3.1400E-09	9.8646E-09	1.8934E-07	0.	8.7340E-09	1.7138E-05	2.0121E-07	3.2624E-06
2.0E-02	1.7128E-05	6.2800E-09	1.9729E-08	3.6997E-07	0.	1.7468E-08	1.7148E-05	3.9372E-07	3.1331E-06
3.0E-02	1.7128E-05	9.4200E-09	2.9594E-08	5.4732E-07	0.	2.6202E-08	1.7158E-05	5.8294E-07	3.0926E-06
4.0E-02	1.7128E-05	1.2560E-08	3.9458E-08	7.2254E-07	0.	3.4936E-08	1.7168E-05	7.7003E-07	3.0439E-06
5.0E-02	1.7128E-05	1.5700E-08	4.9323E-08	8.9616E-07	0.	4.3670E-08	1.7178E-05	9.5553E-07	3.0416E-06
6.0E-02	1.7128E-05	1.8840E-08	5.9188E-08	1.0660E-06	0.	5.2404E-08	1.7188E-05	1.1398E-06	3.0233E-06
7.0E-02	1.7128E-05	2.1980E-08	6.9052E-08	1.2398E-06	0.	6.1138E-08	1.7197E-05	1.3230E-06	3.0079E-06
8.0E-02	1.7128E-05	2.5120E-08	7.8917E-08	1.4102E-06	0.	6.9872E-08	1.7207E-05	1.5052E-06	2.9946E-06
9.0E-02	1.7128E-05	2.8260E-08	8.8781E-08	1.5799E-06	0.	7.8606E-08	1.7217E-05	1.6867E-06	2.9828E-06
1.0E-01	1.7128E-05	3.1400E-08	9.8646E-08	1.7488E-06	0.	8.7340E-08	1.7227E-05	1.8679E-06	2.9723E-06
2.0E-01	1.7128E-05	6.2800E-08	1.9729E-07	3.4105E-06	0.	1.7468E-07	1.7326E-05	3.6460E-06	2.9300E-06
3.0E-01	1.7129E-05	9.4200E-08	2.9594E-07	5.0394E-06	0.	2.6202E-07	1.7424E-05	5.3956E-06	2.8625E-06
4.0E-01	1.7129E-05	1.2560E-07	3.9458E-07	6.6469E-06	0.	3.4936E-07	1.7523E-05	7.1219E-06	2.8337E-06
5.0E-01	1.7129E-05	1.5700E-07	4.9323E-07	8.2306E-06	0.	4.3670E-07	1.7622E-05	8.8323E-06	2.8114E-06
6.0E-01	1.7129E-05	1.8839E-07	5.9188E-07	9.8176E-06	0.	5.2404E-07	1.7721E-05	1.0930E-05	2.7932E-06
7.0E-01	1.7129E-05	2.1979E-07	6.9052E-07	1.1386E-05	0.	6.1138E-07	1.7820E-05	1.2217E-05	2.7778E-06
8.0E-01	1.7130E-05	2.5119E-07	7.8917E-07	1.2946E-05	0.	6.9872E-07	1.7919E-05	1.3596E-05	2.7444E-06
9.0E-01	1.7130E-05	2.8258E-07	8.8781E-07	1.4497E-05	0.	7.8606E-07	1.8018E-05	1.5566E-05	2.7327E-06
1.0E+00	1.7130E-05	3.1398E-07	9.8646E-07	1.6042E-05	0.	8.7340E-07	1.8117E-05	1.7229E-05	2.7421E-06
2.0E+00	1.7136E-05	6.2786E-07	1.9729E-06	3.1213E-05	0.	1.7468E-06	1.9109E-05	3.3588E-05	2.6728E-06
3.0E+00	1.7146E-05	9.4153E-07	2.9594E-06	4.6056E-05	0.	2.6202E-06	2.0105E-05	4.9618E-05	2.6323E-06
4.0E+00	1.7159E-05	1.2549E-06	3.9458E-06	6.0685E-05	0.	3.4936E-06	2.1105E-05	6.5434E-05	2.6038E-06
5.0E+00	1.7176E-05	1.5678E-06	4.9323E-06	7.5156E-05	0.	4.3670E-06	2.2108E-05	8.1091E-05	2.5812E-06
6.0E+00	1.7197E-05	1.8802E-06	5.9187E-06	8.9500E-05	0.	5.2404E-06	2.3116E-05	9.6621E-05	2.5629E-06
7.0E+00	1.7222E-05	2.1920E-06	6.9051E-06	1.0374E-04	0.	6.1138E-06	2.4127E-05	1.1205E-04	2.5475E-06
8.0E+00	1.7251E-05	2.5030E-06	7.8916E-06	1.1789E-04	0.	6.9872E-06	2.5142E-05	1.2733E-04	2.5341E-06
9.0E+00	1.7283E-05	2.8133E-06	8.8780E-06	1.3196E-04	0.	7.8606E-06	2.6161E-05	1.4263E-04	2.5223E-06
1.0E+01	1.7319E-05	3.1226E-06	9.8644E-06	1.4596E-04	0.	8.7340E-06	2.7183E-05	1.5782E-04	2.5117E-06
2.0E+01	1.7869E-05	6.1446E-06	1.9729E-05	2.8321E-04	0.	1.7468E-05	3.7598E-05	3.0682E-04	2.4416E-06
3.0E+01	1.8727E-05	8.9832E-06	2.9592E-05	4.1718E-04	0.	2.6202E-05	4.8320E-05	4.5236E-04	2.3999E-06
4.0E+01	1.9818E-05	1.1585E-05	3.9456E-05	5.4901E-04	0.	3.4936E-05	5.9274E-05	5.9553E-04	2.3696E-06
5.0E+01	2.1064E-05	1.3929E-05	4.9319E-05	6.7926E-04	0.	4.3670E-05	7.0383E-05	7.3686E-04	2.3459E-06
6.0E+01	2.2392E-05	1.6019E-05	5.9182E-05	8.0824E-04	0.	5.2404E-05	8.1574E-05	8.7666E-04	2.3254E-06
7.0E+01	2.3748E-05	1.7877E-05	6.9045E-05	9.3617E-04	0.	6.1138E-05	9.2793E-05	1.0152E-03	2.3082E-06
8.0E+01	2.5090E-05	1.9534E-05	7.8907E-05	1.0632E-03	0.	6.9872E-05	1.0400E-04	1.1526E-03	2.2930E-06
9.0E+01	2.6395E-05	2.1024E-05	8.8770E-05	1.1894E-03	0.	7.8606E-05	1.1516E-04	1.2891E-03	2.2796E-06
1.0E+02	2.7648E-05	2.2376E-05	9.8631E-05	1.3150E-03	0.	8.7340E-05	1.2628E-04	1.4247E-03	2.2675E-06
2.0E+02	3.7501E-05	3.2291E-05	1.9724E-04	2.5429E-03	0.	1.7468E-04	2.3474E-04	2.7499E-03	2.1883E-06
3.0E+02	4.4785E-05	3.9736E-05	2.9581E-04	3.7380E-03	0.	2.6202E-04	3.4060E-04	4.0398E-03	2.1432E-06
4.0E+02	5.0950E-05	4.6023E-05	3.9437E-04	4.9117E-03	0.	3.4936E-04	4.4532E-04	5.3071E-03	2.1116E-06
5.0E+02	5.6398E-05	5.1544E-05	4.9289E-04	6.0696E-03	0.	4.3670E-04	5.4929E-04	6.5578E-03	2.0874E-06
6.0E+02	6.1327E-05	5.6525E-05	5.9139E-04	7.2148E-03	0.	5.2404E-04	6.5272E-04	7.7954E-03	2.0678E-06
7.0E+02	6.5860E-05	6.1099E-05	6.8987E-04	8.3495E-03	0.	6.1138E-04	7.5573E-04	9.0220E-03	2.0513E-06
8.0E+02	7.0084E-05	6.5357E-05	7.8833E-04	9.4753E-03	0.	6.9872E-04	8.5841E-04	1.0239E-02	2.0371E-06
9.0E+02	7.4056E-05	6.9357E-05	8.8675E-04	1.0593E-02	0.	7.8606E-04	9.6081E-04	1.1449E-02	2.0245E-06
1.0E+03	7.3337E-05	7.3337E-05	9.8516E-04	1.1704E-02	0.	8.7340E-04	1.0585E-03	1.2651E-02	2.0134E-06
2.0E+03	1.0371E-04	1.0371E-04	1.9679E-03	2.2538E-02	0.	1.7468E-03	2.0716E-03	2.4388E-02	1.9407E-06
3.0E+03	1.2702E-04	1.2702E-04	2.9484E-03	3.3043E-02	0.	2.6202E-03	3.0754E-03	3.5790E-02	1.8987E-06
4.0E+03	1.4667E-04	1.4667E-04	3.9266E-03	4.3335E-02	0.	3.4936E-03	4.0732E-03	4.6975E-02	1.8691E-06
5.0E+03	1.6399E-04	1.6399E-04	4.9026E-03	5.3468E-02	0.	4.3670E-03	5.0666E-03	5.7999E-02	1.8462E-06
6.0E+03	1.7964E-04	1.7964E-04	5.8764E-03	6.3476E-02	0.	5.2404E-03	6.0561E-03	6.8896E-02	1.8275E-06
7.0E+03	1.9403E-04	1.9403E-04	6.8481E-03	7.3378E-02	0.	6.1138E-03	7.0422E-03	7.9686E-02	1.8118E-06
8.0E+03	2.0743E-04	2.0743E-04	7.8177E-03	8.3191E-02	0.	6.9872E-03	8.0251E-03	9.0385E-02	1.7982E-06
9.0E+03	2.2001E-04	2.2001E-04	8.7852E-03	9.2925E-02	0.	7.8606E-03	9.0052E-03	1.0101E-01	1.7862E-06

SIGW SIGS KI KS
5.800E+07 1.000E-03 4.000E+00 4.000E+00

FREQ.	RE(ZW)	IM(ZW)	RE(ZS)	IM(ZS)	RE(ZI)	IM(ZI)	RE(Z)	IM(Z)	INDUCTANCE
1.0E+04	2.3191E-04	2.3191E-04	9.7506E-03	1.0259E-01	0.	8.7340E-03	9.9825E-03	1.1156E-01	1.7755E-06
2.0E+04	3.2797E-04	3.2797E-04	1.9293E-02	1.9650E-01	0.	1.7468E-02	1.9620E-02	2.1429E-01	1.7093E-06
3.0E+04	4.0168E-04	4.0168E-04	2.8637E-02	2.8714E-01	0.	2.6202E-02	2.9038E-02	3.1374E-01	1.6644E-06
4.0E+04	4.6382E-04	4.6382E-04	3.7789E-02	3.7567E-01	0.	3.4936E-02	3.8253E-02	4.1107E-01	1.6356E-06
5.0E+04	5.1857E-04	5.1857E-04	4.6754E-02	4.6264E-01	0.	4.3670E-02	4.7273E-02	5.0683E-01	1.6133E-06
6.0E+04	5.6807E-04	5.6807E-04	5.5535E-02	5.4836E-01	0.	5.2404E-02	5.6103E-02	6.0134E-01	1.5951E-06
7.0E+04	6.1358E-04	6.1358E-04	6.4133E-02	6.3306E-01	0.	6.1138E-02	6.4747E-02	6.9481E-01	1.5798E-06
8.0E+04	6.5595E-04	6.5595E-04	7.2552E-02	7.1689E-01	0.	6.9872E-02	7.3208E-02	7.8741E-01	1.5669E-06
9.0E+04	6.9574E-04	6.9574E-04	8.0793E-02	7.9995E-01	0.	7.8606E-02	8.1489E-02	8.7925E-01	1.5549E-06
1.0E+05	7.3337E-04	7.3337E-04	8.8858E-02	8.8233E-01	0.	8.7340E-02	8.9592E-02	9.7041E-01	1.5444E-06
2.0E+05	1.0371E-03	1.0371E-03	1.6011E-01	1.6801E+00	0.	1.7468E-01	1.6114E-01	1.8598E+00	1.4768E-06
3.0E+05	1.2702E-03	1.2702E-03	2.1490E-01	2.4478E+00	0.	2.6202E-01	2.1617E-01	2.7111E+00	1.4303E-06
4.0E+05	1.4667E-03	1.4667E-03	2.5389E-01	3.1971E+00	0.	3.4936E-01	2.5535E-01	3.5480E+00	1.4117E-06
5.0E+05	1.6399E-03	1.6399E-03	2.7755E-01	3.9339E+00	0.	4.3670E-01	2.7919E-01	4.3722E+00	1.3917E-06
6.0E+05	1.7964E-03	1.7964E-03	2.8630E-01	4.6615E+00	0.	5.2404E-01	2.8809E-01	5.1874E+00	1.3760E-06
7.0E+05	1.9403E-03	1.9403E-03	2.8048E-01	5.3824E+00	0.	6.1138E-01	2.8242E-01	5.9957E+00	1.3632E-06
8.0E+05	2.0743E-03	2.0743E-03	2.6041E-01	6.0981E+00	0.	6.9872E-01	2.6248E-01	6.7989E+00	1.3526E-06
9.0E+05	2.2001E-03	2.2001E-03	2.2639E-01	6.8102E+00	0.	7.8606E-01	2.2859E-01	7.5984E+00	1.3437E-06
1.0E+06	2.3191E-03	2.3191E-03	1.7872E-01	7.5196E+00	0.	8.7340E-01	1.8104E-01	8.3953E+00	1.3361E-06
2.0E+06	3.2797E-03	3.2797E-03	-9.9214E-01	1.4613E+01	0.	1.7468E+00	-9.8886E-01	1.6363E+01	1.3021E-06
3.0E+06	4.0168E-03	4.0168E-03	-3.2596E+00	2.1954E+01	0.	2.6202E+00	-3.2555E+00	2.4578E+01	1.3039E-06
4.0E+06	4.6382E-03	4.6382E-03	-6.4176E+00	2.9712E+01	0.	3.4936E+00	-6.4129E+00	3.3210E+01	1.3214E-06
5.0E+06	5.1857E-03	5.1857E-03	-1.0303E+01	3.7942E+01	0.	4.3670E+00	-1.0298E+01	4.2314E+01	1.3669E-06
6.0E+06	5.6807E-03	5.6807E-03	-1.4793E+01	4.6651E+01	0.	5.2404E+00	-1.4787E+01	5.1897E+01	1.3766E-06
7.0E+06	6.1358E-03	6.1358E-03	-1.9795E+01	5.5828E+01	0.	6.1138E+00	-1.9789E+01	6.1948E+01	1.4085E-06
8.0E+06	6.5595E-03	6.5595E-03	-2.5242E+01	6.5452E+01	0.	6.9872E+00	-2.5235E+01	7.2446E+01	1.4413E-06
9.0E+06	6.9574E-03	6.9574E-03	-3.1081E+01	7.5502E+01	0.	7.8606E+00	-3.1074E+01	8.3369E+01	1.4743E-06
1.0E+07	7.3337E-03	7.3337E-03	-3.7271E+01	8.5955E+01	0.	8.7340E+00	-3.7264E+01	9.4696E+01	1.5071E-06
2.0E+07	1.0371E-02	1.0371E-02	-1.1350E+02	2.0907E+02	0.	1.7468E+01	-1.1349E+02	2.2655E+02	1.8028E-06
3.0E+07	1.2702E-02	1.2702E-02	-2.0792E+02	3.5869E+02	0.	2.6202E+01	-2.0790E+02	3.8490E+02	2.0420E-06
4.0E+07	1.4667E-02	1.4667E-02	-3.1515E+02	5.2876E+02	0.	3.4936E+01	-3.1514E+02	5.6372E+02	2.2430E-06
5.0E+07	1.6399E-02	1.6399E-02	-4.3251E+02	7.1586E+02	0.	4.3670E+01	-4.3249E+02	7.5955E+02	2.4177E-06
6.0E+07	1.7964E-02	1.7964E-02	-5.5831E+02	9.1770E+02	0.	5.2404E+01	-5.5829E+02	9.7012E+02	2.5733E-06
7.0E+07	1.9403E-02	1.9403E-02	-6.9139E+02	1.1326E+03	0.	6.1138E+01	-6.9137E+02	1.1938E+03	2.7142E-06
8.0E+07	2.0743E-02	2.0743E-02	-8.3089E+02	1.3594E+03	0.	6.9872E+01	-8.3087E+02	1.4293E+03	2.8435E-06
9.0E+07	2.2001E-02	2.2001E-02	-9.7614E+02	1.5970E+03	0.	7.8606E+01	-9.7612E+02	1.6757E+03	2.9632E-06
1.0E+08	2.3191E-02	2.3191E-02	-1.1266E+03	1.8447E+03	0.	8.7340E+01	-1.1266E+03	1.9321E+03	3.0750E-06
2.0E+08	3.2797E-02	3.2797E-02	-2.8476E+03	4.7612E+03	0.	1.7468E+02	-2.8476E+03	4.9359E+03	3.9279E-06
3.0E+08	4.0168E-02	4.0168E-02	-4.8414E+03	8.2776E+03	0.	2.6202E+02	-4.8414E+03	8.5397E+03	4.5304E-06
4.0E+08	4.6382E-02	4.6382E-02	-7.0200E+03	1.2239E+04	0.	3.4936E+02	-7.0199E+03	1.2589E+04	5.0089E-06
5.0E+08	5.1857E-02	5.1857E-02	-9.3377E+03	1.6562E+04	0.	4.3670E+02	-9.3377E+03	1.6999E+04	5.4109E-06
6.0E+08	5.6807E-02	5.6807E-02	-1.1767E+04	2.1191E+04	0.	5.2404E+02	-1.1766E+04	2.1715E+04	5.7600E-06
7.0E+08	6.1358E-02	6.1358E-02	-1.4287E+04	2.6086E+04	0.	6.1138E+02	-1.4287E+04	2.6698E+04	6.0701E-06
8.0E+08	6.5595E-02	6.5595E-02	-1.6885E+04	3.1219E+04	0.	6.9872E+02	-1.6885E+04	3.1918E+04	6.3499E-06
9.0E+08	6.9574E-02	6.9574E-02	-1.9550E+04	3.6567E+04	0.	7.8606E+02	-1.9550E+04	3.7353E+04	6.6055E-06
1.0E+09	7.3337E-02	7.3337E-02	-2.2274E+04	4.2111E+04	0.	8.7340E+02	-2.2274E+04	4.2985E+04	6.8412E-06
2.0E+09	1.0371E-01	1.0371E-01	-5.1659E+04	1.0581E+05	0.	1.7468E+03	-5.1659E+04	1.0756E+05	8.5591E-06
3.0E+09	1.2702E-01	1.2702E-01	-8.3261E+04	1.8017E+05	0.	2.6202E+03	-8.3261E+04	1.8279E+05	9.6971E-06
4.0E+09	1.4667E-01	1.4667E-01	-1.1597E+05	2.6192E+05	0.	3.4936E+03	-1.1597E+05	2.6542E+05	1.0561E-05
5.0E+09	1.6399E-01	1.6399E-01	-1.4929E+05	3.4938E+05	0.	4.3670E+03	-1.4929E+05	3.5374E+05	1.1260E-05
6.0E+09	1.7964E-01	1.7964E-01	-1.8296E+05	4.4146E+05	0.	5.2404E+03	-1.8296E+05	4.4670E+05	1.1849E-05
7.0E+09	1.9403E-01	1.9403E-01	-2.1680E+05	5.3743E+05	0.	6.1138E+03	-2.1680E+05	5.4354E+05	1.2358E-05
8.0E+09	2.0743E-01	2.0743E-01	-2.5071E+05	6.3675E+05	0.	6.9872E+03	-2.5071E+05	6.4373E+05	1.2807E-05
9.0E+09	2.2001E-01	2.2001E-01	-2.8462E+05	7.3900E+05	0.	7.8606E+03	-2.8462E+05	7.4686E+05	1.3207E-05

SIGW SIGS KI KS
 5.800E+07 1.000E-03 4.000E+00 4.000E+00

FREQ.	RE(ZW)	IM(ZW)	RE(ZS)	IM(ZS)	RE(ZI)	IM(ZI)	RE(Z)	IM(Z)	INDUCTANCE
1.0E+10	2.3191E-01	2.3191E-01	-3.1849E+05	8.4387E+05	0.	8.7340E+03	-3.1849E+05	8.5260E+05	1.3570E-05
2.0E+10	3.2797E-01	3.2797E-01	-6.4997E+05	1.9940E+06	0.	1.7468E+04	-6.4997E+05	2.0115E+06	1.6007E-05
3.0E+10	4.0168E-01	4.0168E-01	-9.6496E+05	3.2614E+06	0.	2.6202E+04	-9.6496E+05	3.2876E+06	1.7441E-05
4.0E+10	4.6382E-01	4.6382E-01	-1.2642E+06	4.6003E+06	0.	3.4936E+04	-1.2642E+06	4.6353E+06	1.8443E-05
5.0E+10	5.1857E-01	5.1857E-01	-1.5498E+06	5.9894E+06	0.	4.3670E+04	-1.5498E+06	6.0331E+06	1.9204E-05
6.0E+10	5.6807E-01	5.6807E-01	-1.8233E+06	7.4163E+06	0.	5.2404E+04	-1.8233E+06	7.4687E+06	1.9811E-05
7.0E+10	6.1358E-01	6.1358E-01	-2.0864E+06	8.8731E+06	0.	6.1138E+04	-2.0864E+06	8.9343E+06	2.0313E-05
8.0E+10	6.5595E-01	6.5595E-01	-2.3402E+06	1.0354E+07	0.	6.9872E+04	-2.3402E+06	1.0424E+07	2.0739E-05
9.0E+10	6.9574E-01	6.9574E-01	-2.5856E+06	1.1856E+07	0.	7.8606E+04	-2.5856E+06	1.1935E+07	2.1106E-05

$$Y = \frac{Y_1 Y_2}{Y_1 + Y_2}$$

SIGW SIGS KI KS
 5.800E+07 1.000E-03 4.000E+00 4.000E+00

FREQ.	RI(YI)	IM(YI)	RE(YS)	IM(YS)	RE(Y)	IM(Y)	CAPACITANCE	RE(GAMMA)	IM(GAMMA)	RE(Z(0))	IM(Z(0))
1.0E+10	0.	2.008E+01	3.437E-02	9.094E-02	3.406E-02	9.059E-02	1.442E-12	3.168E-01	2.968E+02	2.872E+03	1.076E+03
2.0E+10	0.	4.016E+01	5.186E-02	1.590E-01	5.145E-02	1.584E-01	1.261E-12	4.450E-01	5.934E+02	3.389E+03	1.098E+03
3.0E+10	0.	6.024E+01	6.586E-02	2.225E-01	6.538E-02	2.217E-01	1.176E-12	5.475E-01	8.900E+02	3.693E+03	1.087E+03
4.0E+10	0.	8.032E+01	7.796E-02	2.835E-01	7.741E-02	2.826E-01	1.125E-12	6.361E-01	1.187E+03	3.906E+03	1.068E+03
5.0E+10	0.	1.004E+02	8.879E-02	3.430E-01	8.819E-02	3.419E-01	1.088E-12	7.155E-01	1.483E+03	4.067E+03	1.047E+03
6.0E+10	0.	1.205E+02	9.871E-02	4.014E-01	9.806E-02	4.001E-01	1.061E-12	7.882E-01	1.780E+03	4.196E+03	1.028E+03
7.0E+10	0.	1.406E+02	1.079E-01	4.588E-01	1.072E-01	4.574E-01	1.040E-12	8.557E-01	2.076E+03	4.303E+03	1.007E+03
8.0E+10	0.	1.606E+02	1.166E-01	5.156E-01	1.158E-01	5.141E-01	1.023E-12	9.191E-01	2.373E+03	4.393E+03	9.880E+02
9.0E+10	0.	1.807E+02	1.247E-01	5.718E-01	1.240E-01	5.701E-01	1.008E-12	9.790E-01	2.669E+03	4.471E+03	9.703E+02

TABLE 3-3

$$\gamma = \frac{Y_1 Y_2}{Y_1 + Y_2}$$

SIGW	SIGS	KI	KS						
5.800E+07	1.000E-04	4.000E+00	4.000E+00						
FREQ.	RE(ZW)	IM(ZW)	RE(ZS)	IM(ZS)	RE(ZI)	IM(ZI)	RE(Z)	IM(Z)	INDUCTANCE
1.0E-02	1.7128E-05	3.1400E-09	9.8646E-09	2.0380E-07	0.	8.7340E-09	1.7138E-05	2.1567E-07	3.4326E-06
2.0E-02	1.7128E-05	6.2800E-09	1.9729E-08	3.9889E-07	0.	1.7468E-08	1.7148E-05	4.2264E-07	3.3633E-06
3.0E-02	1.7128E-05	9.4200E-09	2.9594E-08	5.9070E-07	0.	2.6202E-08	1.7158E-05	6.2632E-07	3.3228E-06
4.0E-02	1.7128E-05	1.2560E-08	3.9458E-08	7.8038E-07	0.	3.4936E-08	1.7168E-05	8.2787E-07	3.2940E-06
5.0E-02	1.7128E-05	1.5700E-08	4.9323E-08	9.6846E-07	0.	4.3670E-08	1.7178E-05	1.0270E-06	3.2717E-06
6.0E-02	1.7128E-05	1.8840E-08	5.9188E-08	1.1553E-06	0.	5.2404E-08	1.7188E-05	1.2265E-06	3.2535E-06
7.0E-02	1.7128E-05	2.1980E-08	6.9052E-08	1.3411E-06	0.	6.1138E-08	1.7197E-05	1.4242E-06	3.2381E-06
8.0E-02	1.7128E-05	2.5120E-08	7.8917E-08	1.5259E-06	0.	6.9872E-08	1.7207E-05	1.6209E-06	3.2247E-06
9.0E-02	1.7128E-05	2.8260E-08	8.8781E-08	1.7100E-06	0.	7.8606E-08	1.7217E-05	1.8169E-06	3.2129E-06
1.0E-01	1.7128E-05	3.1400E-08	9.8646E-08	1.8934E-06	0.	8.7340E-08	1.7227E-05	2.0121E-06	3.2024E-06
2.0E-01	1.7128E-05	6.2800E-08	1.9729E-07	3.6997E-06	0.	1.7468E-07	1.7326E-05	3.9372E-06	3.1331E-06
3.0E-01	1.7129E-05	9.4200E-08	2.9594E-07	5.4732E-06	0.	2.6202E-07	1.7424E-05	5.8294E-06	3.0926E-06
4.0E-01	1.7129E-05	1.2560E-07	3.9458E-07	7.2254E-06	0.	3.4936E-07	1.7523E-05	7.7003E-06	3.0639E-06
5.0E-01	1.7129E-05	1.5700E-07	4.9323E-07	8.9670E-06	0.	4.3670E-07	1.7622E-05	9.5553E-06	3.0416E-06
6.0E-01	1.7129E-05	1.8839E-07	5.9187E-07	1.0685E-05	0.	5.2404E-07	1.7721E-05	1.1398E-05	3.0233E-06
7.0E-01	1.7129E-05	2.1979E-07	6.9051E-07	1.2398E-05	0.	6.1138E-07	1.7820E-05	1.3230E-05	3.0079E-06
8.0E-01	1.7130E-05	2.5119E-07	7.8916E-07	1.4102E-05	0.	6.9872E-07	1.7919E-05	1.5052E-05	2.9946E-06
9.0E-01	1.7130E-05	2.8258E-07	8.8780E-07	1.5799E-05	0.	7.8606E-07	1.8018E-05	1.6867E-05	2.9828E-06
1.0E+00	1.7130E-05	3.1398E-07	9.8644E-07	1.7488E-05	0.	8.7340E-07	1.8117E-05	1.8675E-05	2.9723E-06
2.0E+00	1.7136E-05	6.2786E-07	1.9728E-06	3.4105E-05	0.	1.7468E-06	1.9109E-05	3.6480E-05	2.9030E-06
3.0E+00	1.7146E-05	9.4153E-07	2.9592E-06	5.0394E-05	0.	2.6202E-06	2.0105E-05	5.3956E-05	2.8624E-06
4.0E+00	1.7159E-05	1.2549E-06	3.9455E-06	6.6469E-05	0.	3.4936E-06	2.1105E-05	7.1218E-05	2.8337E-06
5.0E+00	1.7176E-05	1.5678E-06	4.9318E-06	8.2386E-05	0.	4.3670E-06	2.2108E-05	8.8321E-05	2.8113E-06
6.0E+00	1.7197E-05	1.8802E-06	5.9181E-06	9.8176E-05	0.	5.2404E-06	2.3115E-05	1.0530E-04	2.7931E-06
7.0E+00	1.7222E-05	2.1920E-06	6.9043E-06	1.1386E-04	0.	6.1138E-06	2.4124E-05	1.2217E-04	2.7776E-06
8.0E+00	1.7251E-05	2.5030E-06	7.8905E-06	1.2946E-04	0.	6.9872E-06	2.5141E-05	1.3895E-04	2.7643E-06
9.0E+00	1.7283E-05	2.8133E-06	8.8767E-06	1.4497E-04	0.	7.8606E-06	2.6159E-05	1.5565E-04	2.7524E-06
1.0E+01	1.7319E-05	3.1226E-06	9.8628E-06	1.6042E-04	0.	8.7340E-06	2.7181E-05	1.7228E-04	2.7419E-06
2.0E+01	1.7869E-05	6.1446E-06	1.9722E-05	3.1213E-04	0.	1.7468E-05	3.7592E-05	3.3575E-04	2.6718E-06
3.0E+01	1.8727E-05	8.9832E-06	2.9578E-05	4.6056E-04	0.	2.6202E-05	4.8306E-05	4.9575E-04	2.6300E-06
4.0E+01	1.9818E-05	1.1585E-05	3.9431E-05	6.0686E-04	0.	3.4936E-05	5.9250E-05	6.5338E-04	2.5997E-06
5.0E+01	2.1064E-05	1.3929E-05	4.9281E-05	7.5156E-04	0.	4.3670E-05	7.0345E-05	8.0916E-04	2.5756E-06
6.0E+01	2.2392E-05	1.6019E-05	5.9128E-05	8.9501E-04	0.	5.2404E-05	8.1520E-05	9.6343E-04	2.5556E-06
7.0E+01	2.3748E-05	1.7877E-05	6.8972E-05	1.0374E-03	0.	6.1138E-05	9.2719E-05	1.1164E-03	2.5383E-06
8.0E+01	2.5090E-05	1.9534E-05	7.8812E-05	1.1789E-03	0.	6.9872E-05	1.0390E-04	1.2683E-03	2.5232E-06
9.0E+01	2.6395E-05	2.1024E-05	8.8649E-05	1.3196E-03	0.	7.8606E-05	1.1504E-04	1.4192E-03	2.5097E-06
1.0E+02	2.7648E-05	2.2376E-05	9.8484E-05	1.4596E-03	0.	8.7340E-05	1.2613E-04	1.5693E-03	2.4976E-06
2.0E+02	3.7501E-05	3.2291E-05	1.9666E-04	2.8322E-03	0.	1.7468E-04	2.3416E-04	3.0391E-03	2.4185E-06
3.0E+02	4.4785E-05	3.9736E-05	2.9455E-04	4.1719E-03	0.	2.6202E-04	3.3933E-04	4.4736E-03	2.3733E-06
4.0E+02	5.0950E-05	4.6023E-05	3.9214E-04	5.4903E-03	0.	3.4936E-04	4.4309E-04	5.8857E-03	2.3418E-06
5.0E+02	5.6398E-05	5.1544E-05	4.8946E-04	6.7929E-03	0.	4.3670E-04	5.4585E-04	7.2811E-03	2.3176E-06
6.0E+02	6.1327E-05	5.6525E-05	5.8649E-04	8.0828E-03	0.	5.2404E-04	6.4781E-04	8.6634E-03	2.2980E-06
7.0E+02	6.5860E-05	6.1099E-05	6.8324E-04	9.3623E-03	0.	6.1138E-04	7.4910E-04	1.0035E-02	2.2815E-06
8.0E+02	7.0084E-05	6.5357E-05	7.7972E-04	1.0633E-02	0.	6.9872E-04	8.4980E-04	1.1397E-02	2.2673E-06
9.0E+02	7.4056E-05	6.9357E-05	8.7592E-04	1.1895E-02	0.	7.8606E-04	9.4998E-04	1.2751E-02	2.2548E-06
1.0E+03	7.3337E-05	7.3337E-05	9.7185E-04	1.3151E-02	0.	8.7340E-04	1.0452E-03	1.4098E-02	2.2437E-06
2.0E+03	1.0371E-04	1.0371E-04	1.9164E-03	2.5434E-02	0.	1.7468E-03	2.0201E-03	2.7284E-02	2.1712E-06
3.0E+03	1.2702E-04	1.2702E-04	2.8348E-03	3.7390E-02	0.	2.6202E-03	2.9618E-03	4.0137E-02	2.1293E-06
4.0E+03	1.4667E-04	1.4667E-04	3.7274E-03	4.9135E-02	0.	3.4936E-03	3.8743E-03	5.2775E-02	2.0999E-06
5.0E+03	1.6399E-04	1.6399E-04	4.5952E-03	6.0724E-02	0.	4.3670E-03	4.7592E-03	6.5255E-02	2.0771E-06
6.0E+03	1.7964E-04	1.7964E-04	5.4379E-03	7.2189E-02	0.	5.2404E-03	5.6174E-03	7.7609E-02	2.0586E-06
7.0E+03	1.9403E-04	1.9403E-04	6.2568E-03	8.3551E-02	0.	6.1138E-03	6.4501E-03	8.9459E-02	2.0431E-06
8.0E+03	2.0743E-04	2.0743E-04	7.0498E-03	9.4826E-02	0.	6.9872E-03	7.2572E-03	1.0202E-01	2.0294E-06
9.0E+03	2.2001E-04	2.2001E-04	7.8193E-03	1.0602E-01	0.	7.8606E-03	8.0393E-03	1.1410E-01	2.0178E-06

SIGW SIGS KI KS
 5.800E+07 1.000E-04 4.000E+00 4.000E+00

FREQ.	RE(ZW)	IM(ZW)	RE(ZS)	IM(ZS)	RE(ZI)	IM(ZI)	RE(Z)	IM(Z)	INDUCTANCE
1.0E+10	2.3191E-01	2.3191E-01	-2.2368E+06	4.2061E+06	0.	8.7340E+03	-2.2368E+06	4.2149E+06	6.7082E-05
2.0E+10	3.2797E-01	3.2797E-01	-5.1776E+06	1.0575E+07	0.	1.7468E+04	-5.1776E+06	1.0593E+07	8.4294E-05
3.0E+10	4.0168E-01	4.0168E-01	-8.3395E+06	1.8010E+07	0.	2.6202E+04	-8.3395E+06	1.8037E+07	9.5687E-05
4.0E+10	4.6382E-01	4.6382E-01	-1.1612E+07	2.6186E+07	0.	3.4936E+04	-1.1612E+07	2.6221E+07	1.0433E-04
5.0E+10	5.1857E-01	5.1857E-01	-1.4945E+07	3.4931E+07	0.	4.3670E+04	-1.4945E+07	3.4975E+07	1.1133E-04
6.0E+10	5.6807E-01	5.6807E-01	-1.8312E+07	4.4139E+07	0.	5.2404E+04	-1.8312E+07	4.4191E+07	1.1722E-04
7.0E+10	6.1358E-01	6.1358E-01	-2.1697E+07	5.3736E+07	0.	6.1138E+04	-2.1697E+07	5.3797E+07	1.2231E-04
8.0E+10	6.5595E-01	6.5595E-01	-2.5089E+07	6.3668E+07	0.	6.9872E+04	-2.5089E+07	6.3738E+07	1.2680E-04
9.0E+10	6.9574E-01	6.9574E-01	-2.8481E+07	7.3893E+07	0.	7.8606E+04	-2.8481E+07	7.3972E+07	1.3081E-04

TABLE 3-3 (Cont'd)

$\gamma = \frac{\gamma_{01}}{\gamma_{02}}$

SIWA SIGS KI KS
 5.800E+07 1.000E-04 4.000E+00 4.000E+00

FREQ.	RI(YI)	IM(YI)	RE(YS)	IM(YS)	RE(Y)	IM(Y)	CAPACITANCE	RE(GAMMA)	IM(GAMMA)	RE(Z(O))	IM(Z(O))
1.0E-02	0.	2.008E-11	1.932E-05	9.350E-07	2.082E-17	2.008E-11	3.196E-10	1.303E-08	1.320E-08	6.574E+02	-6.492E+02
2.0E-02	0.	4.016E-11	1.974E-05	9.761E-07	8.152E-17	4.016E-11	3.196E-10	1.833E-08	1.879E-08	4.678E+02	-4.564E+02
3.0E-02	0.	6.024E-11	1.999E-05	1.001E-06	1.811E-16	6.024E-11	3.196E-10	2.232E-08	2.315E-08	3.843E+02	-3.706E+02
4.0E-02	0.	8.032E-11	2.017E-05	1.020E-06	3.189E-16	8.032E-11	3.196E-10	2.563E-08	2.690E-08	3.349E+02	-3.191E+02
5.0E-02	0.	1.004E-10	2.032E-05	1.035E-06	4.948E-16	1.004E-10	3.196E-10	2.850E-08	3.026E-08	3.014E+02	-2.839E+02
6.0E-02	0.	1.205E-10	2.044E-05	1.047E-06	7.083E-16	1.205E-10	3.196E-10	3.105E-08	3.334E-08	2.768E+02	-2.577E+02
7.0E-02	0.	1.406E-10	2.054E-05	1.058E-06	9.592E-16	1.406E-10	3.196E-10	3.336E-08	3.623E-08	2.578E+02	-2.373E+02
8.0E-02	0.	1.606E-10	2.063E-05	1.067E-06	1.247E-15	1.606E-10	3.196E-10	3.547E-08	3.897E-08	2.426E+02	-2.208E+02
9.0E-02	0.	1.807E-10	2.071E-05	1.075E-06	1.573E-15	1.807E-10	3.196E-10	3.742E-08	4.157E-08	4.157E-08	-2.071E+02
1.0E-01	0.	2.008E-10	2.078E-05	1.083E-06	1.935E-15	2.008E-10	3.196E-10	3.923E-08	4.408E-08	2.195E+02	-1.954E+02
2.0E-01	0.	4.016E-10	2.127E-05	1.134E-06	7.561E-15	4.016E-10	3.196E-10	5.270E-08	6.602E-08	1.644E+02	-1.312E+02
3.0E-01	0.	6.024E-10	2.157E-05	1.166E-06	1.678E-14	6.024E-10	3.196E-10	6.147E-08	8.538E-08	1.417E+02	-1.020E+02
4.0E-01	0.	8.032E-10	2.178E-05	1.189E-06	2.953E-14	8.032E-10	3.196E-10	6.778E-08	1.038E-07	1.293E+02	-8.439E+01
5.0E-01	0.	1.004E-09	2.195E-05	1.208E-06	4.578E-14	1.004E-09	3.196E-10	7.257E-08	1.219E-07	1.214E+02	-7.228E+01
6.0E-01	0.	1.205E-09	2.209E-05	1.224E-06	6.551E-14	1.205E-09	3.196E-10	7.633E-08	1.398E-07	1.161E+02	-6.335E+01
7.0E-01	0.	1.406E-09	2.221E-05	1.237E-06	8.868E-14	1.406E-09	3.196E-10	7.938E-08	1.578E-07	1.123E+02	-5.647E+01
8.0E-01	0.	1.606E-09	2.231E-05	1.249E-06	1.153E-13	1.606E-09	3.196E-10	8.190E-08	1.757E-07	1.094E+02	-5.098E+01
9.0E-01	0.	1.807E-09	2.241E-05	1.259E-06	1.453E-13	1.807E-09	3.196E-10	8.403E-08	1.938E-07	1.072E+02	-4.649E+01
1.0E+00	0.	2.008E-09	2.249E-05	1.269E-06	1.787E-13	2.008E-09	3.196E-10	8.587E-08	2.118E-07	1.055E+02	-4.276E+01
2.0E+00	0.	4.016E-09	2.306E-05	1.334E-06	6.969E-13	4.016E-09	3.196E-10	9.720E-08	3.949E-07	9.834E+01	-2.419E+01
3.0E+00	0.	6.024E-09	2.341E-05	1.375E-06	1.545E-12	6.024E-09	3.196E-10	1.045E-07	5.796E-07	9.622E+01	-1.733E+01
4.0E+00	0.	8.032E-09	2.364E-05	1.405E-06	2.717E-12	8.032E-09	3.196E-10	1.110E-07	7.644E-07	9.518E+01	-1.379E+01
5.0E+00	0.	1.004E-08	2.386E-05	1.429E-06	4.209E-12	1.004E-08	3.196E-10	1.172E-07	9.488E-07	9.452E+01	-1.163E+01
6.0E+00	0.	1.205E-08	2.403E-05	1.449E-06	6.018E-12	1.205E-08	3.196E-10	1.232E-07	1.133E-06	9.405E+01	-1.018E+01
7.0E+00	0.	1.406E-08	2.417E-05	1.466E-06	8.143E-12	1.406E-08	3.196E-10	1.292E-07	1.317E-06	9.368E+01	-9.135E+00
8.0E+00	0.	1.606E-08	2.429E-05	1.481E-06	1.058E-11	1.606E-08	3.196E-10	1.351E-07	1.500E-06	9.339E+01	-8.350E+00
9.0E+00	0.	1.807E-08	2.440E-05	1.495E-06	1.333E-11	1.807E-08	3.196E-10	1.411E-07	1.683E-06	9.314E+01	-7.738E+00
1.0E+01	0.	2.008E-08	2.450E-05	1.507E-06	1.639E-11	2.008E-08	3.196E-10	1.470E-07	1.865E-06	9.292E+01	-7.247E+00
2.0E+01	0.	4.016E-08	2.518E-05	1.592E-06	6.377E-11	4.016E-08	3.195E-10	2.082E-07	3.677E-06	9.159E+01	-5.038E+00
3.0E+01	0.	6.024E-08	2.560E-05	1.646E-06	1.411E-10	6.023E-08	3.195E-10	2.723E-07	5.470E-06	9.184E+01	-4.309E+00
4.0E+01	0.	8.032E-08	2.590E-05	1.685E-06	2.479E-10	8.030E-08	3.195E-10	3.393E-07	7.250E-06	9.030E+01	-3.946E+00
5.0E+01	0.	1.004E-07	2.614E-05	1.717E-06	3.838E-10	1.004E-07	3.195E-10	4.086E-07	9.020E-06	8.988E+01	-3.727E+00
6.0E+01	0.	1.205E-07	2.634E-05	1.743E-06	5.483E-10	1.204E-07	3.195E-10	4.799E-07	1.078E-05	8.953E+01	-3.577E+00
7.0E+01	0.	1.406E-07	2.651E-05	1.767E-06	7.414E-10	1.405E-07	3.194E-10	5.527E-07	1.255E-05	8.923E+01	-3.463E+00
8.0E+01	0.	1.606E-07	2.666E-05	1.787E-06	9.628E-10	1.606E-07	3.194E-10	6.269E-07	1.421E-05	8.876E+01	-3.371E+00
9.0E+01	0.	1.807E-07	2.679E-05	1.807E-06	1.212E-09	1.806E-07	3.194E-10	7.022E-07	1.601E-05	8.873E+01	-3.292E+00
1.0E+02	0.	2.008E-07	2.691E-05	1.822E-06	1.490E-09	2.007E-07	3.194E-10	7.785E-07	1.776E-05	8.851E+01	-3.222E+00
2.0E+02	0.	4.016E-07	2.773E-05	1.938E-06	5.775E-09	4.011E-07	3.192E-10	1.594E-06	3.493E-05	8.713E+01	-2.724E+00
3.0E+02	0.	6.024E-07	2.823E-05	2.012E-06	1.274E-08	6.012E-07	3.189E-10	2.516E-06	5.194E-05	8.614E+01	-2.354E+00
4.0E+02	0.	8.032E-07	2.868E-05	2.069E-06	2.233E-08	8.009E-07	3.187E-10	3.541E-06	6.648E-05	8.580E+01	-2.028E+00
5.0E+02	0.	1.004E-06	2.989E-05	2.114E-06	3.448E-08	1.000E-06	3.184E-10	4.669E-06	8.536E-05	8.540E+01	-1.724E+00
6.0E+02	0.	1.205E-06	2.913E-05	2.153E-06	4.917E-08	1.199E-06	3.181E-10	5.899E-06	1.019E-04	8.507E+01	-1.432E+00
7.0E+02	0.	1.406E-06	2.934E-05	2.187E-06	6.633E-08	1.397E-06	3.177E-10	7.230E-06	1.184E-04	8.480E+01	-1.148E+00
8.0E+02	0.	1.606E-06	2.953E-05	2.218E-06	8.595E-08	1.595E-06	3.174E-10	8.659E-06	1.348E-04	8.458E+01	-8.711E-01
9.0E+02	0.	1.807E-06	2.969E-05	2.246E-06	1.080E-07	1.792E-06	3.170E-10	1.019E-05	1.512E-04	8.438E+01	-5.988E-01
1.0E+03	0.	2.008E-06	2.984E-05	2.272E-06	1.324E-07	1.989E-06	3.165E-10	1.178E-05	1.675E-04	8.421E+01	-3.171E-01
2.0E+03	0.	4.016E-06	3.084E-05	2.462E-06	5.008E-07	3.911E-06	3.112E-10	3.300E-05	3.268E-04	8.327E+01	2.226E+00
3.0E+03	0.	6.024E-06	3.146E-05	2.596E-06	1.073E-06	5.730E-06	3.040E-10	6.249E-05	4.803E-04	8.296E+01	6.627E+00
4.0E+03	0.	8.032E-06	3.192E-05	2.707E-06	1.816E-06	7.421E-06	2.953E-10	9.917E-05	6.280E-04	8.294E+01	6.928E+00
5.0E+03	0.	1.004E-05	3.228E-05	2.804E-06	2.696E-06	8.967E-06	2.854E-10	1.420E-04	7.697E-04	8.309E+01	9.145E+00
6.0E+03	0.	1.205E-05	3.258E-05	2.891E-06	3.681E-06	1.036E-05	2.748E-10	1.899E-04	9.052E-04	8.337E+01	1.129E+01
7.0E+03	0.	1.406E-05	3.284E-05	2.973E-06	4.741E-06	1.160E-05	2.637E-10	2.421E-04	1.034E-03	8.374E+01	1.336E+01
8.0E+03	0.	1.606E-05	3.308E-05	3.050E-06	5.849E-06	1.268E-05	2.523E-10	2.975E-04	1.158E-03	8.419E+01	1.537E+01
9.0E+03	0.	1.807E-05	3.326E-05	3.123E-06	6.983E-06	1.362E-05	2.409E-10	3.555E-04	1.275E-03	8.499E+01	1.732E+01

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SIGW SIGS KI KS
5.800E+07 1.000E-04 4.000E+00 4.000E+00

FREQ.	RI(YI)	IM(YI)	RE(YI)	IM(YI)	RE(YI)	IM(YI)	CAPACITANCE	RE(GAMMA)	IM(GAMMA)	RE(Z(O))	IM(Z(O))
1.0E+04	0.	2.008E-05	3.345E-05	3.194E-06	8.122E-06	1.443E-05	2.296E-10	4.153E-04	1.386E-03	8.525E+01	1.920E+01
2.0E+04	0.	4.016E-05	3.469E-05	3.815E-06	1.783E-05	1.755E-05	1.397E-10	1.025E-03	2.249E-03	9.224E+01	3.533E+01
3.0E+04	0.	6.024E-05	3.545E-05	4.369E-06	2.369E-05	1.707E-05	9.057E-11	1.551E-03	2.841E-03	9.996E+01	4.787E+01
4.0E+04	0.	8.032E-05	3.601E-05	4.897E-06	2.715E-05	1.609E-05	6.401E-11	1.982E-03	3.308E-03	1.075E+02	5.816E+01
5.0E+04	0.	1.004E-04	3.646E-05	5.410E-06	2.934E-05	1.524E-05	4.852E-11	2.344E-03	3.712E-03	1.144E+02	6.693E+01
6.0E+04	0.	1.205E-04	3.683E-05	5.916E-06	3.084E-05	1.463E-05	3.880E-11	2.653E-03	4.078E-03	1.214E+02	7.464E+01
7.0E+04	0.	1.406E-04	3.715E-05	6.416E-06	3.194E-05	1.421E-05	3.231E-11	2.922E-03	4.420E-03	1.270E+02	8.154E+01
8.0E+04	0.	1.606E-04	3.744E-05	6.912E-06	3.278E-05	1.395E-05	2.776E-11	3.161E-03	4.746E-03	1.338E+02	8.782E+01
9.0E+04	0.	1.807E-04	3.770E-05	7.406E-06	3.345E-05	1.382E-05	2.443E-11	3.375E-03	5.059E-03	1.396E+02	9.360E+01
1.0E+05	0.	2.008E-04	3.794E-05	7.897E-06	3.400E-05	1.378E-05	2.193E-11	3.568E-03	5.363E-03	1.450E+02	9.896E+01
2.0E+05	0.	4.016E-04	3.983E-05	1.272E-05	3.708E-05	1.590E-05	8.165E-11	4.829E-03	8.183E-03	1.900E+02	1.394E+02
3.0E+05	0.	6.024E-04	4.148E-05	1.738E-05	3.901E-05	1.950E-05	1.035E-11	5.404E-03	1.092E-02	2.244E+02	1.677E+02
4.0E+05	0.	8.032E-04	4.315E-05	2.181E-05	4.079E-05	2.337E-05	9.299E-12	5.870E-03	1.367E-02	2.529E+02	1.902E+02
5.0E+05	0.	1.004E-03	4.488E-05	2.601E-05	4.256E-05	2.721E-05	8.662E-12	6.117E-03	1.646E-02	2.775E+02	2.093E+02
6.0E+05	0.	1.205E-03	4.665E-05	2.998E-05	4.435E-05	3.093E-05	8.205E-12	6.205E-03	1.929E-02	2.994E+02	2.261E+02
7.0E+05	0.	1.406E-03	4.844E-05	3.374E-05	4.614E-05	3.451E-05	7.845E-12	6.406E-03	2.214E-02	3.192E+02	2.412E+02
8.0E+05	0.	1.606E-03	5.023E-05	3.732E-05	4.793E-05	3.793E-05	7.547E-12	6.497E-03	2.502E-02	3.374E+02	2.550E+02
9.0E+05	0.	1.807E-03	5.201E-05	4.072E-05	4.970E-05	4.122E-05	7.290E-12	6.569E-03	2.792E-02	3.543E+02	2.678E+02
1.0E+06	0.	2.008E-03	5.378E-05	4.378E-05	5.146E-05	4.438E-05	7.063E-12	6.627E-03	3.063E-02	3.701E+02	2.798E+02
2.0E+06	0.	4.016E-03	7.014E-05	7.107E-05	6.770E-05	7.099E-05	5.649E-12	6.927E-03	6.026E-02	4.932E+02	3.728E+02
3.0E+06	0.	6.024E-03	8.434E-05	9.256E-05	8.179E-05	9.228E-05	4.896E-12	7.004E-03	8.988E-02	5.836E+02	4.405E+02
4.0E+06	0.	8.032E-03	9.700E-05	1.111E-04	9.436E-05	1.107E-04	4.404E-12	7.203E-03	1.196E-01	6.576E+02	4.996E+02
5.0E+06	0.	1.004E-02	1.086E-04	1.277E-04	1.058E-04	1.272E-04	4.050E-12	7.304E-03	1.492E-01	7.215E+02	5.428E+02
6.0E+06	0.	1.205E-02	1.193E-04	1.430E-04	1.165E-04	1.424E-04	3.778E-12	7.394E-03	1.789E-01	7.783E+02	5.845E+02
7.0E+06	0.	1.406E-02	1.293E-04	1.572E-04	1.264E-04	1.566E-04	3.561E-12	7.476E-03	2.066E-01	8.298E+02	6.222E+02
8.0E+06	0.	1.606E-02	1.388E-04	1.706E-04	1.359E-04	1.700E-04	3.382E-12	7.553E-03	2.383E-01	8.771E+02	6.566E+02
9.0E+06	0.	1.807E-02	1.478E-04	1.834E-04	1.448E-04	1.827E-04	3.231E-12	7.625E-03	2.680E-01	9.211E+02	6.885E+02
1.0E+07	0.	2.008E-02	1.564E-04	1.955E-04	1.534E-04	1.948E-04	3.101E-12	7.693E-03	2.977E-01	9.623E+02	7.183E+02
2.0E+07	0.	4.016E-02	2.290E-04	2.980E-04	2.257E-04	2.971E-04	2.364E-12	8.250E-03	5.945E-01	1.282E+03	9.464E+02
3.0E+07	0.	6.024E-02	2.875E-04	3.811E-04	2.839E-04	3.801E-04	2.016E-12	8.691E-03	8.912E-01	1.516E+03	1.109E+03
4.0E+07	0.	8.032E-02	3.384E-04	4.541E-04	3.346E-04	4.529E-04	1.802E-12	9.071E-03	1.188E+00	1.706E+03	1.240E+03
5.0E+07	0.	1.004E-01	3.842E-04	5.203E-04	3.802E-04	5.190E-04	1.652E-12	9.412E-03	1.484E+00	1.870E+03	1.352E+03
6.0E+07	0.	1.205E-01	4.263E-04	5.816E-04	4.223E-04	5.803E-04	1.539E-12	9.726E-03	1.781E+00	2.015E+03	1.449E+03
7.0E+07	0.	1.406E-01	4.657E-04	6.392E-04	4.615E-04	6.379E-04	1.450E-12	1.002E-02	2.078E+00	2.145E+03	1.537E+03
8.0E+07	0.	1.606E-01	5.028E-04	6.939E-04	4.985E-04	6.924E-04	1.378E-12	1.029E-02	2.374E+00	2.265E+03	1.616E+03
9.0E+07	0.	1.807E-01	5.380E-04	7.460E-04	5.336E-04	7.445E-04	1.317E-12	1.055E-02	2.671E+00	2.376E+03	1.689E+03
1.0E+08	0.	2.008E-01	5.717E-04	7.961E-04	5.672E-04	7.945E-04	1.265E-12	1.080E-02	2.967E+00	2.480E+03	1.757E+03
2.0E+08	0.	4.016E-01	8.540E-04	1.224E-03	8.488E-04	1.222E-03	9.727E-13	1.289E-02	5.931E+00	3.279E+03	2.266E+03
3.0E+08	0.	6.024E-01	1.082E-03	1.579E-03	1.076E-03	1.577E-03	8.366E-13	1.457E-02	8.895E+00	3.853E+03	2.620E+03
4.0E+08	0.	8.032E-01	1.280E-03	1.895E-03	1.274E-03	1.892E-03	7.529E-13	1.603E-02	1.186E+01	4.317E+03	2.897E+03
5.0E+08	0.	1.004E+00	1.459E-03	2.184E-03	1.452E-03	2.181E-03	6.944E-13	1.734E-02	1.482E+01	4.712E+03	3.129E+03
6.0E+08	0.	1.205E+00	1.623E-03	2.454E-03	1.617E-03	2.452E-03	6.503E-13	1.856E-02	1.779E+01	5.059E+03	3.329E+03
7.0E+08	0.	1.406E+00	1.778E-03	2.710E-03	1.771E-03	2.707E-03	6.156E-13	1.969E-02	2.075E+01	5.371E+03	3.506E+03
8.0E+08	0.	1.606E+00	1.923E-03	2.954E-03	1.916E-03	2.951E-03	5.871E-13	2.077E-02	2.371E+01	5.655E+03	3.664E+03
9.0E+08	0.	1.807E+00	2.061E-03	3.189E-03	2.054E-03	3.185E-03	5.633E-13	2.178E-02	2.667E+01	5.918E+03	3.809E+03
1.0E+09	0.	2.008E+00	2.194E-03	3.415E-03	2.186E-03	3.411E-03	5.430E-13	2.276E-02	2.964E+01	6.162E+03	3.942E+03
2.0E+09	0.	4.016E+00	3.307E-03	5.389E-03	3.298E-03	5.385E-03	4.285E-13	3.096E-02	5.926E+01	8.006E+03	4.898E+03
3.0E+09	0.	6.024E+00	4.209E-03	7.072E-03	4.199E-03	7.067E-03	3.749E-13	3.760E-02	8.889E+01	9.298E+03	5.519E+03
4.0E+09	0.	8.032E+00	4.996E-03	8.597E-03	4.985E-03	8.591E-03	3.418E-13	4.340E-02	1.185E+02	1.032E+04	5.985E+03
5.0E+09	0.	1.004E+01	5.708E-03	1.002E-02	5.697E-03	1.001E-02	3.187E-13	4.865E-02	1.481E+02	1.118E+04	6.357E+03
6.0E+09	0.	1.205E+01	6.365E-03	1.136E-02	6.353E-03	1.135E-02	3.012E-13	5.349E-02	1.778E+02	1.192E+04	6.667E+03
7.0E+09	0.	1.406E+01	6.980E-03	1.265E-02	6.968E-03	1.264E-02	2.874E-13	5.802E-02	2.074E+02	1.258E+04	6.933E+03
8.0E+09	0.	1.606E+01	7.561E-03	1.389E-02	7.548E-03	1.388E-02	2.761E-13	6.230E-02	2.370E+02	1.318E+04	7.165E+03
9.0E+09	0.	1.807E+01	8.114E-03	1.509E-02	8.101E-03	1.508E-02	2.666E-13	6.637E-02	2.666E+02	1.372E+04	7.370E+03

SIGW SIGS KI KS
 5.800E+07 1.000E-04 4.000E+00 4.000E+00

FREQ.	RI(YI)	IM(YI)	RE(YS)	IM(YS)	RE(Y)	IM(Y)	CAPACITANCE	RE(GAMMA)	IM(GAMMA)	RE(Z(0))	IM(Z(0))
1.0E+10	0.	2.008E+01	8.643E-03	1.625E-02	8.629E-03	1.624E-02	2.585E-13	7.027E-02	2.962E+02	1.423E+04	7.554E+03
2.0E+10	0.	4.016E+01	1.310E-02	2.675E-02	1.308E-02	2.674E-02	2.128E-13	1.031E-01	5.924E+02	1.788E+04	8.743E+03
3.0E+10	0.	6.024E+01	1.671E-02	3.608E-02	1.669E-02	3.606E-02	1.913E-13	1.297E-01	8.886E+02	2.030E+04	9.388E+03
4.0E+10	0.	8.032E+01	1.985E-02	4.477E-02	1.983E-02	4.475E-02	1.781E-13	1.529E-01	1.185E+03	2.213E+04	9.803E+03
5.0E+10	0.	1.004E+02	2.270E-02	5.305E-02	2.267E-02	5.302E-02	1.688E-13	1.738E-01	1.481E+03	2.361E+04	1.009E+04
6.0E+10	0.	1.205E+02	2.531E-02	6.101E-02	2.529E-02	6.099E-02	1.618E-13	1.931E-01	1.777E+03	2.487E+04	1.031E+04
7.0E+10	0.	1.406E+02	2.776E-02	6.875E-02	2.773E-02	6.872E-02	1.562E-13	2.111E-01	2.073E+03	2.595E+04	1.047E+04
8.0E+10	0.	1.606E+02	3.007E-02	7.630E-02	3.004E-02	7.626E-02	1.517E-13	2.281E-01	2.369E+03	2.690E+04	1.059E+04
9.0E+10	0.	1.807E+02	3.226E-02	8.369E-02	3.223E-02	8.365E-02	1.479E-13	2.442E-01	2.666E+03	2.775E+04	1.069E+04

TABLE 3-4

Y = Y_i

SIW 5.800E+07 SI85 1.000E-02 KI 4.000E+00 KS 4.000E+00

FREQ.	RE(ZW)	IM(ZW)	RE(ZS)	IM(ZS)	RE(ZI)	IM(ZI)	RE(Z)	IM(Z)	INDUCTANCE
1.0E-02	1.7128E-05	3.1400E-09	9.8644E-09	1.7488E-07	0.	8.7340E-09	1.7138E-05	1.8675E-07	2.9723E-06
2.0E-02	1.7128E-05	6.2800E-09	1.9729E-08	3.4105E-07	0.	1.7468E-08	1.7148E-05	3.6480E-07	2.9630E-06
3.0E-02	1.7128E-05	9.4200E-09	2.9594E-08	5.0394E-07	0.	2.6202E-08	1.7158E-05	5.3956E-07	2.8629E-06
4.0E-02	1.7128E-05	1.2560E-08	3.9458E-08	6.6469E-07	0.	3.4936E-08	1.7168E-05	7.1219E-07	2.8337E-06
5.0E-02	1.7128E-05	1.5700E-08	4.9323E-08	8.2386E-07	0.	4.3670E-08	1.7178E-05	8.8323E-07	2.8114E-06
6.0E-02	1.7128E-05	1.8840E-08	5.9188E-08	9.8176E-07	0.	5.2404E-08	1.7188E-05	1.0530E-06	2.7932E-06
7.0E-02	1.7128E-05	2.1980E-08	6.9052E-08	1.1386E-06	0.	6.1138E-08	1.7197E-05	1.2217E-06	2.7778E-06
8.0E-02	1.7128E-05	2.5120E-08	7.8917E-08	1.2946E-06	0.	6.9872E-08	1.7207E-05	1.3896E-06	2.7644E-06
9.0E-02	1.7128E-05	2.8260E-08	8.8781E-08	1.4497E-06	0.	7.8606E-08	1.7217E-05	1.5566E-06	2.7527E-06
1.0E-01	1.7128E-05	3.1400E-08	9.8644E-08	1.6042E-06	0.	8.7340E-08	1.7227E-05	1.7229E-06	2.7421E-06
2.0E-01	1.7128E-05	6.2800E-08	1.9729E-07	3.1213E-06	0.	1.7468E-07	1.7326E-05	3.3506E-06	2.6729E-06
3.0E-01	1.7129E-05	9.4200E-08	2.9594E-07	4.6056E-06	0.	2.6202E-07	1.7424E-05	4.9618E-06	2.6323E-06
4.0E-01	1.7129E-05	1.2560E-07	3.9458E-07	6.0685E-06	0.	3.4936E-07	1.7523E-05	6.5435E-06	2.6038E-06
5.0E-01	1.7129E-05	1.5700E-07	4.9323E-07	7.5156E-06	0.	4.3670E-07	1.7622E-05	8.1093E-06	2.5813E-06
6.0E-01	1.7129E-05	1.8839E-07	5.9188E-07	8.9500E-06	0.	5.2404E-07	1.7721E-05	9.6625E-06	2.5630E-06
7.0E-01	1.7129E-05	2.1979E-07	6.9052E-07	1.0374E-05	0.	6.1138E-07	1.7820E-05	1.1205E-05	2.5474E-06
8.0E-01	1.7130E-05	2.5119E-07	7.8917E-07	1.1789E-05	0.	6.9872E-07	1.7919E-05	1.2739E-05	2.5343E-06
9.0E-01	1.7130E-05	2.8258E-07	8.8781E-07	1.3196E-05	0.	7.8606E-07	1.8018E-05	1.4265E-05	2.5225E-06
1.0E+00	1.7130E-05	3.1398E-07	9.8644E-07	1.4596E-05	0.	8.7340E-07	1.8117E-05	1.5783E-05	2.5120E-06
2.0E+00	1.7136E-05	6.2786E-07	1.9729E-06	2.8321E-05	0.	1.7468E-06	1.9109E-05	3.0696E-05	2.4427E-06
3.0E+00	1.7146E-05	9.4153E-07	2.9594E-06	4.1718E-05	0.	2.6202E-06	2.0105E-05	4.5280E-05	2.4022E-06
4.0E+00	1.7159E-05	1.2549E-06	3.9458E-06	5.4901E-05	0.	3.4936E-06	2.1105E-05	5.9650E-05	2.3734E-06
5.0E+00	1.7176E-05	1.5678E-06	4.9323E-06	6.7926E-05	0.	4.3670E-06	2.2109E-05	7.3861E-05	2.3511E-06
6.0E+00	1.7197E-05	1.8807E-06	5.9188E-06	8.0824E-05	0.	5.2404E-06	2.3116E-05	8.7945E-05	2.3320E-06
7.0E+00	1.7222E-05	2.1920E-06	6.9052E-06	9.3617E-05	0.	6.1138E-06	2.4127E-05	1.0192E-04	2.3174E-06
8.0E+00	1.7251E-05	2.5030E-06	7.8917E-06	1.0632E-04	0.	6.9872E-06	2.5142E-05	1.1581E-04	2.3048E-06
9.0E+00	1.7283E-05	2.8133E-06	8.8781E-06	1.1894E-04	0.	7.8606E-06	2.6161E-05	1.2962E-04	2.2922E-06
1.0E+01	1.7319E-05	3.1226E-06	9.8644E-06	1.3150E-04	0.	8.7340E-06	2.7183E-05	1.4336E-04	2.2816E-06
2.0E+01	1.7869E-05	6.1444E-06	1.9729E-05	2.5429E-04	0.	1.7468E-05	3.7599E-05	2.7790E-04	2.2119E-06
3.0E+01	1.8727E-05	8.9832E-06	2.9594E-05	3.7380E-04	0.	2.6202E-05	4.8321E-05	4.0890E-04	2.1697E-06
4.0E+01	1.9818E-05	1.1585E-05	3.9458E-05	4.9117E-04	0.	3.4936E-05	5.9276E-05	5.3769E-04	2.1394E-06
5.0E+01	2.1064E-05	1.3929E-05	4.9323E-05	6.0696E-04	0.	4.3670E-05	7.0386E-05	6.6456E-04	2.1193E-06
6.0E+01	2.2392E-05	1.6019E-05	5.9188E-05	7.2148E-04	0.	5.2404E-05	8.1579E-05	7.8990E-04	2.0953E-06
7.0E+01	2.3748E-05	1.7877E-05	6.9052E-05	8.3495E-04	0.	6.1138E-05	9.2799E-05	9.1396E-04	2.0780E-06
8.0E+01	2.5090E-05	1.9534E-05	7.8917E-05	9.4752E-04	0.	6.9872E-05	1.0401E-04	1.0369E-03	2.0629E-06
9.0E+01	2.6395E-05	2.1024E-05	8.8781E-05	1.0593E-03	0.	7.8606E-05	1.1518E-04	1.1589E-03	2.0494E-06
1.0E+02	2.7648E-05	2.2374E-05	9.8644E-05	1.1704E-03	0.	8.7340E-05	1.2629E-04	1.2801E-03	2.0373E-06
2.0E+02	3.7501E-05	3.2291E-05	1.9729E-04	2.2537E-03	0.	1.7468E-04	2.3479E-04	2.4607E-03	1.9501E-06
3.0E+02	4.4785E-05	3.9736E-05	2.9594E-04	3.3042E-03	0.	2.6202E-04	3.4071E-04	3.6059E-03	1.9130E-06
4.0E+02	5.0950E-05	4.6023E-05	3.9458E-04	4.3333E-03	0.	3.4936E-04	4.4551E-04	4.7207E-03	1.8815E-06
5.0E+02	5.6398E-05	5.1544E-05	4.9323E-04	5.3466E-03	0.	4.3670E-04	5.4960E-04	5.8348E-03	1.8573E-06
6.0E+02	6.1327E-05	5.6525E-05	5.9188E-04	6.3472E-03	0.	5.2404E-04	6.5316E-04	6.9277E-03	1.8376E-06
7.0E+02	6.5860E-05	6.1099E-05	6.9044E-04	7.3373E-03	0.	6.1138E-04	7.5632E-04	8.0098E-03	1.8211E-06
8.0E+02	7.0084E-05	6.5357E-05	7.8909E-04	8.3184E-03	0.	6.9872E-04	8.5918E-04	9.0825E-03	1.8069E-06
9.0E+02	7.4056E-05	6.9357E-05	8.8772E-04	9.2916E-03	0.	7.8606E-04	9.6177E-04	1.0147E-02	1.7944E-06
1.0E+03	7.7337E-05	7.3337E-05	9.8634E-04	1.0258E-02	0.	8.7340E-04	1.0597E-03	1.1205E-02	1.7833E-06
2.0E+03	1.0371E-04	1.0371E-04	1.9725E-03	1.9645E-02	0.	1.7468E-03	2.0762E-03	2.1496E-02	1.7106E-06
3.0E+03	1.2702E-04	1.2702E-04	2.9584E-03	2.8704E-02	0.	2.6202E-03	3.0854E-03	3.1451E-02	1.6685E-06
4.0E+03	1.4667E-04	1.4667E-04	3.9441E-03	3.7549E-02	0.	3.4936E-03	4.0908E-03	4.1189E-02	1.6389E-06
5.0E+03	1.6399E-04	1.6399E-04	4.9294E-03	4.6236E-02	0.	4.3670E-03	5.0936E-03	5.0767E-02	1.6160E-06
6.0E+03	1.7964E-04	1.7964E-04	5.9150E-03	5.4796E-02	0.	5.2404E-03	6.0944E-03	6.0214E-02	1.5973E-06
7.0E+03	1.9403E-04	1.9403E-04	6.9001E-03	6.3251E-02	0.	6.1138E-03	7.0944E-03	6.9559E-02	1.5815E-06
8.0E+03	2.0743E-04	2.0743E-04	7.8851E-03	7.1614E-02	0.	6.9872E-03	8.0924E-03	7.8811E-02	1.5679E-06
9.0E+03	2.2001E-04	2.2001E-04	8.8699E-03	7.9903E-02	0.	7.8606E-03	9.0899E-03	8.7983E-02	1.5559E-06

SIGW SIGS KI KS
 5.800E+07 1.000E-02 4.000E+00 4.000E+00

FREQ.	RE(ZW)	IM(ZW)	RE(ZS)	IM(ZS)	RE(ZI)	IM(ZI)	RE(Z)	IM(Z)	INDUCTANCE
1.0E+10	2.3191E-01	2.3191E-01	-2.7938E+04	1.3382E+05	0.	8.7340E+03	-2.7938E+04	1.4256E+05	2.2688E-06
2.0E+10	3.2797E-01	3.2797E-01	-4.8727E+04	2.9192E+05	0.	1.7468E+04	-4.8727E+04	3.0939E+05	2.4621E-06
3.0E+10	4.0168E-01	4.0168E-01	-6.6055E+04	4.5630E+05	0.	2.6202E+04	-6.6055E+04	4.0250E+05	2.9597E-06
4.0E+10	4.6382E-01	4.6382E-01	-8.1261E+04	6.2402E+05	0.	3.4936E+04	-8.1261E+04	6.5895E+05	2.6219E-06
5.0E+10	5.1857E-01	5.1857E-01	-9.4999E+04	7.9390E+05	0.	4.3670E+04	-9.4999E+04	8.3757E+05	2.6661E-06
6.0E+10	5.6807E-01	5.6807E-01	-1.0764E+05	9.6533E+05	0.	5.2404E+04	-1.0764E+05	1.0177E+06	2.6993E-06
7.0E+10	6.1358E-01	6.1358E-01	-1.1935E+05	1.1379E+06	0.	6.1138E+04	-1.1935E+05	1.1990E+06	2.7261E-06
8.0E+10	6.5595E-01	6.5595E-01	-1.3049E+05	1.3114E+06	0.	6.9872E+04	-1.3049E+05	1.3813E+06	2.7480E-06
9.0E+10	6.9574E-01	6.9574E-01	-1.4118E+05	1.4857E+06	0.	7.8606E+04	-1.4118E+05	1.5643E+06	2.7662E-06

SIGW SIGS KI KS
 5.800E+07 1.000E-02 4.000E+00 4.000E+00

FREQ.	RI(YI)	IM(YI)	RE(YS)	IM(YS)	RE(Y)	IM(Y)	CAPACITANCE	RE(GAMMA)	IM(GAMMA)	RE(Z(O))	IM(Z(O))
1.0E+10	0.	2.008E+01	1.339E-01	6.273E-01	0.	2.008E+01	3.196E-10	1.650E+02	-1.700E+03	8.466E+01	8.217E+00
2.0E+10	0.	4.016E+01	1.977E-01	1.168E+00	0.	4.016E+01	3.196E-10	2.767E+02	-3.536E+03	8.804E+01	6.891E+00
3.0E+10	0.	6.024E+01	2.478E-01	1.694E+00	0.	6.024E+01	3.196E-10	3.682E+02	-5.404E+03	8.971E+01	6.112E+00
4.0E+10	0.	8.032E+01	2.904E-01	2.210E+00	0.	8.032E+01	3.196E-10	4.477E+02	-7.289E+03	9.079E+01	5.974E+00
5.0E+10	0.	1.004E+02	3.282E-01	2.722E+00	0.	1.004E+02	3.196E-10	5.192E+02	-9.180E+03	9.146E+01	5.172E+00
6.0E+10	0.	1.205E+02	3.626E-01	3.230E+00	0.	1.205E+02	3.196E-10	5.847E+02	-1.109E+04	9.204E+01	4.694E+00
7.0E+10	0.	1.406E+02	3.942E-01	3.735E+00	0.	1.406E+02	3.196E-10	6.453E+02	-1.300E+04	9.248E+01	4.591E+00
8.0E+10	0.	1.606E+02	4.240E-01	4.237E+00	0.	1.606E+02	3.196E-10	7.028E+02	-1.491E+04	9.283E+01	4.379E+00
9.0E+10	0.	1.807E+02	4.526E-01	4.738E+00	0.	1.807E+02	3.196E-10	7.579E+02	-1.683E+04	9.313E+01	4.194E+00

SIGW SIGS KI KS
 5.800E+07 1.000E-03 4.000E+00 4.000E+00

FREQ.	RE(ZW)	IM(ZW)	RE(ZS)	IM(ZS)	RE(ZI)	IM(ZI)	RE(Z)	IM(Z)	INDUCTANCE
1.0E+10	2.3191E-01	2.3191E-01	-3.1849E+05	8.4387E+05	0.	8.7340E+03	-3.1849E+05	8.5260E+05	1.3570E-05
2.0E+10	3.2797E-01	3.2797E-01	-6.4997E+05	1.9940E+06	0.	1.7468E+04	-6.4997E+05	2.0115E+06	1.6007E-05
3.0E+10	4.0168E-01	4.0168E-01	-9.6496E+05	3.2614E+06	0.	2.6202E+04	-9.6496E+05	3.2876E+06	1.7441E-05
4.0E+10	4.6382E-01	4.6382E-01	-1.2642E+06	4.6003E+06	0.	3.4936E+04	-1.2642E+06	4.6353E+06	1.8443E-05
5.0E+10	5.1857E-01	5.1857E-01	-1.5498E+06	5.9894E+06	0.	4.3670E+04	-1.5498E+06	6.0331E+06	1.9204E-05
6.0E+10	5.6807E-01	5.6807E-01	-1.8233E+06	7.4163E+06	0.	5.2404E+04	-1.8233E+06	7.4687E+06	1.9811E-05
7.0E+10	6.1358E-01	6.1358E-01	-2.0864E+06	8.8731E+06	0.	6.1138E+04	-2.0864E+06	8.9343E+06	2.0313E-05
8.0E+10	6.5595E-01	6.5595E-01	-2.3402E+06	1.0354E+07	0.	6.9872E+04	-2.3402E+06	1.0424E+07	2.0739E-05
9.0E+10	6.9574E-01	6.9574E-01	-2.5856E+06	1.1856E+07	0.	7.8606E+04	-2.5856E+06	1.1935E+07	2.1106E-05

SIGM SIGS KI KS
 5.800E+07 1.000E-03 4.000E+00 4.000E+00

FREQ.	RI(YI)	IM(YI)	RE(YI)	IM(YI)	RE(YI)	IM(Y)	CAPACITANCE	RE(GAMMA)	IM(GAMMA)	RE(ZI0))	IM(ZI0))
1.0E+10	0.	2.008E+01	3.437E-02	9.094E-02	0.	2.008E+01	3.196E-10	7.601E+02	-4.207E+03	2.095E+02	3.709E+01
2.0E+10	0.	4.016E+01	5.186E-02	1.590E-01	0.	4.016E+01	3.196E-10	1.434E+03	-9.101E+03	2.266E+02	3.571E+01
3.0E+10	0.	6.024E+01	6.586E-02	2.225E-01	0.	6.024E+01	3.196E-10	2.044E+03	-1.422E+04	2.361E+02	3.303E+01
4.0E+10	0.	8.032E+01	7.796E-02	2.835E-01	0.	8.032E+01	3.196E-10	2.608E+03	-1.947E+04	2.424E+02	3.247E+01
5.0E+10	0.	1.004E+02	8.879E-02	3.430E-01	0.	1.004E+02	3.196E-10	3.136E+03	-2.481E+04	2.471E+02	3.123E+01
6.0E+10	0.	1.205E+02	9.871E-02	4.014E-01	0.	1.205E+02	3.196E-10	3.639E+03	-3.022E+04	2.500E+02	3.017E+01
7.0E+10	0.	1.406E+02	1.079E-01	4.588E-01	0.	1.406E+02	3.196E-10	4.110E+03	-3.567E+04	2.530E+02	2.924E+01
8.0E+10	0.	1.606E+02	1.166E-01	5.156E-01	0.	1.606E+02	3.196E-10	4.509E+03	-4.117E+04	2.555E+02	2.842E+01
9.0E+10	0.	1.807E+02	1.247E-01	5.718E-01	0.	1.807E+02	3.196E-10	5.002E+03	-4.671E+04	2.580E+02	2.766E+01

SIGW SIGS KI KS
 5.800E+07 1.000E-04 4.000E+00 4.000E+00

FREQ.	RE(ZW)	IM(ZW)	RE(ZS)	IM(ZS)	RE(ZI)	IM(ZI)	RE(Z)	IM(Z)	INDUCTANCE
1.0E+10	2.3191E-01	2.3191E-01	-2.2368E+04	4.2061E+06	0.	8.7340E+03	-2.2368E+06	4.2149E+06	6.7082E-05
2.0E+10	3.2797E-01	3.2797E-01	-5.1776E+04	1.0575E+07	0.	1.7468E+04	-5.1776E+06	1.0593E+07	8.4294E-05
3.0E+10	4.0168E-01	4.0168E-01	-8.3395E+04	1.8010E+07	0.	2.6202E+04	-8.3395E+06	1.8037E+07	9.5687E-05
4.0E+10	4.6382E-01	4.6382E-01	-1.1612E+07	2.6186E+07	0.	3.4936E+04	-1.1612E+07	2.6221E+07	1.0433E-04
5.0E+10	5.1857E-01	5.1857E-01	-1.4945E+07	3.4931E+07	0.	4.3670E+04	-1.4945E+07	3.4975E+07	1.1135E-04
6.0E+10	5.6807E-01	5.6807E-01	-1.8312E+07	4.4139E+07	0.	5.2404E+04	-1.8312E+07	4.4191E+07	1.1722E-04
7.0E+10	6.1358E-01	6.1358E-01	-2.1697E+07	5.3736E+07	0.	6.1138E+04	-2.1697E+07	5.3797E+07	1.2231E-04
8.0E+10	6.5595E-01	6.5595E-01	-2.5089E+07	6.3668E+07	0.	6.9872E+04	-2.5089E+07	6.3738E+07	1.2680E-04
9.0E+10	6.9574E-01	6.9574E-01	-2.8481E+07	7.3893E+07	0.	7.8606E+04	-2.8481E+07	7.3972E+07	1.3061E-04

SIGW SIGS KI KS
 5.800E+07 1.000E-04 4.000E+00 4.000E+00

FREQ.	RI(YI)	IM(YI)	RE(YS)	IM(YS)	RE(Y)	IM(Y)	CAPACITANCE	RE(GAMMA)	IM(GAMMA)	RE(Z(0))	IM(Z(0))
1.0E+10	0.	2.008E+01	8.643E-03	1.625E-02	0.	2.008E+01	3.196E-10	2.364E+03	-9.498E+03	4.730E+02	1.177E+02
2.0E+10	0.	4.016E+01	1.310E-02	2.675E-02	0.	4.016E+01	3.196E-10	4.904E+03	-2.120E+04	5.279E+02	1.221E+02
3.0E+10	0.	6.024E+01	1.671E-02	3.608E-02	0.	6.024E+01	3.196E-10	7.434E+03	-3.379E+04	5.609E+02	1.234E+02
4.0E+10	0.	8.032E+01	1.985E-02	4.477E-02	0.	8.032E+01	3.196E-10	9.931E+03	-4.695E+04	5.846E+02	1.237E+02
5.0E+10	0.	1.004E+02	2.270E-02	5.305E-02	0.	1.004E+02	3.196E-10	1.239E+04	-6.054E+04	6.030E+02	1.234E+02
6.0E+10	0.	1.205E+02	2.531E-02	6.101E-02	0.	1.205E+02	3.196E-10	1.482E+04	-7.445E+04	6.180E+02	1.230E+02
7.0E+10	0.	1.406E+02	2.776E-02	6.875E-02	0.	1.406E+02	3.196E-10	1.720E+04	-8.864E+04	6.307E+02	1.224E+02
8.0E+10	0.	1.606E+02	3.007E-02	7.630E-02	0.	1.606E+02	3.196E-10	1.955E+04	-1.031E+05	6.416E+02	1.217E+02
9.0E+10	0.	1.807E+02	3.226E-02	8.369E-02	0.	1.807E+02	3.196E-10	2.187E+04	-1.177E+05	6.511E+02	1.210E+02

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APPENDIX A

TABLE OF SYMBOLS

\vec{A}	-	Magnetic vector potential associated with $\nabla \cdot \vec{B} = 0$
\vec{A}^*	-	Magnetic vector potential associated with $\nabla \cdot \vec{E} = 0$
A_z	-	Component of vector potential in the z direction associated with $\nabla \cdot \vec{B} = 0$
A_z	-	Total vector potential in the z direction; i.e., sum of transverse magnetic and electric potentials
A_z^*	-	Component of vector potential in the z direction associated with $\nabla \cdot \vec{E} = 0$
a_0	-	Constant associated with principle mode transverse magnetic waves
$\overline{a_0^2}$	-	Constant associated with principle mode transverse magnetic waves
\vec{a}_z	-	Unit vector in the z-direction
a_ν	-	Constant associated with transverse magnetic waves
a_ν^*	-	Constant associated with transverse electric waves
\vec{B}	-	Magnetic field vector (webers/meter ²)
B_r	-	Component of magnetic field in the radial direction (webers/meter ²)
B_θ	-	Component of magnetic field in the theta direction (webers/meter ²)
B_z	-	Component of magnetic field in the z direction (webers/meter ²)
b_ν	-	Constant associated with transverse electric waves
$\overline{b_\nu}$	-	Constant associated with transverse electric waves
b_0	-	Constant associated with transverse electric waves
$\overline{b_0}$	-	Constant associated with transverse electric waves

TABLE OF SYMBOLS (CONT'D)

\vec{E}	- Electric field vector (volts/meter)
E_r	- Component of electric field in the radial direction (volts/meter)
E_θ	- Component of electric field in the theta direction (volts/meter)
E_z	- Component of electric field in the z direction (volts/meter)
\vec{H}	- Magnetic intensity vector (ampere-turns/meter)
h	- Propagation factor in the z direction ($\exp(tihz)$). Note $\exp(-ihz)$ is propagating in the -z direction
i	- $= \sqrt{-1}$
\vec{J}_{true}	- Electric current density due to change motion (amperes/meter ³)
K	- Relative dielectric constant
\vec{M}	- Magnetization vector
n_i	- Inside radius of hollow insulator
n_o	- Outside radius of hollow insulator
\vec{P}	- Electric polarization vector
R	- Product form solution for radial variation ($R = R(n)$)
r	- Cylindrical radial coordinate
t	- Time
Z_{outside}	- Impedance of hollow conductor driven from the outside
Z_{inside}	- Impedance of hollow conductor driven from the inside
Z_ν	- General circular function; i.e., J_ν, Y_ν, H_ν or linear combinations. Note that prime; i.e., Z'_ν , denotes differentiation with respect to the whole argument

TABLE OF SYMBOLS (CONT'D)

z_T	- Cylindrical coordinate in the z(cartesian coordinate) direction	
A_ν	- Constant associated with integer ν	} Note prime, i.e., J'_ν indicates differentiation with respect to the whole argument
J_ν	- Bessel function of order ν	
Y_ν	- Neumann function of order ν	
$H_\nu^{(1)}$	- Hankel function of 1st kind of order ν	
$H_\nu^{(2)}$	- Hankel function of 2nd kind of order ν	
α	- Real number (general)	
β	- Real number (general)	
η	- Real number (general)	
ξ	- Real number (general)	
μ	- permeability of medium in henries/meter	
μ_0	- Permeability of free space = $4\pi \times 10^{-7}$ henries/meter	
ϵ_0	- Dielectric constant of free space = $(1/36\pi) \times 10^{-9}$ farads/meter	
ρ_{true}	- Actual electric charge (coulombs/meter ³)	
σ	- Conductivity (mhos/meter)	
χ	- Electric susceptibility	
ϵ	- Dielectric constant ($\epsilon = K\epsilon_0$)	
ω	- Frequency in radians/second	
ϕ	- Scalar potential associated with $\nabla \cdot \vec{B} = 0$	
ϕ^*	- Scalar potential associated with $\nabla \cdot \vec{E} = 0$	
γ	- Propagation constant for planar, homogeneous propagation ($\gamma = -i\omega\mu(\sigma + i\omega\epsilon)$)	

TABLE OF SYMBOLS (CONT'D)

Θ	- Product form solution for theta variation ($\Theta \equiv \Theta(\theta)$)
ν	- Integer
λ	- $= \sqrt{\gamma^2 - h^2}$
θ	- Cylindrical angular coordinate
ω	- Radian frequency = $2\pi f$

APPENDIX B

MAXWELL'S EQUATIONS

Maxwell's equations in differential form are:

(Reference 1, p. 159)

$$\nabla \times \vec{B} = \mu_0 (\vec{j}_{\text{true}} + \dot{\vec{P}} + \nabla \times \vec{M} + \epsilon_0 \dot{\vec{E}}), \quad (\text{B-1})$$

$$\nabla \times \vec{E} = -\dot{\vec{B}}, \quad (\text{B-2})$$

$$\nabla \cdot \vec{B} = 0 \quad (\text{B-3})$$

$$\nabla \cdot \vec{E} = \frac{1}{\epsilon_0} (\rho_{\text{true}} - \nabla \cdot \vec{P}), \quad (\text{B-4})$$

$$\nabla \cdot \vec{j}_{\text{true}} + \dot{\rho}_{\text{true}} = 0; \quad (\text{B-5})$$

where: $\mu_0 = 4\pi \times 10^{-7}$ henries/meter,

$\epsilon_0 = \frac{1}{36\pi} \times 10^{-9}$ farads/meter,

\vec{P} is the electric polarization vector

\vec{M} is the magnetization vector

\vec{j}_{true} is measured in amperes/meter³

ρ_{true} is measured in coulombs/meter³

\vec{E} is measured in volts/meter

\vec{B} is measured in webers/meter²

If it is further assumed that the medium is simple, so that

$$\vec{j}_{\text{true}} = \sigma \vec{E},$$

$$\vec{P} = \epsilon_0 (K-1) \vec{E} = \epsilon_0 \chi \vec{E};$$

where: σ is the conductivity in mhos/meter,
 K is the specific inductive capacity; i.e., $\epsilon = K\epsilon_0$,
 χ is the electric susceptibility,

and if $\mu = \mu_0$, so that $\vec{M} = \vec{0}$; i.e.,

$$\vec{B} = \mu\vec{H} = \mu_0(\vec{H} + \vec{M});$$

where: \vec{H} is the field intensity in ampere-turns/meter.

then Maxwell's equations reduce to

$$\nabla \times \vec{B} = \mu_0 (\sigma\vec{E} + \epsilon\dot{\vec{E}}), \quad (\text{B-6})$$

$$\nabla \times \vec{E} = -i\omega\vec{B}, \quad (\text{B-7})$$

$$\nabla \cdot \vec{B} = 0, \quad (\text{B-8})$$

$$\nabla \cdot \vec{E} = 0. \quad (\text{B-9})$$

Define the Fourier transform as

$$g(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(t) \epsilon^{-i\omega t} dt. \quad (\text{B-10})$$

The inverse relationship to equation B-10 is

$$f(t) = \int_{-\infty}^{\infty} g(\omega) \epsilon^{i\omega t} d\omega. \quad (\text{B-11})$$

(1) Assuming there are no sources in the region where the equations are to be valid.

Note that the transform relations B-10 and B-11 are consistent with sinusoidal variations of the form $\exp(i\omega t)$.

Finally, take the Fourier transforms of equations B-6 through B-9 so that the time-independent Maxwell's equations become

$$\nabla \times \vec{B} = \mu_0 (\sigma + i\omega\epsilon)\vec{E}, \quad (\text{B-12})$$

$$\nabla \times \vec{E} = -i\omega\vec{B}, \quad (\text{B-13})$$

$$\nabla \cdot \vec{B} = 0, \quad (\text{B-14})$$

$$\nabla \cdot \vec{E} = 0;$$

where: The symbology \vec{B} and \vec{E} now represent the Fourier transforms of their respective time-varying counterparts.

APPENDIX C
POTENTIAL EQUATIONS

Equations B-14 and B-15 indicate that both of the vectors, \vec{B} and \vec{E} , are solenoidal; that is, they both may be expressed as the curl of another vector; i.e.,

$$\vec{B} = \nabla \times A, \quad (C-1)$$

$$\vec{E} = -\nabla \times \vec{A}^*; \quad (C-2)$$

where: \vec{A} and \vec{A}^* are not the same vectors.

The vectors \vec{A} and \vec{A}^* can both be selected so as to satisfy the same differential equation. Since the derivations of this equation are similar, the following analysis is limited to the vector \vec{A} . From equations B-13 and C-1,

$$\begin{aligned} \nabla \times \vec{E} &= -i\omega\vec{B} = -i\omega\nabla \times \vec{A} \\ \Rightarrow \nabla \times (\vec{E} + i\omega\vec{A}) &= \vec{0}. \end{aligned} \quad (C-3)$$

Therefore, since the curl of the gradient is zero

$$\begin{aligned} -\nabla\varphi &= \vec{E} + i\omega\vec{A} \\ \Rightarrow \vec{E} &= -\nabla\varphi - i\omega\vec{A} \end{aligned} \quad (C-4)$$

Substituting equation C-4 into equation B-14,

$$\begin{aligned} \nabla \cdot (-\nabla\varphi - i\omega\vec{A}) &= 0, \\ \Rightarrow \nabla^2\varphi + i\omega\nabla \cdot \vec{A} &= 0. \end{aligned} \quad (C-5)$$

From equations B-12 and C-4,

$$\begin{aligned}\nabla \times \vec{B} &= \nabla \times \nabla \times \vec{A} = \nabla(\nabla \cdot \vec{A}) - \nabla^2 \vec{A} = \mu_0 (\sigma + i\omega\epsilon) \vec{E} \\ \Rightarrow \nabla^2 \vec{A} - \nabla(\nabla \cdot \vec{A}) &= -\mu_0 (\sigma + i\omega\epsilon) (-\nabla\varphi - i\omega\vec{A}) \\ \Rightarrow \nabla^2 \vec{A} - \nabla(\nabla \cdot \vec{A}) &= \mu_0 (\sigma + i\omega\epsilon) \nabla\varphi + i\omega\mu_0 (\sigma + i\omega\epsilon) \vec{A}\end{aligned}\quad (C-6)$$

If $\nabla \cdot \vec{A}^{(1)}$, which has not been determined, is chosen so that

$$\begin{aligned}\nabla \cdot \vec{A} &= -\mu_0 (\sigma + i\omega\epsilon) \varphi, \\ \Rightarrow \varphi &= -\frac{1}{\mu_0 (\sigma + i\omega\epsilon)} \nabla \cdot \vec{A},\end{aligned}\quad (C-7)$$

then φ and A will separate in equations C-5 and C-6. The result is

$$\nabla^2 \varphi - i\omega\mu_0 (\sigma + i\omega\epsilon) \varphi = 0 \quad (C-8)$$

$$\nabla^2 \vec{A} - i\omega\mu_0 (\sigma + i\omega\epsilon) \vec{A} = 0 \quad (C-9)$$

Equations C-1, C-4 and C-7 indicate that the complete electromagnetic field vectors (\vec{E} and \vec{B}) can be determined from knowledge of the vector potential \vec{A} . A similar result is true for the vector \vec{A}^* (and corresponding φ^*), which must satisfy equations C-8 and C-7. In this case, the roles of \vec{B} and \vec{E} are reversed; i.e.,

$$\vec{E} = -\nabla \times \vec{A}^*, \quad (C-10)$$

⁽¹⁾ φ and \vec{A} ; thus, φ and $\nabla \cdot \vec{A}$ are independent.

$$\vec{B} = -\nabla\varphi^* - \mu_0(\sigma + i\omega\epsilon)\vec{A}^*, \quad (\text{C-11})$$

$$\varphi^* = -\frac{1}{i\omega} \nabla \cdot \vec{A}^*. \quad (\text{C-12})$$

Define a complex propagation constant, γ , by

$$\gamma^2 = -i\omega\mu_0(\sigma + i\omega\epsilon).$$

Then both \vec{A} and \vec{A}^* must satisfy the vector Helmholtz equation (see equation C-9)

$$(\nabla^2 + \gamma^2)\vec{A}^* = \vec{0}. \quad (\text{C-13})$$

APPENDIX D

TRANSVERSE ELECTRIC AND MAGNETIC WAVES

Suppose that the vectors \vec{A} and \vec{A}^* are restricted to single components; i.e.,

$$\vec{A} = \vec{a}_z A_z \quad (D-1)$$

$$\vec{A} = \vec{a}_z A_z^* \quad (D-2)$$

On the surface, equations D-1 and D-2 appear to be unduly restrictive. However, Stratton (Reference 2, p. 351) states

"The electromagnetic field obtained by superimposing the partial fields derived from A_z and $A_z^{*(1)}$ is of such generality that one can satisfy a prescribed set of boundary conditions on any cylindrical surface whose generating elements are parallel to the z-axis."

Equations 29 and 30 then are sufficient then to describe propagation along transmission lines, waveguides, buried cables, etc.

For present purposes, it is adequate to consider only cylindrical coordinates. The vectors A_z and A_z^* must satisfy equation C-13. Thus, the equation, in cylindrical coordinates, to be satisfied is (Reference 6, p. 116)

$$\left(\frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2} + \gamma^2 \right) A_z^{(*)} = 0 \quad (D-3)$$

(1) Stratton makes this statement in connection with the Hertz components π_z and π_z^* ; however, these vectors are very simply related to A_z and A_z^* .

From equations C-2, C-4, C-7, C-10, C-11 and C-12, the electric and magnetic fields associated with A_z and A_z^* are

$$E_r = -\frac{i\omega}{\gamma} \frac{\partial^2 A_z}{\partial r \partial z}, \quad (D-4)$$

$$E_\theta = -\frac{i\omega}{\gamma} \frac{1}{r} \frac{\partial^2 A_z}{\partial \theta \partial z}, \quad (D-5)$$

$$E_z = -i\omega A_z - \frac{i\omega}{\gamma} \frac{\partial^2 A_z}{\partial z^2}, \quad (D-6)$$

$$B_r = \frac{1}{r} \frac{\partial A_z}{\partial \theta}, \quad (D-7)$$

$$B_\theta = -\frac{\partial A_z}{\partial r}, \quad (D-8)$$

$$B_z = 0, \quad (D-9)$$

$$E_r^* = -\frac{1}{r} \frac{\partial A_z^*}{\partial \theta}, \quad (D-10)$$

$$E_\theta^* = \frac{\partial A_z^*}{\partial r}, \quad (D-11)$$

$$E_z^* = 0, \quad (D-12)$$

$$B_r^* = \frac{1}{i\omega} \frac{\partial^2 A_z^*}{\partial r \partial z}, \quad (D-13)$$

$$B_\theta^* = \frac{1}{i\omega} \frac{1}{r} \frac{\partial^2 A_z^*}{\partial \theta \partial z}, \quad (D-14)$$

$$B_z^* = \frac{\gamma^2}{i\omega} A_z^* + \frac{1}{i\omega} \frac{\partial^2 A_z^*}{\partial z^2}. \quad (D-15)$$

Equations D-4 - D-9, due to the fact that $B_z = 0$, are known as transverse magnetic waves. Equations D-10 - D-15, due to the fact that $E_z = 0$, are known as transverse electric waves. As previously mentioned, a general solution is made up of sums of these two wave types.

One approach to the solution of equation D-3 is by way of a Fourier transform with respect to z , so that

$$A_z^{(*)}(r, \theta, z) = \int_{-\infty}^{\infty} A_z^{(*)}(r, \theta, h) \epsilon^{ihz} d_z. \quad (D-16)$$

Equation D-16 is similar to equation B-10 except that the variable h has been substituted for the variable ω . Alternately, one can consider elementary solutions of the form

$$A_z^{(*)}(r, \theta, z) = A_z^{(*)}(r, \theta) \epsilon^{\pm ihz} \quad (D-17)$$

A note of caution is due here when considering numerical values for h . If it is assumed that the form $\exp(ihz)$ is to represent a wave travelling in the positive z direction, then if

$$h = \alpha + i\beta, \quad (D-18)$$

it follows that

$$\epsilon^{ihz} = \epsilon^{-\beta z} \epsilon^{i\alpha z}.$$

The elementary time wave, at any fixed r is then

$$\epsilon^{-\beta z} \epsilon^{i\alpha z} \epsilon^{i\omega t} = \epsilon^{-\beta z} \epsilon^{i(\alpha z + \omega t)}.$$

If then this wave is to travel in the positive z direction, it follows that

$$\alpha < 0 \quad (D-19)$$

Furthermore, for $z > 0$, the wave must attenuate; thus,

$$\beta \geq 0. \quad (D-20)$$

The value of h for a wave travelling in the positive z -direction must lie in the second or third quadrant. Alternately, if it is assumed that e^{-ihz} is a wave travelling in the positive z -direction, then the corresponding value of h must be in the first or fourth quadrants.

In any case the differential equation which governs the behavior of $A_Z^{(*)}(r, \theta, h)$ can be obtained from equation D-3 and is

$$\left(\frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} + (\gamma^2 - h^2) \right) A_Z^{(*)}(r, \theta, h) = 0 \quad (D-21)$$

Looking for the typical product form solutions, assume that

$$A_Z^{(*)}(r, \theta, h) = R\Theta,$$

so that equation D-21 becomes

$$\frac{r}{R} \frac{d}{dr} r \frac{dR}{dr} + r^2(\gamma^2 - h^2) + \frac{1}{\Theta} \frac{d^2\Theta}{d\theta^2} = 0 \quad (D-22)$$

Equation D-22 separates into the two equations

$$\frac{1}{\Theta} \frac{d^2\Theta}{d\theta^2} = -\nu^2, \quad (D-23)$$

$$\frac{r}{R} \frac{d}{dr} \left(r \frac{dR}{dr} \right) + r^2 (\gamma^2 - h^2) = +\nu^2; \quad (D-24)$$

where: ν is integer, if the solutions are to be periodic in θ (cylindrical symmetry)

The general solution to equation D-23 is

$$\theta = A_\nu e^{i\nu\theta}; \quad \nu = 0, \underline{+1}, \underline{+2}, \dots \quad (D-25)$$

Equation D-24 is Bessel's equation, which has various solutions depending upon convenience and the particular boundary conditions. Three common, independent solutions to Bessel's equation are

$$R \sim J_\nu(\lambda r), \quad (D-26)$$

$$R \sim Y_\nu(\lambda r), \quad (D-27)$$

$$R \sim H_\nu^{(1)}(\lambda r) \text{ or } H_\nu^{(2)}(\lambda r); \quad (D-28)$$

$$\text{where: } H_\nu^{(1)}(\lambda r) = J_\nu(\lambda r) + iY_\nu(\lambda r),$$

$$H_\nu^{(2)}(\lambda r) = J_\nu(\lambda r) - iY_\nu(\lambda r),$$

$$i = \sqrt{-1},$$

$$\lambda^2 = \gamma^2 - h^2.$$

Complete solutions to Bessel's equation can be formed from linear combinations of equations D-26, D-27 and D-28. In this process, it is necessary to observe that J_ν exists at the origin,

while Y_ν (and thus H_ν) has a logarithmic singularity at the origin. Therefore, if the origin is to be included, the solution consists only of the functions J_ν .

If the Hankel function solutions are to be used (in regions which preclude the origin), it is necessary to distinguish between the two forms, one of these functions represents inward travelling waves, while the other function represents outward travelling waves. This determination can be made from the asymptotic forms for the Hankel functions, which are (references 1, 2, 3)

$$H_\nu^{(1)}(\lambda r) \longrightarrow \sqrt{\frac{2}{\pi \lambda r}} e^{i\left(\lambda r - \frac{2\nu+1}{4}\pi\right)}, \quad (D-29)$$

$$r \rightarrow \infty$$

$$H_\nu^{(2)}(\lambda r) \longrightarrow \sqrt{\frac{2}{\pi \lambda r}} e^{-i\left(\lambda r - \frac{2\nu+1}{4}\pi\right)}. \quad (D-30)$$

$$r \rightarrow \infty$$

Assume that

$$\lambda = \sqrt{\gamma^2 - h^2} = \eta + i\xi,$$

so that

$$i\lambda r = -\xi r + i\eta.$$

With the time variation $e^{i\omega t}$, then $H_\nu^{(2)}(\lambda r)$ will represent an outward travelling wave, if

$$\eta > 0,$$

$$\xi < 0.$$

In other words, if $\lambda = \sqrt{\gamma^2 - h^2}$ is, numerically speaking, in the fourth quadrant, then $H_\nu^{(2)}(\lambda r)$ is an outward travelling wave and $H_\nu^{(1)}(\lambda r)$ is an inward travelling wave. On the other hand, if

$$\eta < 0,$$

$$\xi > 0,$$

so that λ , numerically speaking, is in the second quadrant, then $H_\nu^{(1)}(\lambda r)$ is an outward travelling wave and $H_\nu^{(2)}(\lambda r)$ is an inward travelling wave.

In view of the previous discussions concerning h and H_ν , it is seen that one must establish conventions regarding outward travelling waves. For the purpose of analysis herein, this convention will be established with respect to the principle value of the square root function. The principle square root is in the first and fourth quadrants; that is, the real part of the principle value is always positive. Therefore, the functions

$$e^{-ihz}, \quad (D-31)$$

and

$$H_\nu^{(2)}(\lambda r) \quad (D-32)$$

represent waves travelling in the positive z direction and waves travelling radially outward, respectively. This is, of course, with respect to the time variation $e^{i\omega t}$, and the principle value of the square root function.

It is now possible to formulate general solutions for Maxwell's equations, in accordance with equations D-4 through D-15, as sums of the elementary solutions previously discussed.

Solutions are to be formed, in general, by summing the two types of potential A_z and A_z^* . The solutions to the potential equation are

$$A_z^T = A_z + A_z^* = \sum_{\nu=-\infty}^{\infty} \left[a_{\nu} Z_{\nu}(\lambda r) + a_{\nu}^* Z_{\nu}(\lambda r) \right] \epsilon^{i\nu\theta} \epsilon^{-ihz}; \quad (D-33)$$

where: a_{ν} and a_{ν}^* are constants associated with A_z and A_z^* , respectively; thus, with transverse magnetic and transverse electric waves respectively,

A_z^T is the total potential,

Z_{ν} is a general circular function; i.e., J_{ν} , Y_{ν} , H_{ν} or combinations.

Substituting equation D-33 into equations D-4 through D-15, the electromagnetic fields are

$$E_r = \sum_{\nu=-\infty}^{\infty} \left[-\frac{\omega\lambda h}{\gamma^2} Z_{\nu}'(\lambda r) a_{\nu} - \frac{i\nu}{r} Z_{\nu}(\lambda r) a_{\nu}^* \right] \epsilon^{i\nu\theta} \epsilon^{-ihz} \quad (D-34)$$

$$E_{\theta} = \sum_{\nu=-\infty}^{\infty} \left[-\frac{i\omega h \nu}{\gamma^2 r} Z_{\nu}(\lambda r) a_{\nu} + \lambda Z_{\nu}'(\lambda r) a_{\nu}^* \right] \epsilon^{i\nu\theta} \epsilon^{-ihz} \quad (D-35)$$

$$E_z = \sum_{\nu=-\infty}^{\infty} \left[-i\omega \frac{(\gamma^2 - h^2)}{\gamma^2} Z_{\nu}(\lambda r) a_{\nu} \right] \epsilon^{i\nu\theta} \epsilon^{-ihz} \quad (D-36)$$

$$B_r = \sum_{\nu=-\infty}^{\infty} \left[\frac{i\nu}{r} Z_{\nu}(\lambda r) a_{\nu} - \frac{h\lambda}{\omega} Z_{\nu}'(\lambda r) a_{\nu}^* \right] \epsilon^{i\nu\theta} \epsilon^{-ihz} \quad (D-37)$$

$$B_{\theta} = \sum_{\nu=-\infty}^{\infty} \left[-\lambda Z_{\nu}'(\lambda r) a_{\nu} + \frac{\nu h}{i\omega} Z_{\nu}(\lambda r) a_{\nu}^* \right] \epsilon^{i\nu\theta} \epsilon^{-ihz} \quad (D-38)$$

$$B_z = \sum_{\nu=-\infty}^{\infty} \left[\frac{\lambda^2}{i\omega} Z_{\nu}(\lambda r) a_{\nu}^* \epsilon^{i\nu\theta} \right] \epsilon^{-ihz} ; \quad (D-39)$$

where: a_{ν} and a_{ν}^* are associated with the transverse magnetic and transverse electric waves, respectively.

The fields are propagating in the positive z direction.

Note, if in the above expressions for the fields, the substitutions

$$\omega \rightarrow -\omega,$$

$$h \rightarrow -h,$$

$$a_{\nu} \rightarrow -\frac{i\gamma^2}{\omega\lambda^2} a_{\nu} ,$$

$$a_{\nu}^* \rightarrow -\frac{i\mu\omega}{\lambda^2} a_{\nu}^* ,$$

are made, then the coefficients agree with those in Stratton (reference 2, p. 524, 525).

APPENDIX E

HOLLOW CONDUCTOR IMPEDANCE

Reference 5, Chapter 11, makes use of the impedance expressions for a hollow conductor, which are quoted in reference 3. It is the purpose of this section to derive these expressions, using the conventions previously discussed. Although reference 3 states that these expressions are "exact", the following discussion points out the several approximations involved.

The geometry of the situation to be considered is illustrated in Figure E-1. It is assumed that only transverse magnetic type waves are present, so that the magnetic field is entirely in the plane of Figure 1; thus, the major current in the conductor is normal to Figure 1.

With reference to equations D-34 through D-39, and the discussions which precede these equations, the "exact" solution requires equating tangential components of electric and magnetic fields at the boundaries between regions 1, 2 and 3. In line with the assumptions of reference 3; in particular, that the conductor is (electromagnetically speaking) very good, then the impedances can be derived considering only the fields in region 2, which are:

$$E_r^2 = - \frac{\omega \lambda_2 h}{\gamma_2} \left[J_0'(\lambda_2 r) a_0^2 + H_0^{(1)'}(\lambda_2 r) \overline{a_0^2} \right] \epsilon^{-ihz} , \quad (E-1)$$

$$E_\theta^2 = 0 , \quad (E-2)$$

$$E_z^2 = - \frac{i\omega \lambda_2^2}{\gamma_2} \left[J_0(\lambda_2 r) a_0^2 + H_0^{(1)}(\lambda_2 r) \overline{a_0^2} \right] \epsilon^{-ihz} , \quad (E-3)$$

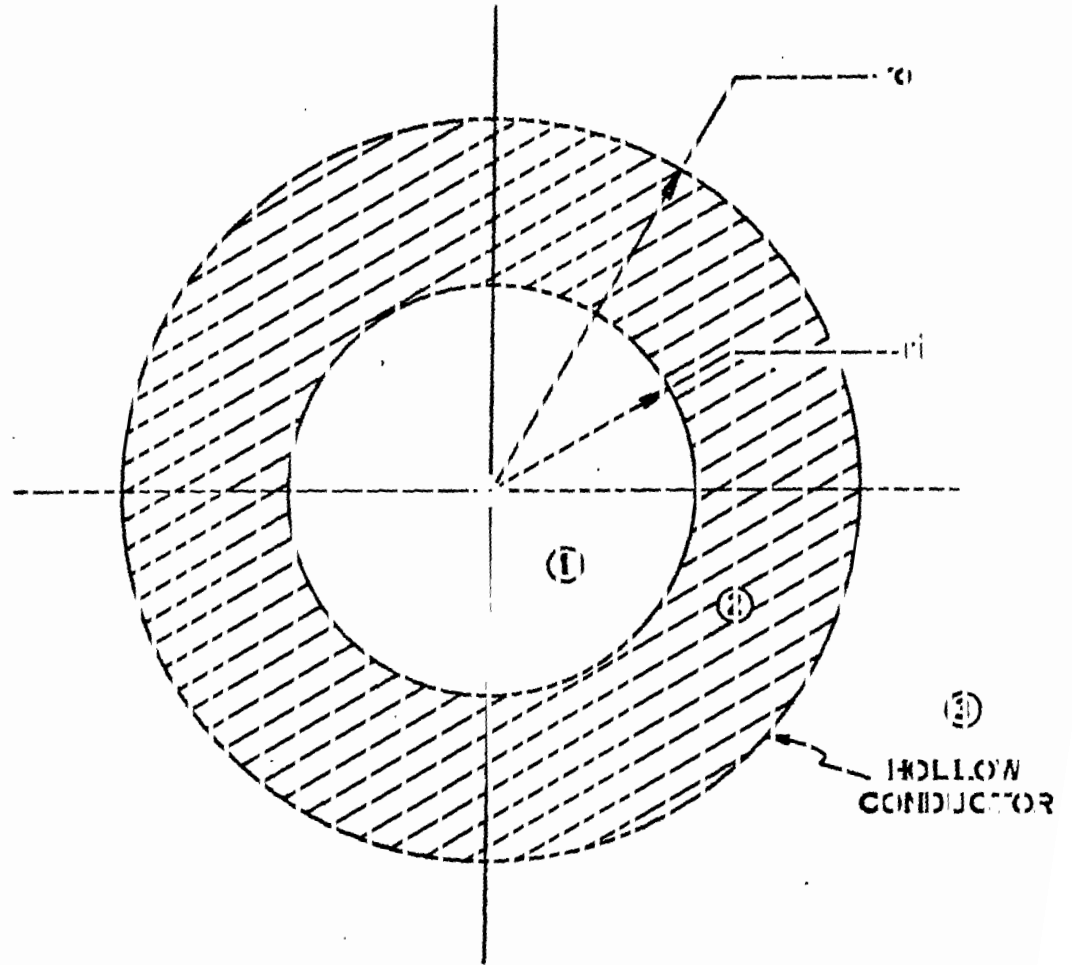


FIGURE E--1

GEOMETRY OF HOLLOW CONDUCTOR

$$B_r^2 = 0 , \quad (E-4)$$

$$B_\theta^2 = - \lambda_2 \left[J_0'(\lambda_2 r) a_0^2 + H_0^{(1)'}(\lambda_2 r) \overline{a_0^2} \right] e^{-ihz} , \quad (E-5)$$

$$B_z^2 = 0 , \quad (E-6)$$

where: a_0^2 and $\overline{a_0^2}$ are constants.

In region 2, neither the origin nor the point at infinity is included. Thus, it is necessary to consider linear sums of two independent solutions of Bessel's equation in equations E-1 through E-6. To be consistent with reference 3, the J_0 and $H_0^{(1)}$ functions have been selected; however, other functions could have been used.

The impedances desired are those seen at the outside and inside surfaces. These impedances are defined as

$$Z_{\text{outside}} = \frac{E_z(r=r_o)}{I_{\text{outside}}} , \quad (E-7)$$

$$Z_{\text{inside}} = \frac{E_z(r=r_i)}{I_{\text{inside}}} , \quad (E-8)$$

where: I_{outside} is the total current in the conductor when driven from the outside,

I_{inside} is the total current in the conductor when driven from the inside.

Consider the Maxwell equations

$$\nabla \times \vec{B} = \mu_0 (\vec{J} + \epsilon \dot{\vec{E}}) \quad (\text{E-9})$$

in region 2. Here, if the conductor is sufficiently good ($\sigma \gg \omega\epsilon$)

$$\nabla \times \vec{B} \approx \mu_0 \vec{J} \quad (\text{E-10})$$

Integrate equation E-10 over the surface of the conductor, i.e.,

$$\int_S \nabla \times \vec{B} \cdot d\vec{A} = \oint_{C_0} \vec{B} \cdot d\vec{l} - \oint_{C_i} \vec{B} \cdot d\vec{l} = \mu_0 \int_S \vec{J} \cdot d\vec{A} = \mu_0 I; \quad (\text{E-11})$$

where: S is the conductor cross-sectional surface,
 C_i is the contour of the inner circumference,
 C_0 is the contour of the outer circumference,
 I is the total conductor current.

It follows from equation E-11 that

$$I = \frac{1}{\mu_0} [2\pi r_0 B_\theta(r_0) - 2\pi r_i B_\theta(r_i)] \quad (\text{E-12})$$

If the conductor is driven from the outside, it is reasonable that $B_\theta(r_i) \approx 0$. Similarly, if the conductor is driven from the inside it is reasonable that $B_\theta(r_0) \approx 0$. Note that $B_\theta = 0 \Rightarrow E_r = 0$, so that no propagating energy appears interior (respectively exterior) to the conductor. It follows from equations E-5 and E-12 that

$$I_{\text{outside}} = -\frac{\lambda_2}{\mu_0} (2\pi r_o) \left[J_0'(\lambda_2 r_o) H_0^{(1)'}(\lambda_2 r_i) - J_0'(\lambda_2 r_i) H_0^{(1)'}(\lambda_2 r_o) \right], \quad (\text{E-13})$$

$$I_{\text{inside}} = \frac{\lambda_2}{\mu_0} (2\pi r_i) \left[J_0'(\lambda_2 r_i) H_0^{(1)'}(\lambda_2 r_o) - J_0'(\lambda_2 r_o) H_0^{(1)'}(\lambda_2 r_i) \right], \quad (\text{E-14})$$

Substituting equations E-13 and E-14 into equations E-7 and E-8, respectively, the impedances are

$$Z_{\text{outside}} = \frac{i\omega\mu_0\lambda_2}{\gamma_2^2 (2\pi r_o)} \left[\frac{J_0(\lambda_2 r_o) H_0^{(1)'}(\lambda_2 r_i) - J_0'(\lambda_2 r_i) H_0^{(1)}(\lambda_2 r_o)}{J_0'(\lambda_2 r_o) H_0^{(1)'}(\lambda_2 r_i) - J_0'(\lambda_2 r_i) H_0^{(1)'}(\lambda_2 r_o)} \right], \quad (\text{E-15})$$

$$Z_{\text{inside}} = \frac{-i\omega\mu_0\lambda_2}{\gamma_2^2 (2\pi r_i)} \left[\frac{J_0(\lambda_2 r_i) H_0^{(1)'}(\lambda_2 r_o) - J_0'(\lambda_2 r_o) H_0^{(1)}(\lambda_2 r_i)}{J_0'(\lambda_2 r_i) H_0^{(1)'}(\lambda_2 r_o) - J_0'(\lambda_2 r_o) H_0^{(1)'}(\lambda_2 r_i)} \right]. \quad (\text{E-16})$$

Consider the coefficient in equations E-15 and E-16, i.e.,

$$\text{coefficient} = \frac{\mu_0}{2\pi r k} \frac{i\omega\lambda_2}{\gamma_2^2} \quad (\text{E-17})$$

Since medium 2 is a good conductor, it follows that

$$\gamma_2^2 = -i\omega\mu_0(\sigma + i\omega\epsilon) \approx -i\omega\mu_0\sigma.$$

Furthermore, since h must remain finite,

$$\lambda_2^2 = \gamma_2^2 - h^2 \approx \gamma_2^2,$$

$$\Rightarrow \lambda_2 \approx \gamma_2 .$$

Thus,

$$\begin{aligned} \text{coefficient} &= \frac{\mu_0}{2\pi r_k} \frac{i\omega}{\gamma_2} = \frac{\mu_0}{2\pi r_k} \frac{i\omega}{\sqrt{-i\omega\mu_0\sigma}}, \\ &= \frac{1}{2\pi r_k} \sqrt{\frac{\mu_0\omega}{\sigma}} \cdot \frac{i\sqrt{2}}{(1-i)}, \\ &= \frac{1}{2\pi r_k} \sqrt{\frac{\omega\mu_0\sigma}{2}} \times \frac{1}{\sigma} \cdot \frac{2i}{1-i}, \\ &= \frac{1}{2\pi r_k} \frac{1}{\sigma\delta}(1-i) = \frac{1}{2\pi r_k} \frac{\sqrt{2}}{\sigma\delta} \times \frac{1}{\sqrt{i}}; \quad (\text{E-18}) \end{aligned}$$

$$\text{where: } \delta = \sqrt{\frac{2}{\omega\mu_0\sigma}}$$

Equation E-18 agrees with reference 3; however, the use of the function $H^{(1)}$ rather than $H^{(2)}$ is unfortunate, since this function becomes infinite as $r_0 \rightarrow \infty$, if the principle value of λ_2 is used. Thus, if these expressions are to be used in the limiting case, then the substitution $\lambda_2 \rightarrow -\lambda_2$ must be used, if the principle value of the square root is to be utilized.

Reference 5 makes use of equation E-16 to derive the impedance of an infinite hollow conductor; i.e., a conductor similar to that shown in Figure E-1 with $r_0 \rightarrow \infty$. When this is done, the preceding remarks concerning $H^{(1)}$ and $H^{(2)}$ are quite relevant. To demonstrate this, the infinite hollow conductor impedance is derived directly in Appendix F.

APPENDIX F

INFINITE HOLLOW CONDUCTOR IMPEDANCE

Consider an infinite hollow conductor. The geometry to be considered is illustrated in Figure F-1, the fields in medium 2 are (symmetric fields, $\nu = 0$, from equations D-34 through D-39)

$$E_r^2 = - \frac{\omega \lambda_2 h}{\gamma_2} H_0^{(2)'} (\lambda_2 r) a_0, \quad (F-1)$$

$$E_\theta^2 = 0, \quad (F-2)$$

$$E_z^2 = - i\omega \frac{\lambda_2^2}{\gamma_2} H_0^{(2)} (\lambda_2 r) a_0, \quad (F-3)$$

$$B_r^2 = 0, \quad (F-4)$$

$$B_\theta^2 = - \lambda_2 H_0^{(2)'} (\lambda_2 r) a_0, \quad (F-5)$$

$$B_z^2 = 0; \quad (F-6)$$

where: the function $H_0^{(2)}$ has been chosen as an outward travelling wave.

The current density (total current, conduction current plus displacement current) in the z-direction, in medium 3, is

$$j_z = (\sigma + i\omega\epsilon) E_z = (-i\omega)(\sigma_2 + i\omega\epsilon_2) \frac{\lambda_2^2}{\gamma_2} H_0^{(2)} (\lambda_2 r) a_0 \quad (F-7)$$

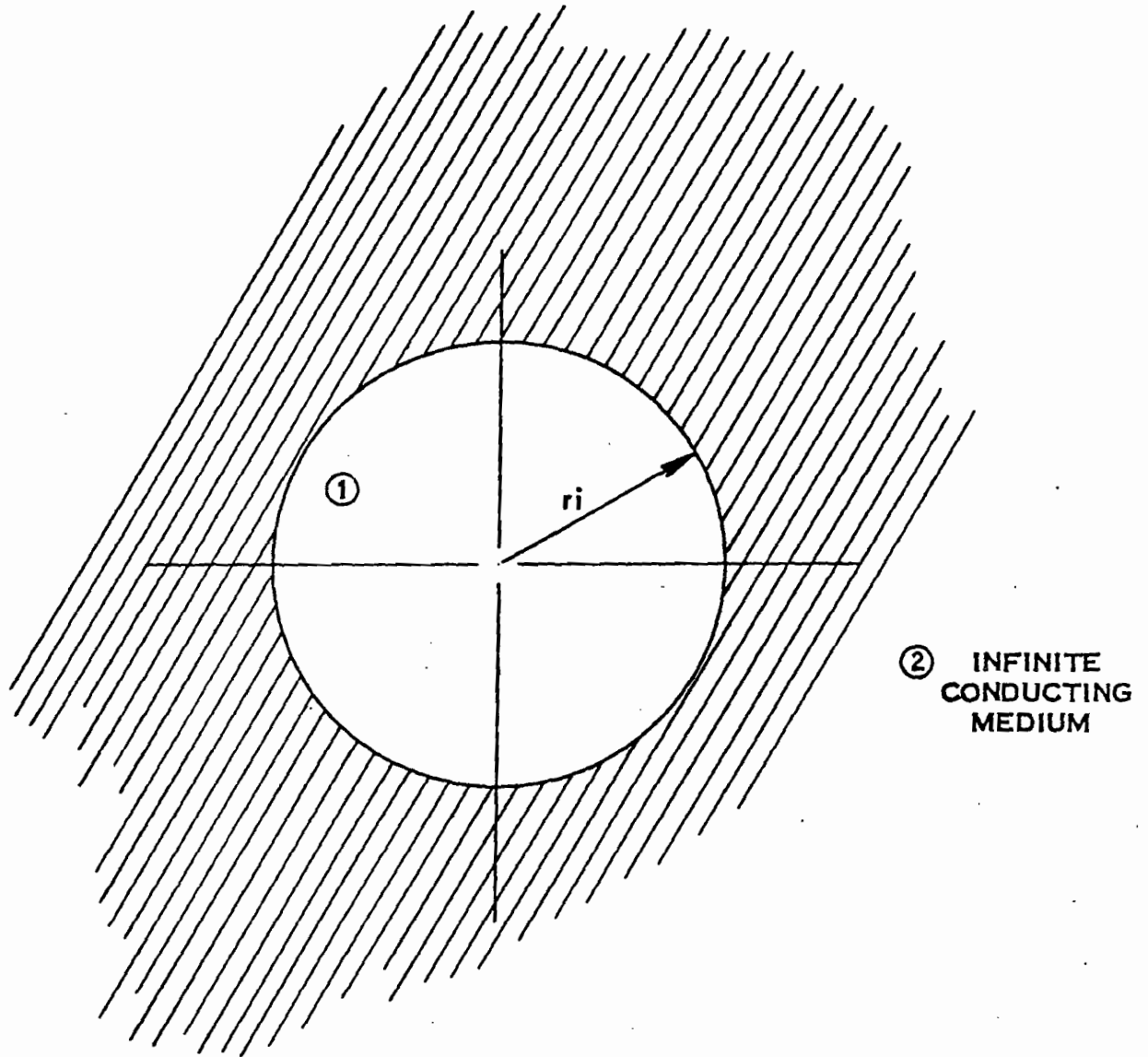


FIGURE F-1

GEOMETRY OF INFINITE HOLLOW CONDUCTING MEDIUM

Therefore the total current is

$$I = (-i\omega)(\sigma_2 + i\omega\epsilon_2) \frac{\lambda_2^2}{\gamma_2} a_0 \int_0^{2\pi} \int_{r_i}^{\infty} H_0^{(2)}(\lambda_2 r) r dr d\theta \quad (\text{F-8})$$

Using the relation (reference 7, 9.1.30, p. 361)

$$\frac{1}{z} \frac{d}{dz} (z Z_\nu(z)) = z^{\nu-1} Z_{\nu-1}(z),$$

equation F-8 can be evaluated as

$$\begin{aligned} I &= (-i\omega)(\sigma_2 + i\omega\epsilon_2) \frac{\lambda_2^2}{\gamma_2} a_0 \times \frac{1}{\lambda_2} 2\pi \int_{\lambda_2 r_i}^{\infty} H_0^{(2)}(\lambda_2 r) (\lambda_2 r) d(\lambda_2 r) \\ &= (i\omega) \frac{(\sigma_2 + i\omega\epsilon_2)}{\gamma_2} 2\pi a_0 H_1^{(2)}(\lambda_2 r_i) \lambda_2 r_i \end{aligned} \quad (\text{F-9})$$

The impedance on the inside surface is therefore

$$\begin{aligned} Z &= \frac{E_z(r_i)}{I} = \frac{-\frac{i\omega\lambda_2^2}{\gamma_2} H_0^{(2)}(\lambda_2 r_i)}{i\omega \frac{(\sigma_2 + i\omega\epsilon_2)}{\gamma_2} 2\pi H_1^{(2)}(\lambda_2 r_i) \lambda_2 r_i} \\ &= \frac{-\lambda_2^2 H_0^{(2)}(\lambda_2 r_i)}{2\pi H_1^{(2)}(\lambda_2 r_i) \lambda_2 r_i (\sigma_2 + i\omega\epsilon_2)} \\ &= \frac{-\lambda_2 H_0^{(2)}(\lambda_2 r_i)}{2\pi r_i (\sigma_2 + i\omega\epsilon_2) H_1^{(2)}(\lambda_2 r_i)} \end{aligned} \quad (\text{F-10})$$

If the conductivity is sufficiently high, then

$$\lambda_2 \rightarrow \gamma_2 \rightarrow \sqrt{-i\omega\mu_0\sigma}.$$

Using the relation (reference 7, 9.1.39, p. 361)

$$H_\nu^{(2)}(z) = -\epsilon^{\nu\pi i} H_\nu^{(1)}(z\epsilon^{\pi i}),$$

equation 96 can be written

$$Z = \frac{\lambda_2}{2\pi r_i \sigma_2} \frac{H_0^{(1)}(-\lambda_2 r_i)}{H_1^{(1)}(-\lambda_2 r_i)} \quad (F-11)$$

Equation F-11 should be compared with the hollow conductor impedance of equation E-16 with $r_o \rightarrow \infty$. That is, when $r_o \rightarrow \infty$ in equation E-16, the non-principle value of the square root should be utilized.

APPENDIX G
CENTER CONDUCTOR IMPEDANCE

Reference 5, Chapter 11 makes use of the impedance of a solid circular conductor. For purposes of completeness, this impedance is derived herein.

The geometry to be considered is illustrated in Figure G-1. From equations D-34 through D-39, the transverse magnetic, angularly symmetric fields in the center conductor (medium 1) are

$$E_r^1 = - \frac{\omega \lambda_1 h}{\gamma_1} J_0'(\lambda_1 r) a_0, \quad (G-1)$$

$$E_\theta^1 = 0, \quad (G-2)$$

$$E_z^1 = - i\omega \frac{\lambda_1^2}{\gamma_1} J_0(\lambda_1 r) a_0, \quad (G-3)$$

$$B_r^1 = 0, \quad (G-4)$$

$$B_\theta^1 = - \lambda_1 J_0'(\lambda_1 r) a_0, \quad (G-5)$$

$$B_z^1 = 0. \quad (G-6)$$

The total current density, including displacement current density, in the center conductor is

$$j_z = (\sigma_1 + i\omega\epsilon_1) E_z = -(\sigma_1 + i\omega\epsilon_1) \frac{i\omega\lambda_1^2}{\gamma_1} J_0'(\lambda_1 r) a_0, \quad (G-7)$$

C-2:

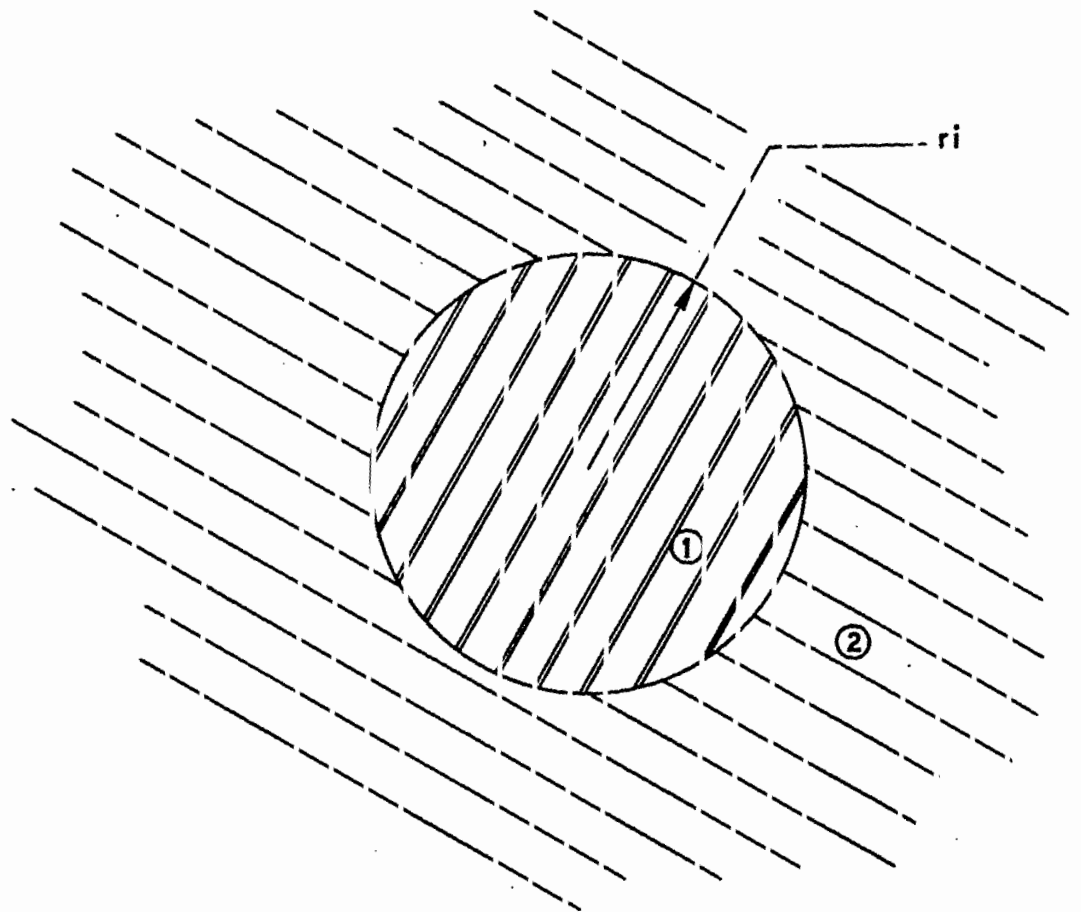


FIGURE G-1

GEOMETRY OF SOLID CENTER CONDUCTOR

so that the total current is

$$I = 2\pi \int_0^{r_i} j_z r dr = -2\pi(\sigma_1 + i\omega\epsilon_1) \frac{i\omega\lambda_1^2}{\gamma_1} \int_0^{r_i} J_0'(\lambda_1 r) r dr \quad (G-8)$$

using the relation (reference 7, 9.1.30, p. 361)

$$\frac{1}{z} \frac{d}{dz} (z Z_\nu(z)) = z^{\nu-1} Z_{\nu-1}(z), \quad (G-9)$$

equation G-8 can be evaluated as

$$I = -2\pi r_i (\sigma + i\omega\epsilon) \frac{i\omega\lambda_1}{\gamma_1} J_1(\lambda_1 r_i) \quad (G-10)$$

The impedance of the center conductor is

$$\begin{aligned} Z^i &= \frac{E_z(r_i)}{I} \\ &= \frac{\lambda_1}{2\pi r_i (\sigma_1 + i\omega\epsilon_1)} \frac{J_0(\lambda_1 r_i)}{J_1(\lambda_1 r_i)} \end{aligned} \quad (G-11)$$

If the center conductor is metallic, then for all frequencies of interest $\sigma \gg i\omega\epsilon$, and indeed σ is very large. Under these conditions, since h must remain finite,

$$\lim_{\sigma \rightarrow \infty} \lambda_1 = \lim_{\sigma \rightarrow \infty} \gamma_1^2 - h^2 = \gamma_1 \approx \sqrt{-i\omega\mu_0\sigma_1}.$$

therefore, for very high conductivities

$$Z^i \approx \frac{\sqrt{-i\omega\mu_0\sigma_1}}{2\pi r_i \sigma_1} \frac{J_0(\sqrt{-i\omega\mu_0\sigma_1} r_i)}{J_1(\sqrt{-i\omega\mu_0\sigma_1} r_i)} \quad (G-12)$$