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ELECTRON RING ACCELERATOR INTERNAL REPORT NUMBER 1

A PHENOMENOLOGICAL DESCRIPTION OF SPARK FORMATION

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The intention of this note is to describe the formation of a spark from the time when appreciable current flows, that is, when a plasma has already formed between the electrodes. The mechanism is imagined as involving the storage of energy in the gas, in the form of electron kinetic energy, excited, ionized, or dissociated molecules, etc., which is manifested as finite resistivity. The energy deposition process stops when the resistance of the gas is substantially below that of the circuit. The time scale is taken as short enough that the ions and atoms can be considered to have infinite mass. We assume that conditions are homogeneous within the spark channel and that the energy outside the channel is zero. The cross sectional area of the channel A is not known, and it is left undetermined and supposed constant in time. The energy is described as a mean energy per molecule $\overline{E}$ which is of the order of (but materially greater than, by virtue of unproductive inelastic processes) the ionization energy times the fractional ionization f.

If W is the energy in the channel and d is its length, we have

$$W = n_o A d \overline{E}$$

and so

$$dW/dt = n_o A d (d\overline{E}/dt)$$
$$= V^2 A/\rho d,$$

where n_o is the molecular number density, ρ is the mean resistivity, and V is the gap voltage. Hence

$$d\bar{E}/dt = V^2/n_o d^2 \rho .$$

If R is the external circuit resistance,

$$V = V_o - iR = V_o - RV/(\rho d/A),$$

where V_o is the open-circuit generator voltage. Therefore

$$d\bar{E}/dt = (V_o{}^2/n_o d^2 \rho)(1 + AR/\rho d)^{-2}.$$

The resistivity is a function of the temperature T and consequently of the mean energy per molecule $\bar{E}$. We suppose that

$$\rho = 10^{-9}(mc^2/n_- e^2)(\nu_+ + \nu_o)$$

where collision frequencies ν refer to ions and neutrals. For the ion component we use Spitzer's[1] formula for singly charged ions and for the neutral collision frequency the formula[2]

$$\nu_o = n_o \sigma_o (8kT/\pi m)^{\frac{1}{2}},$$

where σ_o is the molecular elastic cross section. It is assumed that $n_+ = n_-$. The result is

$$\rho = 6.5 \times 10^3 LT^{-3/2} + 2.4 \times 10^9 \ \sigma_o T^{\frac{1}{2}}/f, \qquad (1)$$

where L is a Coulomb logarithm of the order of 3 and f is the fractional ionization. This can be reasonably well fitted by a formula of the form

$$f = \alpha e^{-\beta/T}$$

(for atmospheric air[3] $\alpha \sim 1.4$, $\beta \sim 5.8 \times 10^4\ ^oK$). Figure 1 shows the resistivity of air at atmospheric density according to (1) with L = 3. Experimental estimates of spark resistivity based on photographic measurement of channel width have given values of 7.2×10^{-3} and 2×10^{-2} ohm-cm for 8 and 35-mil-long sparks.

At final temperatures of present interest the ionic and neutral components of the resistivity are about equal, though during the formative stage of the spark the second term dominates. In view of the great analytical simplification which results when the first term is dropped, we shall suppose it to be negligible, recognizing that some error may be incurred in the latter stages of formation. We also assume as stated previously

$$\bar{E} = E_o f.$$

Then

$$d\bar{E}/dt = E_o df/dt \sim \rho_o E_o\ d/dt(1/\rho) = (V_o^2/n_o d^2)(1/\rho)(1 + AR/\rho d)^{-2}$$

or

$$ds/d\tau = s(1 + s)^{-2}, \qquad (2)$$

where $\rho_o \equiv 2.4 \times 10^9\ \sigma_o T^{\frac{1}{2}}$, s is the normalized conductance $AR/\rho d$, and $\tau \equiv t/(\rho_o E_o n_o d^2/V_o^2)$. The quantity ρ_o is of course not strictly constant, since it depends on $T^{\frac{1}{2}}$. Reference to figure 1, however, shows that most of the spark formation occurs over a fairly restricted range of temperatures, so that in first approximation an effective value of this parameter can be used. With $\sigma_o = 7 \times 10^{-16} cm^2$, $T = 10^4\ ^oK$ we have $\rho_o = 1.7 \times 10^{-4}$ ohm-cm (note: at $10^4\ ^oK$, $f \sim 0.004$, so that $\rho \sim 0.04$ ohm-cm due to neutrals only, the ionic contribution at this temperature being about 0.02 ohm-cm). If one takes $E_o = 30$ ev $= 5 \times 10^{-18}$ joule and $V_o/d = 30$ kV/cm, the characteristic time $t_o = t/\tau$ is about 3×10^{-11} sec.

Figure 2 shows a solution of (2). It is seen that if the pulse duration greatly exceeds about $20t_o$, one may expect a reasonably square pulse, attaining the order of 90% of the supply voltage at $t/t_o = 70$. Pulse distortion is severe if the duration is of the order of $20t_o$ or less. Experimental results with an air spark with $d \sim 20$ mils at one atmosphere show an effective rise time of the order of 2 nsec, implying a corresponding value of t_o of about 10^{-10} sec, about three times the value given above. It should be noted, however, that the figure of 30 ev per secondary electron holds only for energetic primaries, the required energy per secondary being higher for low-energy primaries. The fit would be satisfactory of E_o were taken to be about 100 ev for $V_o/d = 30$ kV/cm.

It should be noted that the breakdown field is a function[4] of the gap length d (figure 3; these figures are for polished electrodes, and a substantial factor for local field enhancement should be allowed). Because of the square-law dependence of $1/t_o$ on breakdown field V_o/d it is easy to explain the superior resolution at 1.5-mil gap spacing observed in air as compared with larger spacings; here the breakdown field is more than twice that at 20 mils so that, using $E_o = 100$ ev, t_o is about 0.2×10^{-10} sec.

Since the characteristic time can be written

$$t_o = (\rho_o E_o/n_o)(n_o d/V_o)^2,$$

the second paranthesis being independent of pressure at breakdown, it is implied that the performance of the switch should improve at increased pressures when the gap separation is maintained constant. This point needs to be checked experimentally.

REFERENCES

1. Spitzer, L.J., Physics of Fully Ionized Gases, Interscience, New York, 1956, p. 84.

2. Drummond, J.E. (ed.), Plasma Physics, McGraw-Hill, New York, 1961, p. 248.

3. Ibid p. 251.

4. Brown, S.C., Basic Data of Plasma Physics, MIT Press, Cambridge, 1959, p. 240.

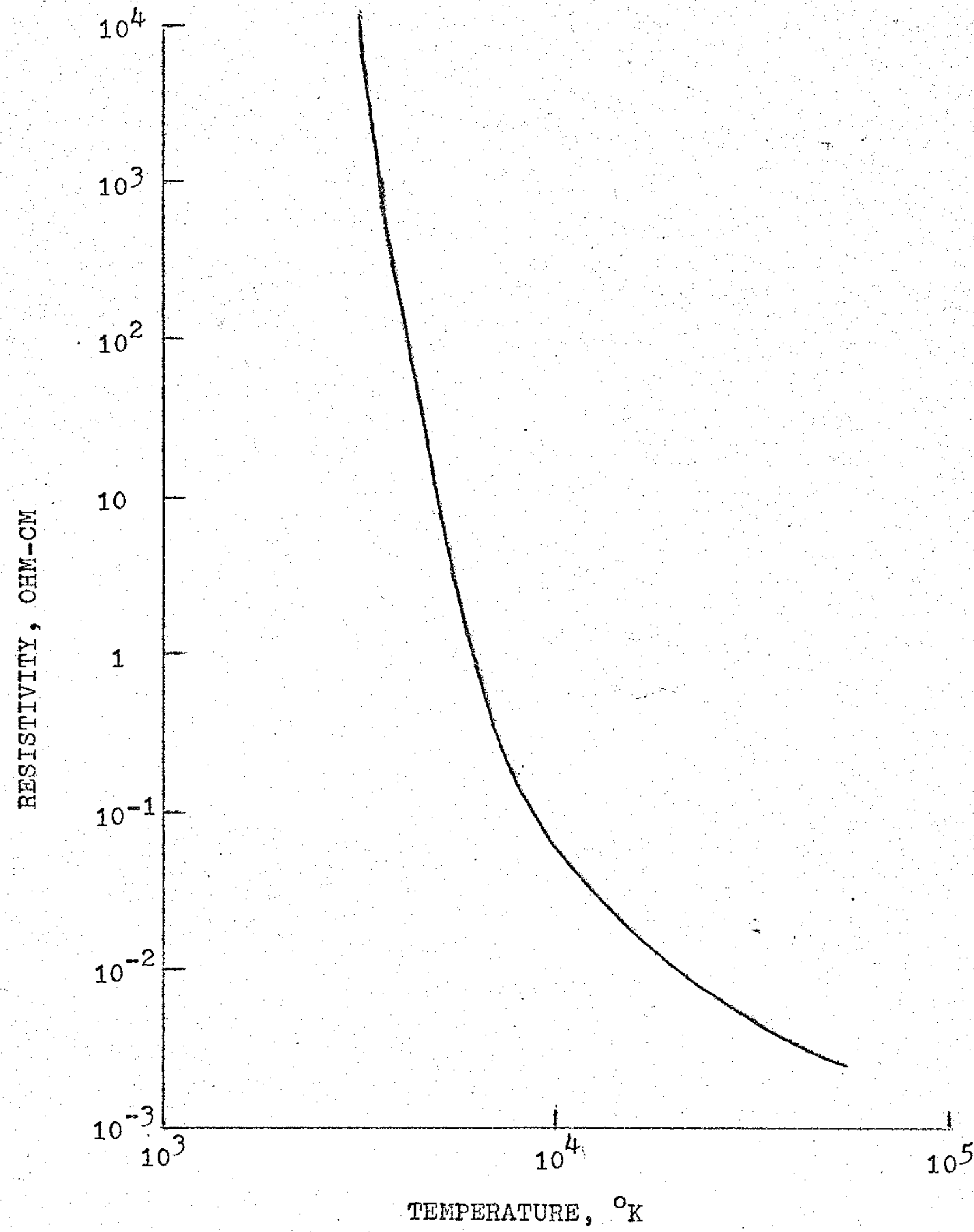

Figure 1. Resistivity of heated air according to (1) with L = 3

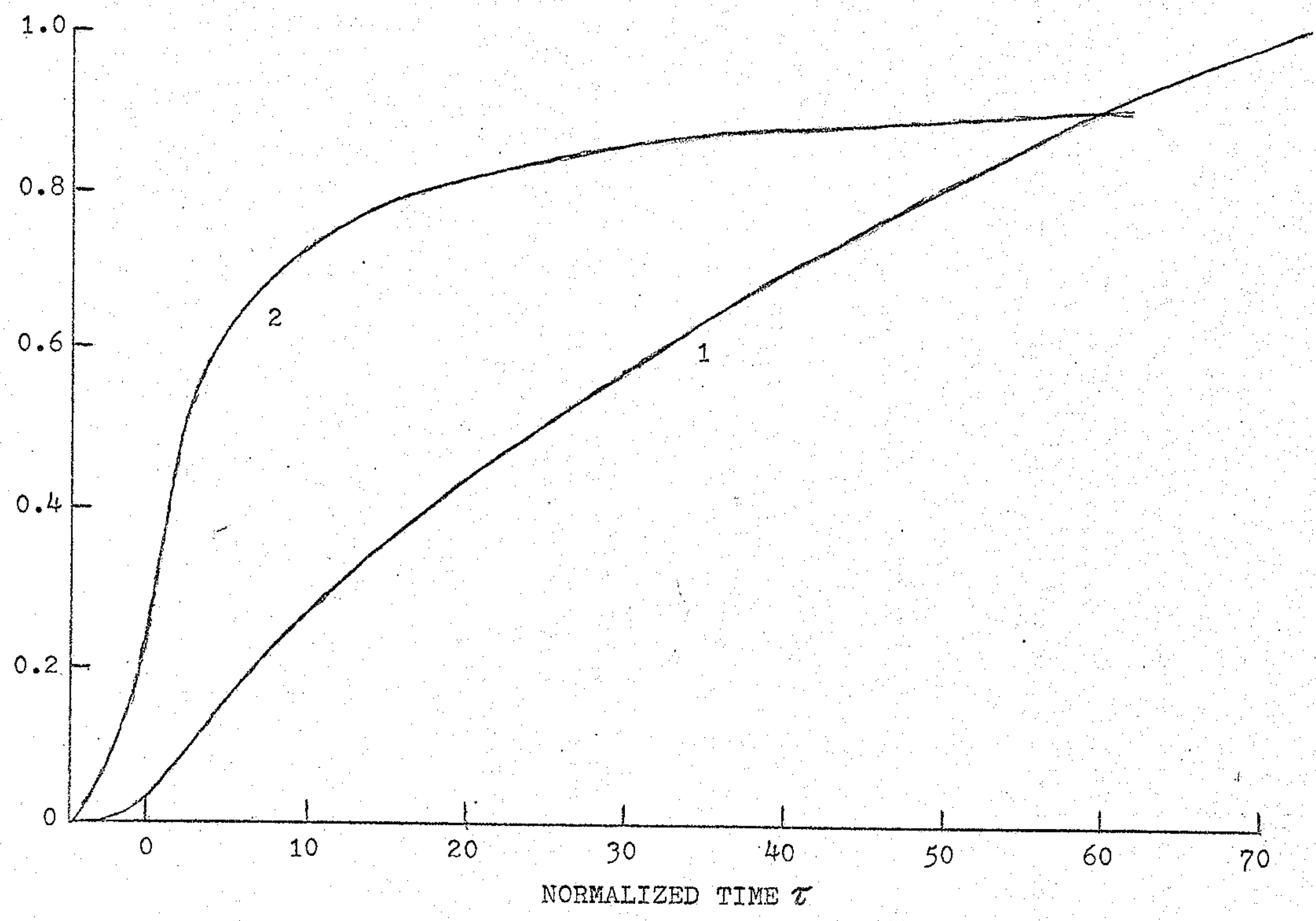

Figure 2. Solution of (2). Curve 1 is the normalized conductance; curve 2 is the normalized load voltage $s/(s + 1)$

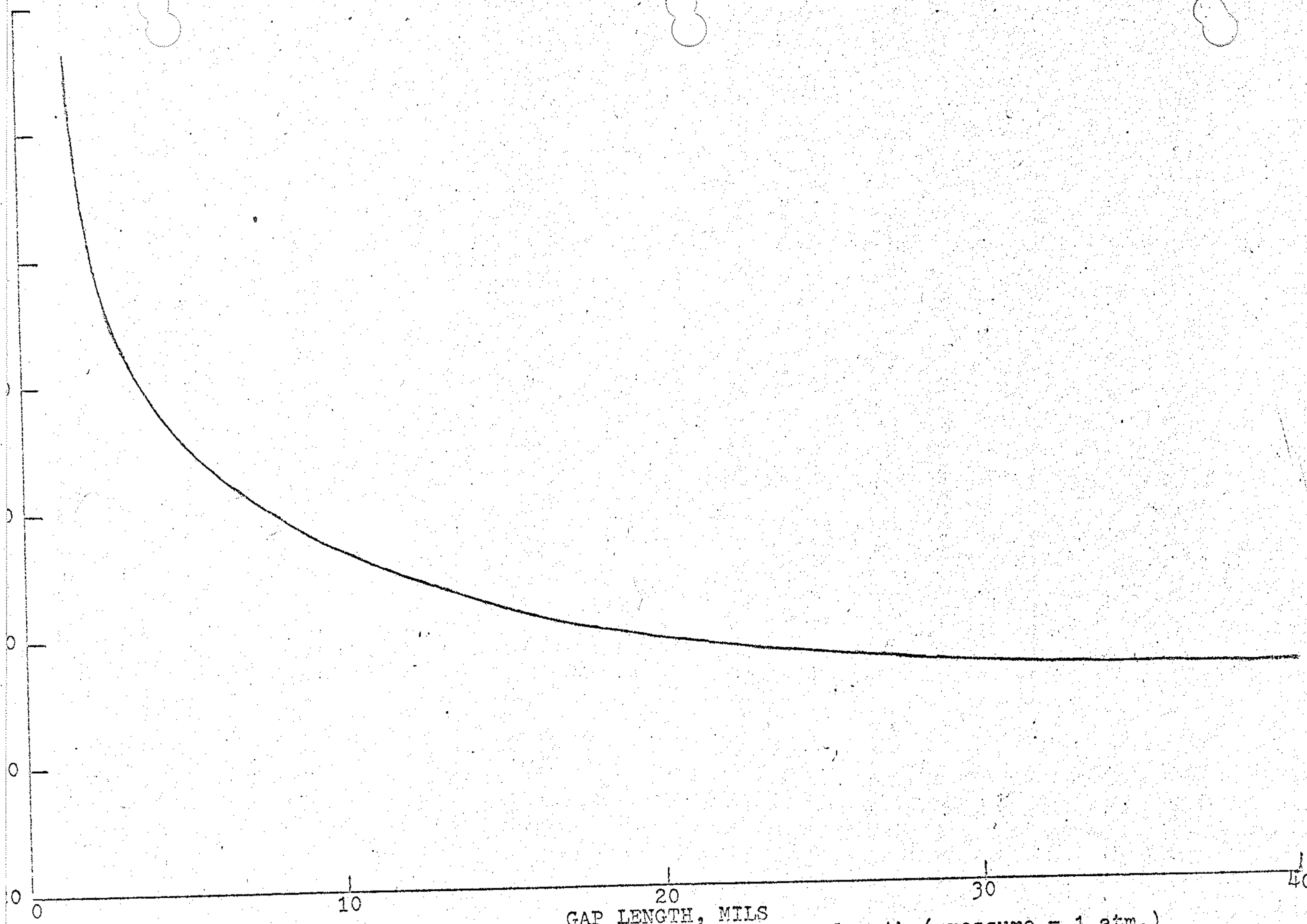

Figure 3. Air breakdown field strength as a function of gap length (pressure = 1 atm.)