EMP INTERACTION NOTE

NOTE 21

21 January 1969

Approximate Method for Calculating the Currents Induced in Underground Cables by a High Altitude Overhead Electromagnetic Field Source

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ABSTRACT

From a previously derived expression for calculating currents induced in long underground cables by radio-frequency electromagnetic fields, a simplified expression to calculate the currents induced by a high altitude EM source directly over the cable is derived. The integral term of the general current solution is reduced to closed form by approximating the field source as a plane wave propagating in a direction normal to the earth's surface. Using this simplified integral term with typical values for the termination impedances of the cable, the reflection terms in the current equation are simplified to reduce the total calculation time for the problem.

I. Introduction

In the area of EMP interactions with surface and underground systems, extensive theoretical and experimental work has been done on the problem of calculating the currents induced in long underground cables by external electromagnetic fields[1,2,3,4]. However, in all the above cases, the theory has been applied only to field sources at the surface of the earth and lying along the axis of the given cable.

In this paper, the special case of a radio frequency* electromagnetic field source directly overhead and high above the underground cable will be considered to derive a simplified solution to the cable current equation. Further simplification of the results will follow from considering only the most common cable termination impedances.

II. The General Cable Current Equation

The general equation used to calculate the current induced in long underground cables by incident CW electromagnetic fields is [2].

$$I(x) = Ke^{-\Gamma x} + Le^{\Gamma x} + \frac{1}{2Z_o}\int_o^d E_o(v)\, e^{-\Gamma|x-v|}\, dv \qquad (1)$$

where I is the current at point x on the cable, d is the length of the cable, K and L are the reflection coefficients at the cable ends x = 0 and x = d, respectively, Γ is the propagation constant for current travelling down the earth-cable transmission system, Z_o is the characteristic impedance of the earth-cable equivalent transmission line, E_o is the incident CW electric field component that is parallel to the cable axis, and v is the dummy variable of integration, which varies over the range of x.

The propagation constant and the characteristic impedance are calculated from these equations:

$$\Gamma = (ZY)^{1/2} \qquad Z_o = (Z/Y)^{1/2} \qquad (2)$$

where Z is the longitudinal impedance per unit length of the earth-cable transmission system and Y is the transverse admittance per unit length. The calculation of these quantities is explained in detail in AFWL-TR-65-94 and in Interaction Note I[2,3].

Also from AFWL-TR-65-94[2], we have the expressions for the reflection coefficients:

*The frequencies of interest here are from 10^2 to 10^8 Hertz.

$$K = \frac{(Z_2 + Z_o)(Z_o - Z_1)e^{\Gamma d} F(o) + (Z_1 - Z_o)(Z_2 - Z_o) F(d)}{(Z_1 + Z_o)(Z_2 + Z_o)e^{\Gamma d} - (Z_1 - Z_o)(Z_2 - Z_o)e^{-\Gamma d}}$$

$$L = \frac{(Z_2 - Z_o)(Z_1 - Z_o)e^{-\Gamma d} F(o) + (Z_1 + Z_o)(Z_o - Z_2) F(d)}{(Z_1 + Z_o)(Z_2 + Z_o)e^{\Gamma d} - (Z_1 - Z_o)(Z_2 - Z_o)e^{-\Gamma d}} \qquad (3)$$

where $F(x) = \frac{1}{2Z_o} \int_o^d E_o\ (v)\ e^{-\Gamma|x-v|}\ dv$ and Z_1 and Z_2 are the termination impedances of the equivalent transmission line at the cable ends $x = 0$ and $x = d$ respectively.

In general, the cable electrical characteristics and dimensions and the electrical characteristics of the earth are known. However, the incident electromagnetic field is usually known only in the air above the ground; a calculation must be performed to determine the field that actually penetrates the interface and reaches the cable ($E_o(v)$). Then the integration in the current equation is performed; in general, a complex numerical integration.

With the above equations and calculation techniques in mind, this theory will now be applied to the case of a high altitude CW field source directly over the cable.

III. The High Altitude Source

The field source considered here will have the following characteristics: It will be vertically above the cable and it will be at a sufficiently high altitude so that the overall effect in the neighborhood of the cable will be that of a plane-wave CW field descending directly perpendicular to the surface of the earth*. Mathematically, this will produce a driving field on the entire length of the cable that is independent of the distance along the cable. That is

$$E_o(v,t) = E' e^{i\omega t}$$

where E' is a constant at a given frequency ω.

If this field is used in the cable current equation, we arrive at the following result for the integral term:

$$F(x) = \frac{1}{2Z_o} \int_o^d E' e^{-\Gamma|x-v|} dv$$

$$= \frac{E'}{2Z_o} \{e^{-\Gamma x} \int_o^x e^{\Gamma v}\ dv + e^{\Gamma x} \int_x^d e^{-\Gamma v}\ dv\}$$

$$= \frac{E'}{2Z_o} \{\frac{e^{-\Gamma x}}{\Gamma} (e^{\Gamma x} - 1) - \frac{e^{\Gamma x}}{\Gamma} (e^{-\Gamma d} - e^{-\Gamma x}))$$

*Also implicit in this treatment are the approximations that the earth in the area of the cable is perfectly flat and the cable is parallel to the surface of the earth.

Since $Z_o \Gamma = Z$, the longitudinal impedance per unit length, the final result for the current induced in our postulated cable is:

$$I(x) = Ke^{-\Gamma x} + Le^{\Gamma x} + \frac{E'}{2Z} \{2 - e^{-\Gamma x} - e^{-\Gamma(d-x)}\} \qquad (4)$$

E' is obtained from the known electromagnetic field in the air (remember that E' is the field that actually reaches the underground cable) from the known relation between the magnitude of a plane electromagnetic wave incident normally on the plane interface between two media and the magnitude of the wave that is transmitted through the interface[5]. When the two media are air and earth respectively we may assume the conductivity of the air is equal to zero and the magnetic permeabilities of the two media are equal. This gives:

$$E' = \frac{2E_i}{1 + \left(\frac{\sigma + i\omega\varepsilon}{i\omega\varepsilon_o}\right)^{1/2}} \qquad (5)$$

where E_i is the electric field incident on the earth, σ is the ground conductivity (mhos per meter), ε is the dielectric permittivity of the earth (henries per meter) and ε_o is the dielectric permittivity of air.

Once the complex impedance and propagation parameters for the cable have been calculated, it is a simple task to calculate the current at every point on the cable.*

If we assume that the cables of interest are hundreds of meters long or longer, another special case simplification may be considered. For a cable with its outer conductor in good electrical contact with the earth along its entire length and for earth conductivities sufficiently high so that

$1/\Gamma_r << d/2$ (Γ_r is the attenuation factor for currents induced in the given cable; the real part of Γ), then for points x on the cable such that

$$\Gamma_r x >> 1 \quad \text{and} \quad \Gamma_r (d - x) >> 1$$

the current is

$$I(x) \cong E'/Z \qquad (6)$$

This approximate current response is exactly the same as that derived for the same cable under the influence of an electromagnetic field generated at the earth's surface; the current frequency response that is the basis of MARS2 computer code for calculating the currents induced in cables by arbitrary electromagnetic pulses[4].

*The ARFH computer code at AFWL performs this calculation on a CDC 6600 (see Section V).

Of course, the more general equation (4) may be applied to pulsed fields by using the equation as a field-current transfer function in an inverse Fourier Integral after Fourier analyzing the input field. This series of operations would allow the calculation of the induced pulsed currents as a function of time from arbitrary high-altitude sources*.

As simple as equation (4) is, we will now see that it can be simplified even further for most practical cases.

IV. Analysis of the Reflection Coefficients

We can see from equation (3) that, if the termination impedances of a transmission line Z_1 and Z_2 are both equal to the line impedance Z_o, then the reflection coefficients K and L are both equal to zero. To state this differently, if you match the termination impedances of a transmission line with the line impedance, then the end reflections are eliminated. This is a condition that is designed into most systems using transmission lines.

But, in the case we are discussing in this paper, the transmission line consists of some complex cable or metal conduit as one conductor and the surrounding earth as the current return path. In general, this is not the normal signal mode for the cable, whether it is a power cable or a communications cable. Therefore, no attempt is usually made to match the impedance to ground at the end of the cable-earth system. In fact, the only attempt to affect the earth-to-ground impedance of an underground cable at all is when the outer metal shield of a cable is "grounded" at an end to protect the cable and connected instrumentation from high surge currents (e.g., a lightning strike). Therefore, in general, there will be mismatches between the earth-cable line impedance and the impedances to ground at the ends.

If the cable line impedance is much greater than the termination impedances ($Z_o >> Z_1$ and $Z_o >> Z_2$) then the reflection coefficients from equation (3) become:

$$K \doteq \frac{e^{\Gamma d}\, F(o) + F(d)}{e^{\Gamma d} - e^{-\Gamma d}}$$

$$L \doteq \frac{e^{-\Gamma d}\, F(o) + F(d)}{e^{\Gamma d} - e^{-\Gamma d}} \tag{7}$$

But

$$F(x) = \frac{1}{2Z_o} \int_o^d E_o(v)\, e^{-\Gamma|x-v|} dv$$

which, from equation (4), equals

$$F(x) = \frac{E'}{2Z} \{2 - e^{-\Gamma x} - e^{-\Gamma(d-x)}\}$$

*This operation is also performed by the ARFH computer code. (See Section V.)

Therefore,

$$F(o) = \frac{E'}{2Z}\{1 - e^{-\Gamma d}\} = F(d)$$

Replacing this value for F(o) and F(d) in equation (7)

$$K = \frac{E'}{2Z}$$

$$L = \frac{E'}{2Z} e^{-\Gamma d}$$

If these values for the reflection coefficients are placed in equation (4), the current induced in a cable with grounded terminations by a high altitude source is:

$$I(x) = \frac{E'}{2Z}\left(e^{-\Gamma x} + e^{-\Gamma(d-x)} + \{2 - e^{-\Gamma x} - e^{-\Gamma(d-x)}\}\right)$$

$$= \frac{E'}{Z}\text{, independent of } x \qquad (8)$$

If the termination impedances are much greater than the line impedance ($Z_1 >> Z_o$ and $Z_2 >> Z_o$), then the reflection coefficients from equation (3) reduce to:

$$K \cong \frac{E'}{2Z}\,\frac{2 - e^{\Gamma d} - e^{-\Gamma d}}{e^{\Gamma d} - e^{-\Gamma d}}$$

$$L \cong \frac{E'}{2Z}\,\frac{2e^{-\Gamma d} - 1 - e^{-2\Gamma d}}{e^{\Gamma d} - e^{-\Gamma d}}$$

Placing these values into equation (4) gives us the current induced in a cable with electrically floating ends by a high altitude source:

$$I(x) = \frac{E'}{Z}\left[1 - \frac{\text{Sinh}\Gamma x + \text{Sinh}\Gamma(d-x)}{\text{Sinh}\Gamma d}\right] \qquad (9)$$

This expression may be further reduced for low frequency driving fields or short cables, where $\Gamma d << 1$. Using this approximation, and the appropriate second order approximation to sinh (x),

$$I(x) \cong \frac{E'}{Z}\left[1 - \frac{\Gamma x + \frac{(\Gamma x)^3}{3!} + \Gamma(d-x) + \frac{[\Gamma(d-x)]^3}{3!}}{\Gamma d + \frac{(\Gamma d)^3}{3!}}\right]$$

$$\cong \frac{E'}{2Z}\left[\frac{\Gamma^3 (d^2x - x^2d)}{\Gamma d}\right]$$

$$= YE'x\,\frac{d-x}{2} \qquad (10)$$

where Y is the transverse admittance per unit length of the cable-earth system (see equation (2)).

When calculating the current induced in underground cables from a high altitude EM source, whether the source is a single frequency (CW) or a pulse field composed of many frequencies, it is very likely that either the approximation of equation (8) or that of equation (9) will hold. For the case of the bare cable with grounded ends, it may be possible that the termination impedances will approach the line impedance for a very limited range of frequencies. However, in the absence of termination impedance measurements, equation (8) may be used for this problem with no more than a factor of two error in the current calculated at the cable ends. At distances from the ends greater than $1/\Gamma_r$, the reflections are negligible regardless of assumed or actual terminations.

Of course, if the impedance to ground at the ends has been measured and is on the order of the line impedance of the cable, then equation (4), with the reflection coefficients calculated by equation (3), should be used.

V. <u>The AFWL Computer Codes CALINA(HA) and ARFH</u>

At the Air Force Weapons Laboratory, a set of two computer codes has been generated to calculate the currents, as a function of time, induced on an underground cable by a known high-altitude source electromagnetic field. This set of codes will accept input electric field waveforms that have known, closed-form Fourier Transforms to calculate the current induced on a given underground cable at a number of equally-spaced points on the cable,including the ends. These codes are presently written in FORTRAN IV for the CDC 6600 computer at AFWL.

The code CALINA (HA) accepts as input the waveform parameters of the input field that are necessary to calculate its Fourier Transform. For instance, the present form of CALINA(HA) considers input electric fields of the form

$$E_i(t) = \sum_{m=1}^{\ell_1} A_m e^{-\alpha_m t} + \sum_{n=1}^{\ell_2} B_n t e^{-\beta_n t}$$

where t is time and A_m, α_m, B_n and β_n are all real coefficients. This field transforms into

$$E_i(\omega) = \sum_{m=1}^{\ell_1} A_m \frac{1}{\alpha_m + i\omega} + \sum_{n=1}^{\ell_2} B_n \frac{1}{(\beta_n + i\omega)^2}$$

where $i = \sqrt{-1}$ and ω is the frequency in radians per second. Therefore, the code reads in the pairs of coefficients A_m and α_m for m=1 through ℓ_1, and B_n and β_n for n=1 through ℓ_2.

After performing the above transform and reading in the given electrical characteristics of the earth and air, CALINA(HA) uses equation (5) to transform the input field into the driving electric field in the earth:

$$E'(\omega) = \frac{2\, E_i\ (\omega)}{1 + (\frac{\sigma\ +\ i\omega\varepsilon}{i\omega\varepsilon_o})^{1/2}}$$

The above two operations are performed for a large number of discreet frequencies through the band of frequencies that describe the input field.

The program ARFH accepts the following input: the driving field $E'(\omega)$ as calculated by CALINA(HA) and the frequency array over which it is calculated; the electrical characteristics of the earth that were previously used in CALINA(HA); the electrical and dimensional characteristics of the given cable, including the termination impedances Z_1 and Z_2; and the cable points and the time array over which the currents will be calculated. At each frequency in the given array, ARFH calculates the current at each given point on the cable by using equation (4) (equation (3) is used to calculate the reflection coefficients K and L). After the currents have been calculated for all given frequencies, a special Inverse Fourier Transform subroutine is used to calculate the current as a function of time at each given cable point.

Remember that these codes are limited by the approximations applied to derive equations (3) and (4); the codes were designed to calculate the currents induced in a long underground cable by a high-altitude source electromagnetic field directly over the cable, subject to the restrictions of Section III. However, these codes provide a fast and useful short cut to a calculation that would normally necessitate a long, complex numerical integration.

VI. Conclusion

In this note, equations are derived to calculate the current induced in underground cables by high altitude CW electromagnetic fields. Using suitable approximations, the integral term of the general cable current equation was reduced to a closed form expression for easy calculation. Also, considering the normal means of terminating a cable to the earth, it was possible to simplify the equations for calculating the reflection coefficients.

These derived CW cable current equations are used as field-current transfer functions for computer codes that calculate the current induced in underground cables by high-altitude pulsed electromagnetic fields, as in the present AFWL high-altitude source cable current code, ARFH.

VII. References

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4. Lt D.R. Marston, *The Theoretical Basis of the MARS2 Cable Current Code*, EMP Interaction Note IV, Air Force Weapons Laboratory (October 1966).

5. J.A. Stratton, *Electromagnetic Theory*, McGraw-Hill Book Co., New York (1941), see page 494.

6. F.J. Sulkowski, *FORPLEX, A Program to Calculate Inverse Fourier Transforms*, EMP Mathematics Note II (10 November 1966).