SCR-622 IN 14

Sandia Corporation

MONOGRAPH

RADIO-FREQUENCY LEAKAGE INTO MISSILES

by

R H Duncan

and

C W Harrison, Jr

APRIL 1963

SCR-622

SANDIA CORPORATION MONOGRAPH

RADIO-FREQUENCY LEAKAGE INTO MISSILES

by

R. H. Duncan
Research Center, New Mexico State University
University Park, New Mexico

and

C. W. Harrison, Jr.
Member of the Technical Staff
Sandia Corporation
Albuquerque, New Mexico

APRIL 1963

ACKNOWLEDGMENT

The writers are indebted to K. S. Kunz, Research Professor, New Mexico State University, for many helpful suggestions.

SUMMARY

Missile functions are often initiated by the detonation, upon command from either a wire or radio circuit, of sensitive electroexplosive devices. The possibility exists of inadvertent detonation of these devices caused by the leakage of radio-frequency energy into the missile from high-power radar or communications facilities. This paper treats the idealized problem of RF energy leakage through a slot in an infinite cylinder as a perturbation on the scattering problem for the same object with no slot. It is shown that interior response depends on three factors: the exterior skin current density from the scattering problem, the transmitting admittance of the slot, and an eigenfunction expansion of the interior field when unit voltage is impressed across the slot. It is then proposed that the form of this solution may be applicable and conceptually useful in treating other problems either theoretically or experimentally.

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

RADIO-FREQUENCY LEAKAGE INTO MISSILES

1. Introduction

There are at least three mechanisms by which electromagnetic radiation can leak into regions surrounded by metallic shields. At low frequencies an imperfectly conducting shield may fail to isolate the shielded region from an externally applied field. King and Harrison (1961) have considered this case in some detail. A second possibility is that the shield may be incomplete. For example, there is a functional requirement for openings (such as access doors) in the outer shell of a missile. If the missile is exposed to the field of a nearby radar or communications antenna, the resulting level of interference with missile circuitry may be objectionable. The circuitry may be that associated with sensitive electroexplosive devices used to initiate missile responses upon command. It is obvious that inadvertent interference with these circuits may affect ordnance reliability. However, many of our comments are applicable to other problems in radio-frequency interference and to equipment other than missiles. A third possibility is that radio-frequency energy will be carried into the missile along umbilical cables. This case is not being considered in this paper.

It must be taken for granted that efforts toward better design of potentially unreliable circuits will continue. Radio-frequency filters, sealed components, coded input schemes, and other techniques will all undoubtedly play a larger role in future design. However, at any given time it will be necessary to adequately assess the level of interference and to know the margin of safety. Designs which are presently adequate may become inadequate if radar power continues to increase.

It is also important to make the point that the art of predicting induced currents must be refined and made accurate. Underestimation of danger leads to a false sense of security. Overestimation may lead to ridiculous precautionary procedures. One occasionally observes "Turn Off Two-Way Radio" signs in the vicinity of highway or industrial projects in which explosives are used. If the same policy were invoked at Cape Canaveral, the countdown procedure for launching a rocket could not begin until all nearby sources of radio frequency were turned off. Excessive caution can be a source of petty annoyance, or it can interfere with the operation of a complicated system.

At first glance the complexity of the problem seems overwhelming. Exact analysis of the environment and internal details of a missile from the point of view of electromagnetic theory is certainly impossible. If one shuns analysis and electrical measurements entirely, the only kind of testing possible is to illuminate the missile with electromagnetic energy and simply observe whether or not the electroexplosive devices (hereafter referred to by the short and popular name, squibs) are affected. The test should be performed in a realistic environment with the illumination being provided by the actual sources which may be suspect. (Of course, one must alter internal details in such a way that an exploding squib does not actually initiate action by the missile.) This test is simple and meaningful if a few squibs are actually exploded. Then one knows that certain circuits must be redesigned, or that the use of less sensitive squibs is indicated. The problem is considerably magnified if the firing of a squib is a rare event. The probability of the type of accident being considered here may be small but nevertheless intolerable. To clearly establish a probability, p, by experimental means requires more than p^{-1} tests. We are lead to reject the purely probabilistic model because it requires an excessive amount of testing which should be repeated for a large variety of missile environments. A last point against this approach is that it provides no knowledge regarding the margin of safety. It may be practically impossible to accidentally fire the squibs in a presently existing design which will be quite unacceptable in the near future.

The next step in evolving a model is to consider the measurement of the currents induced in the squib circuits. Separate bench tests of the squibs can be made. It is a simple matter now to guarantee that all squibs in a given sample are fired and to estimate the mean, variance, and probability distribution of the required firing current. Conclusions are then reached by comparing these data with the currents induced in the missile circuits with the missile in a realistic environment.

We shall not dwell on the details of setting confidence limits and the somewhat subjective problem of safety recommendations as we intend to concentrate on deterministic models for describing the induced currents. The reader should be warned that another important aspect of the problem will not be treated here. We have not made any distinction between transient and steady-state situations. It is well known that a squib can be subjected to a large current for a short period of time without firing. However, the construction of a theoretical model for the steady-state case is a necessary prelude to the transient solution. Finally, it may seem that we have relegated statistical aspects of the problem to consideration of quality-control data on squibs. We are not ruling out error and variance analysis of the field measurements of induced currents. It should be obvious that these measurements can be carried out on a sample of several missiles and it would be prolix to dwell on this point.

Leaving experimental refinements aside, further development of a meaningful test program must lead toward an understanding of the induced currents in terms of the interaction of the missile with its electromagnetic environment. This part of the problem is the most difficult because of the infinite variety of possible complicated situations. However, in spite of the complications, a great deal of conceptually useful information can be gleaned from careful application of physical principles. The next section of this paper provides a brief review and critique of various attempts to organize induced squib current data. Then we turn attention to the solution of a representative idealized boundary value problem. Idealized problems have certain features which would survive in complicated problems if they could be explicitly solved. Realizing this, we propose a rather general semiempirical approach to the problem in the final part of the paper.

2. Review of Previous Work

In this section we shall attempt to present a brief review of those ideas and experiments which have been used in attempts to obtain a satisfactory model for explaining induced squib currents. Many technical details are omitted. In some cases, variations or refinements of the methods exist and we have taken the liberty of giving only bare essentials. We have included a few general statements regarding experimental data. Actual data have been omitted since most of it is too particular to be relevant to a general discussion.

The approach which we recommend in a later section of this paper has not yet been subjected to experimental tests. The names of the methods in the following outline were coined by the authors for their own convenience.

a. Mutual Impedance Method. This method is based on the well-known fact that the electromagnetic field equations for linear homogeneous media can be manipulated to provide an equivalent circuit description of the interaction between sources and sinks of electromagnetic energy. As a simple example, let the current and voltage at the terminals of an antenna be I_1, V_1, respectively. Denote similar quantities at a pair of squib terminals inside a nearby missile by I_2, V_2. Then,

$$\begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} Y_{11} & Y_{12} \\ Y_{12} & Y_{22} \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix}. \tag{2.1}$$

If the squib impedance is Z_L,

$$I_2 = \frac{Y_{12} V_1}{1 - Y_{22} Z_L}. \tag{2.2}$$

The attractive simplicity of (2.2) leads to the deception that progress can be made by measuring the admittance coefficient Y_{12}. (The diagonal elements, Y_{11} and Y_{22}, are not difficult to measure.) Admittances are dependent on position and environmental details. Off diagonal terms in an admittance matrix are notoriously difficult to estimate theoretically in all but the simplest problems. One would be just as well off to move the missile about and directly catalog squib current as a function of position and orientation of the missile. Either procedure is likely to generate coffers of unexplained data. There may be special situations or aspects of the problem in which mutual impedance concepts can be used to advantage. However, we do not favor this method as the basis for an over-all attack.

b. Transfer Function Method. The method consists of taking data on squib current and field intensity to define a transfer function

$$Y(f) = \left(\frac{I_s}{\overline{E}}\right)_{measured}, \tag{2.3}$$

where f is the frequency of the illuminating field, and E is the field intensity for a given polarization. Measurements of $\overline{E}$ are usually made at a single point in the field with the missile removed. It is then supposed that if the missile is immersed in a field of different magnitude the squib current will be $I_s = Y(f)E$. The method has both a fallacy and a correct limit. The fallacy is that a point measurement of $\overline{E}$ is not always adequate. Rather, the distribution of $\overline{E}$ determines the interior response of an object placed in the field. However, except in the immediate vicinity of sharp points and edges in the environment, an electromagnetic field $(\overline{E}, \overline{H})$ does not vary much over distances short compared to a wavelength. Heuristically, the method should provide an adequate description of the interior response of objects which are uniformly illiminated. If sharp points and edges are an appreciable fraction of a wavelength away from the object, Y(f) should be insensitive to environmental details.

c. Radiation Pattern Method.* This method is based on the realization that if radiation can leak into a missile it can also certainly leak out. The squib is removed and a transmitter is attached to the circuit terminals. The resulting radiation pattern, $G(\theta, \phi)$, is then measured. (Absolute field intensity measurements must be made; the usual relative pattern will not serve.) Now the reciprocity theorem can be invoked to calculate squib current for the case of plane-wave illumination. Omitting impedance mismatch factors which are required in practice and assuming an incident plane wave of intensity S, the absorbed power is given by

$$P_s = SA(\theta, \phi),$$

where

$$A(\theta, \phi) = \frac{\lambda^2}{4\pi} G(\theta, \phi). \tag{2.4}$$

The relation between $A(\theta, \phi)$ and $G(\theta, \phi)$ shown in (2.4) is established in many texts for efficient antennas. It can also be shown tc hold for inefficient antennas (Appendix A). It is necessary to make this point since much of the energy injected into the squib circuit in the transmitting mode may be absorbed inside the missile.

Difficulties with the model can be attributed to the fact that in a complicated environment, the external transmitted field can not be interpreted as a radiation far field and the illuminating field will rarely be a plane wave. From a fundamental point of view, improvement of the approach will have to involve a study of the interaction of antennas with near fields and surface waves, the latter because many of the measurements must, of necessity, be carried out over a partially conducting ground. Finally, the model is undoubtedly valid for application to inflight interference with circuits inside missiles and aircraft.

d. Antenna Equivalent Circuit Method.† This method is distinct from those discussed above in that it is entirely theoretical; no measurements are involved. It is also the most difficult to discuss briefly, involving, as it does, several intricate papers dealing with explicit problems. Basically, the approach is to find the parameters for the Thévenin equivalent circuit for a receiving antenna, namely: V_{oc}, the open circuit voltage, and Z_T, the transmitting impedance. If these parameters are known and a load impedance is presumed to be part of a simple circuit connected across the leak (antenna terminals), the load current can be readily calculated. The procedure is likely to overestimate the current delivered to squibs in practical situations. Practical circuits are often located at a considerable distance from the actual leak; interfering energy propagates in some complicated way from the leak to the circuit location. This process inevitably involves some attenuation. Focusing effects in the interior are not anticipated in the wavelength region of interest.

Theory of this type has been successful in order-of-magnitude calculations. It provides nominal agreement with the slope of some experimental curves of squib current versus frequency. Both linear antenna theory and slot antenna theory have been used in particular situations. In one application folded

*G. E. Galos (Ordnance Mission, White Sands Missile Range), unpublished information.

†C. W. Harrison, Jr., (Sandia Corporation) unpublished information.

dipole theory is used to estimate the change in the squib current caused by attaching a control cable to a missile, assuming that the interior response of the missile without the cable is known.

This approach can be shown to be an approximation to the more general method to be described later in the paper.

3. Scattering by Slotted Cylinders

It has been pointed out that radio-frequency interference problems are too complicated to allow detailed theoretical solutions. The role of theory in this case is to provide organizing concepts as a basis for economical and meaningful experimentation.

The main problem we wish to present is that of scattering of a cylindrical wave by an infinite slotted cylinder of infinite conductivity. Coordinates for the problem are the conventional (ρ, ϕ, z) circular cylinder coordinates. The slot is totally circumferential in the region $|z| < w$ at $\rho = a$, the radius of the cylinder. The choice of an infinite cylinder need not be a source of concern since it can be shown that the effective length and, hence, the open circuit voltage, of an infinite cylindrical receiving antenna is finite (Appendix B). Also, we will discuss modification of the treatment to obtain an approximate solution for a finite cylinder. Cylindrical wave excitation enables us to avoid the complications of asymmetry in the azimuthal coordinate during a preliminary investigation of the problem. This feature of the idealized problem can also be revised. In what follows, those symbols which are used without explicit definition are conventional in the literature of electromagnetic theory.

We assume, for simplicity, that the electric vector of the incoming field is parallel to the cylinder axis. Time dependence of $\exp(j\omega t)$ is assumed, but suppressed. Consistent with this latter assumption, $H_0^{(1)}(k\rho)$ and $H_0^{(2)}(k\rho)$ are chosen to represent the incoming and reflected waves, respectively. In addition to these waves, there will be a diffracted wave because of the slot. The electric field in the slot will be an unknown function, $f(z)$. It is convenient to redefine this function as $Vf_0(z)$ where $f_0(z)$ is normalized to unity on the interval $|z| < w$. The electric field vanishes at the surface of the cylinder everywhere outside the slot. We also normalize the incident field to E_0 volts per meter at the surface of the cylinder. If $F_0(\alpha)$ is the Fourier transform of $f_0(z)$,

$$E_z^0(\rho, z) = E_0\left[\frac{H_0^{(1)}(k\rho)}{H_0^{(1)}(ka)} - \frac{H_0^{(2)}(k\rho)}{H_0^{(2)}(ka)}\right] + \frac{V}{2\pi}\int_{-\infty}^{+\infty} \frac{H_0^{(2)}(\beta\rho)}{H_0^{(2)}(\beta a)} F_0(\alpha)\, e^{-j\alpha z}\, d\alpha \tag{3.1}$$

represents the field outside the cylinder and satisfies the boundary conditions at $\rho = a$ (Stratton, 1941). Similarly

$$E_z^i(\rho, z) = \frac{V}{2\pi}\int_{-\infty}^{+\infty} \frac{J_0(\beta\rho)}{J_0(\beta a)} F_0(\alpha)\, e^{-j\alpha z}\, d\alpha \tag{3.2}$$

represents the field inside the cylinder and satisfies the boundary conditions at $\rho = a$.

In the above expressions we have defined β as follows:

$$\beta^2 = k^2 - \alpha^2 ;$$

$$ph\beta = 0, \ |\alpha| < k; \qquad (3.3)$$

$$ph\beta = -\pi/2, \ |\alpha| > k.$$

The contour of integration is along the real α axis with downward and upward indentations at $\alpha = -k$ and $\alpha = +k$, respectively.

Expressions (3.1) and (3.2) are tautologically equal at $\rho = a$. A second condition is required if the solution is to yield any information. We require continuity of H_ϕ through the slot. Modes of the magnetic field can be found by applying the operator

$$-\frac{jk^2}{\mu\omega(k^2 - \alpha^2)}\frac{\partial}{\partial\rho} \qquad (3.4)$$

to each mode of E_z with the understanding that $\alpha = 0$ for the incident and reflected Hankel functions. The continuity condition on H_ϕ results in

$$\frac{4E_0}{\pi Z_0 KD(K)} - \frac{V}{2\pi a} I_w^0(z) = \frac{V}{2\pi a} I_w^i(z), \qquad (3.5)$$

where we have abbreviated by letting

$$ka = K, \ Z_0 = \sqrt{\mu/\epsilon},$$

$$D(K) = J_0^2(K) + Y_0^2(K),$$

$$I_w^0(z) = \frac{K}{Z_0}\int_{-\infty}^{+\infty} \frac{H_1^{(2)}(\beta a)}{j\beta H_0(\beta a)} F_0(\alpha)\, e^{-j\alpha z}\, d\alpha, \qquad (3.6)$$

$$I_w^i(z) = \frac{K}{Z_0}\int_{-\infty}^{+\infty} \frac{jJ_1(\beta a)}{\beta J_0(\beta a)} F_0(\alpha)\, e^{-j\alpha z}\, d\alpha. \qquad (3.7)$$

Equation (3.5) is an integral equation on the interval $|z| < w$ for the function $f_0(z)$. The first term is the surface current density on a cylinder in a scattering problem for a cylinder with no slot. $I_w^0(z)$ and $I_w^i(z)$ are the total outer and inner currents on a transmitting antenna with unit voltage impressed across the slot (Duncan, 1962)* by means of an idealized generator. In the slot these quantities are not actually currents but no ambiguity results from using the same functional notation on the entire z domain.

---

*A reference manual of data for the infinite cylindrical antenna is in preparation.

We shall not attempt a rigorous solution of the integral equation. A convenient procedure is to assume a form for $f_0(z)$, require that (3.5) be satisfied at $z = 0$, and solve for the parameter V. We obtain

$$V = \frac{8E_0 a}{Z_0 KD(K)} \frac{1}{I_w^0(0) + I_w^i(0)} . \qquad (3.8)$$

A convenient choice for $f_0(z)$ is

$$f_0(z) = \frac{1}{2W}, \quad |z| < w,$$

$$= 0, \text{ otherwise;} \qquad (3.9)$$

with

$$F_0(\alpha) = \sin \alpha w/\alpha w .$$

A somewhat better choice (Wait, 1959) which satisfies appropriate edge conditions near $z = \pm w$ is

$$f_0(z) = \frac{1}{\pi\sqrt{w^2 - z^2}}, \quad |z| < w$$

$$= 0, \text{ otherwise;} \qquad (3.10)$$

with

$$F_0(\alpha) = J_0(\alpha w).$$

Final numerical results are not very sensitive to the choice of $f_0(z)$; we use (3.9) in the analysis.

We now turn attention to the evaluation of $I_w^0(0)$. If we set $F_0(\alpha) = 1$ in (3.6) we obtain

$$I_\delta^0(z) = \frac{K}{Z_0} \int_{-\infty}^{+\infty} \frac{H_1(\beta a)}{j\beta H_0(\beta a)} e^{-j\alpha z} d\alpha, \qquad (3.11)$$

the outer current on a δ-gap transmitting antenna for which the boundary value of E_z is given by $E_z = -\delta(z)$. From the form of (3.6),

$$I_w^0(z) = \int_{-\infty}^{+\infty} I_\delta^0(z_0) f(z - z_0) dz_0 . \qquad (3.12)$$

Since all of the functions involved are even and $f_0(z)$ vanishes for $z > w$,

$$I_w^0(0) = 2\int_0^w I_\delta^0(z_0) f(z_0) dz_0 . \qquad (3.13)$$

Thus, the net effect of the window function is to require a smoothed version of the current distribution for a δ-gap model. The latter has a short-range logarithmic singularity at the origin. If w exceeds the range of the singularity, z_s, and is not too large, the value of $I_w^0(z)$ is not very sensitive to the choice of $f_0(z)$, it being understood, of course, that $f_0(z)$ must be an integrable function. Since $f_0(z)$ is not actually known in the present problem we shall obtain $I_w^0(0)$ by the graphical smoothing procedure which also serves to define z_s. Letting Y_s represent the admittance corresponding to the smoothed δ-gap current function,

$$V \simeq \frac{8E_0 a}{Z_0 KD(K)} \left[\frac{1}{Y_s + I_w^i(0)}\right], \quad z_s < w << \lambda. \tag{3.14}$$

This approximation is incorrect in the limit of very narrow slots. If $w < z_s$, the logarithmic nature of $I_\delta(z)$ must be taken into account explicitly. It is known that for small z

$$I_\delta^0(z) + I_\delta^i(z) \approx -j \frac{4K}{Z_0} \ell n \; kz. \tag{3.15}$$

When this is averaged over the window function,

$$I_w^0(0) + I_w^0(0) \approx -j \frac{4K}{Z_0} \ell n \; kw, \quad w < z_s. \tag{3.16}$$

Preliminary to evaluating $I_w^i(0)$ consider the interior current for the δ-gap model,

$$I_\delta^i(z) = \frac{K}{Z_0} \int_{-\infty}^{+\infty} \frac{jJ_1(\beta a)}{\beta J_0(\beta a)} e^{-j\alpha z} d\alpha. \tag{3.17}$$

The integrand of (3.17) is continuous across the branch cut. Poles of the integrand are at $\beta_n a = \xi_n$, where (ξ_n) are the ordered zeros of $J_0(\chi)$. If the tube is below cutoff, all poles are on the imaginary axis at

$$\alpha_n = \pm \frac{j}{a} \sqrt{\xi_n^2 - K^2} = \pm \frac{j\gamma_n}{a}. \tag{3.18}$$

If K is greater than some ξ_n, there will be propagating modes corresponding to poles along the real axis in the interval $-k < \alpha < +k$. We define downward indentations at the poles in the interval $-k < \alpha < 0$, upward indentations for those in the interval $0 < \alpha < k$. The contour may be closed in the lower or upper half-planes corresponding to $z > 0$ and $z < 0$, respectively. The result is

$$I_\delta^i(z) = j \frac{2\pi K}{Z_0} \sum_{n=1}^{n=\infty} \frac{\exp\left(-\frac{\gamma_n |z|}{a}\right)}{\sqrt{\xi_n^2 - K^2}}. \tag{3.19}$$

From (3.7) and (3.11) we have

$$I_w^i(0) = 2 \int_0^w I_\delta^i(z_0) f(z_0) dz_0 . \tag{3.20}$$

From (3.19) and (3.20), $I_w^i(0)$ is given by

$$I_w^i(0) = j\,\frac{2\pi Ka}{Z_0 w}\sum_{n=0}^{n=\infty}\frac{1-e^{-\frac{\gamma_n w}{a}}}{\left(\xi_n^2 - K^2\right)} . \tag{3.21}$$

The rate of convergence of (3.21) can be improved. First use the Fourier inversion theorem to obtain

$$\frac{K}{Z_0}\,\frac{jJ_1(\beta a)}{\beta J_0(\beta a)} = \frac{1}{2\pi}\int_{-\infty}^{+\infty} I_\delta^i(z)\,e^{j\alpha z}\,dz. \tag{3.22}$$

Then set $\alpha = 0$ in (3.22) to obtain

$$\frac{jKJ_1(K)}{Z_0 kJ_0(K)} = \frac{1}{\pi}\int_0^{\infty} I_\delta^i(z)dz \tag{3.23}$$

and use (3.19) to calculate the right-hand side of (3.23). The result is the useful identity

$$\frac{J_1(K)}{KJ_0(K)} = \sum_{n=0}^{n=\infty}\frac{2}{\left(\xi_n^2 - K^2\right)} . \tag{3.24}$$

This may be combined with (3.21) to give

$$I_w^0(0) = \frac{j\pi Ka}{Z_0 w}\left[\frac{J_1(K)}{KJ_0(K)} - \sum_{n=0}^{n=\infty}\frac{2\,e^{-\frac{\gamma_n w}{a}}}{\left(\xi_n^2 - K^2\right)}\right] . \tag{3.25}$$

The series in (3.25) is now rapidly convergent and convenient for computational purposes except near resonance. If K is nearly equal to some ξ_r, it is only necessary to keep the dominant term in (3.21). The exponential can also be expanded to low order to give

$$I_w^i(0) \approx j\,\frac{2\pi K}{Z_0}\,\frac{1}{\sqrt{\xi_r^2 - K^2}} . \tag{3.26}$$

Components of the interior field can be computed by the residue theorem from (3.2). The presence of $F_0(\alpha)$ will cause the integral to diverge on the infinite arc used to close the contour unless $|z| > w$ no matter which window function is used. If field components are desired in the region $|z| < w$, they can be computed by convolution of the window function and the residue series for field components in the δ-gap problem just as in the above treatment of $I_w^i(0)$. The results for $|z| > w$ are:

$$E_z^i(\rho, z) = \frac{V}{a}\sum_{n=1}^{n=\infty}\frac{\xi_n J_0\left(\xi_n \frac{\rho}{a}\right)F_0(\alpha_n)}{J_1(\xi_n)\sqrt{\xi_n^2 - K^2}}\; e^{-\frac{\gamma_n |z|}{a}} , \tag{3.27}$$

$$H_\phi^i(\rho, z) = j \frac{kV}{Z_0} \sum_{n=1}^{n=\infty} \frac{J_1\left(\xi_n \frac{\rho}{a}\right) F_0(\alpha_n)}{J_1(\xi_n) \sqrt{\xi_n^2 - K^2}} e^{-\frac{\gamma_n |z|}{a}}, \tag{3.28}$$

$$E_\rho^i(\rho, z) = (\pm) \frac{V}{a} \sum_{n=1}^{n=\infty} \frac{J_1\left(\xi_n \frac{\rho}{a}\right) F_0(\alpha_n)}{J_1(\xi_n)} e^{-\frac{\gamma_n |z|}{a}} . \tag{3.29}$$

As a criterion of interior response we calculate the interior skin current density, $\mathcal{J}(z)$, from (3.28) by evaluating $H_\phi(a, z)$. Then,

$$\mathcal{J}(z) = j \frac{kV}{Z_0} \sum_{n=1}^{n=\infty} \frac{F_0(\alpha_n)}{\sqrt{\xi_n^2 - K^2}} e^{-\frac{\gamma_n |z|}{a}} . \tag{3.30}$$

The interior response is finite at resonance in spite of the appearance of (3.30). At resonance, $I_w^i(0)$, as given by (3.26), is much larger than Y_s. The result is that V approaches zero in just the right way to maintain finite fields in the interior. With either window function, $F_0(\alpha_n)$ becomes unity for the resonant term. Keeping only the dominant resonant terms in the expressions for V and $\mathcal{J}(z)$ gives

$$\mathcal{J}_r(z) \simeq \frac{4E_0}{\pi Z_0 KD(K)}, \quad K = \xi_r . \tag{3.31}$$

Note that $\mathcal{J}_r(z)$ does not contain z since $\exp\left(-\frac{\gamma_n |z|}{a}\right)$ becomes unity at resonance. At low frequency the damping factors insure that the field does not penetrate far into the tube. If the frequency is above the first resonant frequency, there will be at least one mode which propagates unattenuated in the z direction. Combination of these notions leads to the suggestive sketch shown in Figure 1 in which the envelope of resonant responses is given by (3.31). Comparison with (3.5) shows that (3.31) is just the outer skin current density in the scattering problem for an object with no slot.

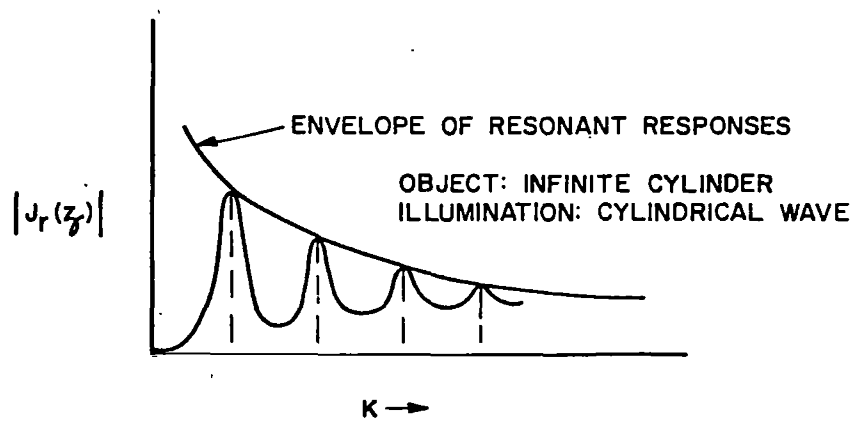

Fig. 1 -- Sketch of interior response versus K

The intimate connection between interior response and the scattering problem for an object with no leaks is a quite general result. Consider the solution of any problem similar to the one above in which an object with a leak is illuminated by a field in which $\overline{E}$ is parallel to the longitudinal axis of the object. Let the distribution of the incident field be entirely arbitrary. Introduce the interior and exterior diffracted fields as perturbations on the scattering problem for the object with no leak, as we have done above. If the leak is small with respect to a wavelength, the phase of the window field is not expected to change appreciably over the window. Introduce an approximate window function, $Vf_0(z, \phi)$. Choose a reasonable form for $f_0(z, \phi)$ and solve for the parameter V, obtaining the analog of (3.14) which contains a factor depending on the scattering problem and a second factor which depends on the interior and exterior admittances when the leak is driven by unit voltage in the transmitting mode. The interior admittance is just the interior skin current density function evaluated at the slot position. Hence, these quantities depend on the interior eigenfunctions and resonant frequencies in precisely the same way. If the problem is not azimuthally symmetric and the object is of finite length, three parameters, one for each dimension, will be required to label the interior eigenfunctions. If the interior is complicated it will not be feasible to find these eigenfunctions explicitly. However, it is sufficient for our purposes to know that they exist in principle. Whatever the nature of the complications, near resonance the terms with resonant denominators dominate the equations analogous to (3.21) and (3.28). If only dominant terms are retained, the resonant denominators cancel, leaving the factor which depends only on the incident and reflected fields evaluated at the slot position,

$$\left|\mathcal{J}_r\right| \simeq \left(H_\phi^{inc} + H_\phi^{ref}\right)_{slot}. \qquad (3.32)$$

Using these ideas we can now sketch the kind of behaviour to be expected of the interior response of a missile. For the infinite cylinder, (3.31) is monotonic. However, for a finite object, we can expect that (3.32) will exhibit resonances associated with the length of the object, as shown in Figure 2. The internal resonances exist in principle; in practice they would have to be located experimentally. An interesting possibility is that of accidental coincidence of internal and external resonances leading to an unusually large interior response. Heretofore, unexplained peaks have occurred in experimental data taken on missiles.

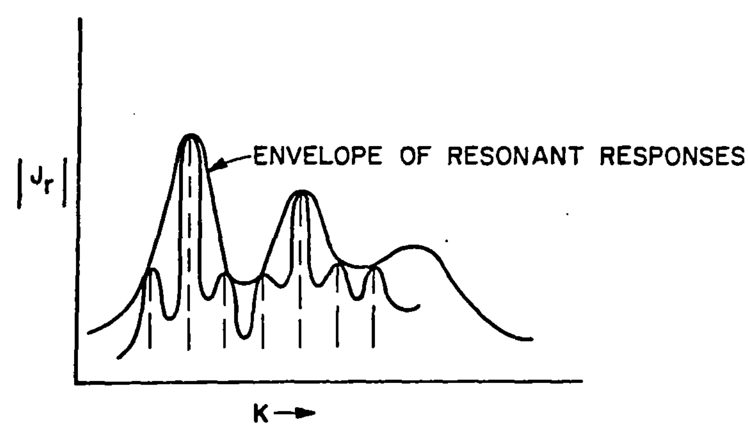

Fig. 2 -- Sketch of interior response of a finite object

An approximate formula accounting for the coupling to a simple circuit inside a cylinder is not difficult to derive. Consider the simple circuit of Figure 3.

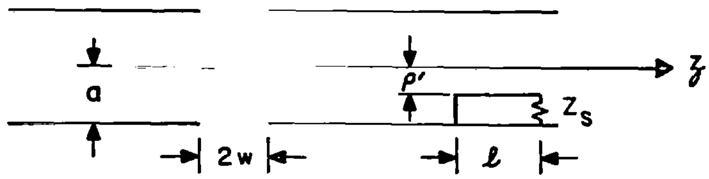

Fig. 3 -- Simple circuit inside slotted cylinder

It will be assumed that the voltage induced in the circuit is given simply by

$$V_i = \oint E \cdot ds = -j\omega\mu \int \overline{H} \cdot \overline{n} da, \tag{3.33}$$

and that the unperturbed fields (for the cylinder with no circuit) can be used in the computation. At resonance, (3.14), (3.26), (3.28), (3.33), and Ohm's law for the circuit combine to give, for the current in the circuit,

$$|I_s| = \left[\frac{4E_0}{\pi Z_0 K D(K)}\right] \frac{J_0\left(\xi_r \frac{\rho'}{a}\right) Z_0 \ell}{J_1(\xi_r) Z_c}. \tag{3.34}$$

Away from resonance, Y_s must be retained in (3.14) and the integration must include all the modes of (3.28). At high frequencies, ξ_r will be one of the large roots of $J_0(\chi)$, and $J_0\left(\xi_r \frac{\rho'}{a}\right)$ will contain several nodes. One can choose ρ' such that (3.34) is identically zero and then obtain an appreciable nonzero result by changing ρ' only slightly. This rather unphysical ambiguity is the price one pays for the simplification achieved by neglecting the higher order interactions between the circuit and the field and treating the circuit as a wire of zero diameter. The formula may, however, be useful at low frequency.

In all of the theory derived so far we have assumed a lossless interior. Under this assumption resonant terms dominate in various expressions so that a great deal of simplification is possible at the resonant frequencies. In problems involving complicated lossy interiors, arbitrary illumination, and finite objects we postulate that

$$|I_s| \simeq \left[H_\phi^{inc} + H_\phi^{ref}\right]_P \left[\frac{1}{Y_s + Y_i}\right] F(P', Z_c, f, L, a). \tag{3.35}$$

In (3.35) P is the leak location, P' is the circuit location, L and a are the length and radius of the object, and f is the applied frequency. $F(P', Z_c, f, L, a)$ is some complicated (and underivable) function characteristic of the interior of the object. In a truly complicated situation all that is left is a way of thinking about the problem which may be useful to the experimentalist. An obvious objection to (3.35) is that it is suggested by a particular case in which there is only a TM field. However, it is known experimentally that the component of the illumination which is polarized in the direction of the longitudinal axis of a missile is the most effective in producing unwanted currents in circuits inside the missile. Also, gross approximations are permissible. Any describing function which proved to be useful and reliable to within, say, ±5 db would represent an improvement over other attempts to construct a theoretical model.

It should be clear by now that this approach is a generalization of the earlier approach by Harrison which was discussed briefly in the introduction. The factor $\left[H_\phi^{inc} + H_\phi^{ref}\right]$ (times $2\pi a$) can be regarded as a constant current generator feeding admittances associated with the slot and the interior of the object. Return to the earlier theory involves regarding the illumination as a plane wave and the object as a linear antenna, connecting the load circuit across the slot, neglect of the interior fields, and, finally, the transformation from an equivalent circuit representation involving a constant current generator to one involving a voltage generator whose open circuit voltage is given by linear antenna theory.

4. Conclusions

We have demonstrated the intimate connection between the interior response of a leaky object illuminated by electromagnetic energy and the scattering solution for a congruent object with no leaks. This part of the paper will be devoted to pointing out some implications of the results for those concerned with electromagnetic interference with ordnance or other industrial problems of radio-frequency interference. The work is applicable in those situations where the location of the leak is known. Access ports on rockets and missiles provide an example.

It has been shown that interior response breaks rather neatly into three factors: the current density for a scattering problem, a factor depending on the slot admittance in the transmitting mode, and the interior response-per-unit voltage. The theory suggests an alternative to present experimental techniques which involve radiating the object and measuring the interior response. Instead it may be advantageous to study the interior response directly by attaching a feeding transmission line to the leak and normalizing the results to unit applied voltage. Measurements of this type can be carefully performed in a laboratory. Slot admittance could be measured in a realistic environment. No other measurements would be required in any situation which allowed even an approximate treatment of the scattering factor. Linear antenna theory may serve well in this regard at low frequency.

Factorization of the interior response is conceptually useful in another way. Suppose that empirical changes are made in the internal layout of a missile which reduce the levels of currents in hazardous circuits. Factorization allows the conclusion that a design improvement for one type of illumination is, in fact, an improvement for any assumed distribution of illumination.

Although we have emphasized the importance of the scattering solution, not much help can be obtained from the literature of scattering theory. Most of existing theory is concerned with far field expressions; current density on the scattering object is the quantity needed here. Although linear antenna theory may be used at low frequencies much of the scattering theory relevant to the hazards-to-ordnance problem is yet to be developed. Thick cylinder theory, both in the transmitting and scattering modes, could make a considerable contribution to the hazards problem. Another fruitful field of research would be the interaction of classical objects with illuminating fields other than plane wave. In particular, the interaction of realistic antennas with surface waves is a much-needed study. It is hoped that these remarks will help ordnance engineers to identify relevant literature should it become available in the future.

In closing, we should not leave the impression that the problem of electromagnetic leakage into missiles can be "solved" in the same sense that a well-defined problem can be solved. One only needs to glance at the complicated interior of a missile and then to imagine an exterior problem containing an electromagnetic jungle called a gantry crane to be convinced of the difficulties. Fortunately, not all situations are that complicated. In any case, the most that can be achieved is an engineering solution involving an intuitive synthesis of theory, experience, and engineering judgment.

APPENDIX A

RECEIVING CROSS SECTION OF INEFFICIENT ANTENNAS

Several texts establish the formula

$$A(\theta, \phi) = \frac{\lambda^2}{4\pi} G(\theta, \phi) \tag{A-1}$$

relating the receiving cross section and gain function of an efficient antenna. Slight modification of a typical derivation (Silver, 1949) suffices to demonstrate that (A-1) is also true for lossy antennas. We define the gain function as

$$G(\theta, \phi) = \frac{P(\theta, \phi)}{P_T / 4\pi}, \tag{A-2}$$

where $P(\theta, \phi)$ is the actual radiated power per unit solid angle and P_T is the total power injected into the antenna terminals. The ratio of $\int P(\theta, \phi)\, d\Omega$ to P_T is the transmitting efficiency, α. The receiving cross section is defined by

$$p(\theta, \phi) = A(\theta, \phi) S, \tag{A-3}$$

where S is the intensity of an incident plane wave and $p(\theta, \phi)$ is the power delivered to a load. From the reciprocity theorem for antenna patterns,

$$\frac{A(\theta, \phi)}{A_M} = \frac{G(\theta, \phi)}{G_M}. \tag{A-4}$$

In (A-4) the factors A_M and G_M serve to normalize the receiving and gain patterns. Denoting the average of $A(\theta, \phi)$ by $\overline{A}$, it is now easy to show that

$$A(\theta, \phi) = \frac{\overline{A}}{\alpha} G(\theta, \phi). \tag{A-5}$$

Now consider two antennas, a and b, with a transmitting energy to b. Under matched conditions,

$$\frac{G_a G_b \overline{A}_b}{\alpha_b} = \frac{16\pi r^2 R_a R_b |i_a|^2}{|V_a|^2}, \tag{A-6}$$

where r is the distance between a and b, R_a and R_b are the antenna resistances, i_a and V_a are the terminal current and voltage, respectively, of antenna a. Reversing the roles of the antennas as transmitter and receiver provides another equation which is derivable from (A-6) merely by interchanging the indices a and b. The ratio of these expressions, together with the relation $i_a V_b = i_b V_b$ from the reciprocity theorem, gives

$$\frac{\overline{A}_a}{\alpha_a} = \frac{\overline{A}_b}{\alpha_b}. \tag{A-7}$$

Average receiving cross section divided by transmitting efficiency is a universal constant for all antennas. Choose the constant as $\lambda^2/4\pi$ in the usual way by considering an efficient short dipole. It is now apparent that (A-1) is true for lossy antennas.

APPENDIX B

EFFECTIVE LENGTH OF AN INFINITE CYLINDRICAL ANTENNA

The basic definition of the effective length (Schelkunoff, Friis, 1952) of an antenna of length 2h is

$$\ell_{eff} = \frac{\int_{-h}^{+h} I(z)dz}{I(0)} , \tag{B-1}$$

where I(z) is the transmitting current distribution along the antenna and I(0) is the input current at the antenna terminals. It is customary in antenna theory to introduce an idealized driving generator such that the electric field in a narrow gap in the antenna surface at $z = 0$ is given by $-V\delta(z)$. The result of this idealization is a short-range singularity in I(z). Strictly speaking, (B-1) is identically zero for the δ-gap theoretical model. The significance of the singularity and means for removing it have been discussed previously (Duncan, 1962). We shall replace (B-1) by

$$\ell_{eff} = \frac{1}{Y_s}\int_{-\infty}^{+\infty} I(z)dz, \tag{B-2}$$

where Y_s is defined by a procedure which smooths out the singularity inherent in the δ-gap model. Values of Y_s corresponding to several values of K are given in the reference. With (3.11) and the Fourier inversion theorem one obtains

$$\frac{K}{Z_0}\,\frac{H_1(\beta a)}{j\beta H_0(\beta a)} = \frac{1}{2\pi}\int_{-\infty}^{+\infty} I(z)\,e^{j\alpha z}\,dz. \tag{B-3}$$

From (B-3) it follows by setting $\alpha = 0$ that

$$\int_{-\infty}^{+\infty} I(z)dz = \frac{2\pi K}{Z_0}\,\frac{H_1(K)}{jkH_0(K)} . \tag{B-4}$$

Now (B-4) can be combined with numerical data for Y_s according to (B-2) to give the effective length of an infinite cylindrical antenna. Curiously, it turns out that $|\ell_{eff}| \simeq \lambda/4$.

REFERENCES

Brown, G. H. and King, R. W. P., "High-Frequency Models in Antenna Investigations," Proc. IRE, 22, pp 457-480, April 1934.

Duncan, R. H., "Theory of the Infinite Cylindrical Antenna Including the Feed-Point Singularity in Antenna Current," J. Research NBS 66D (Radio Prop.) No. 2, pp 181-188, March-April 1962.

King, R. W. P. and Harrison, C. W., Jr., "Cylindrical Shields," IRE Transactions on Antennas and Propagation, AP-9, No. 2, pp 166-170, March 1961.

Schelkunoff, S. A. and Friis, H. T., Antennas Theory and Practice, John Wiley and Sons, Inc., p 301, 1952.

Silver, S., Microwave Antenna Theory and Design, McGraw-Hill Book Company, Inc., p 45, 1949.

Stratton, J. A., Electromagnetic Theory, McGraw-Hill Book Company, Inc., p 360, 1941.

Wait, J. R., Electromagnetic Radiation From Cylindrical Structures, Pergammon Press, 1959.

Issued by
Technical Information Division
Sandia Corporation
Albuquerque, New Mexico