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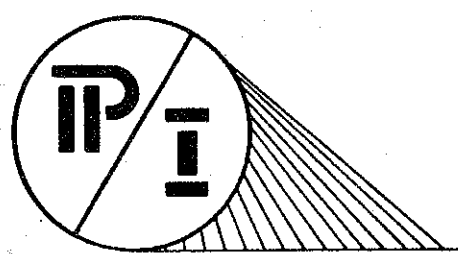
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Summary

This report contains an analytical formulation for J. C. Martin's treatment of the way in which the statistical fluctuation of the electrical breakdown strength in solids is related to the volume dependence of the mean breakdown field. The principal result is a relation between σ' and m: where σ' is the standard deviation of the breakdown field strength for a given sample and m is the constant in the volume-dependence expression

$$\overline{F} = k/(v)^{1/m}$$

relating the mean breakdown field strength, $\overline{F}$, to the volume, v (k and m are constants for a given material).

In addition, some expressions for non-uniform fields are derived. An equivalent volume is defined so that the non-uniform field problem can be treated in the same way as the uniform field case.

1. Introduction

The theory of electrical breakdown in solids has been the subject of extensive investigation (References 1 and 2 are reviews of the literature). The process can be divided somewhat arbitrarily into thermal and intrinsic breakdown.

Thermal breakdown is brought on by the rise of temperature resulting from joule heating of the material in which conduction currents are flowing. This kind of breakdown is characterized by voltages which are applied continuously or for long pulse

lengths. To be precise, one must talk about a breakdown voltage for a given geometrical arrangement rather than a breakdown field strength. The reason is that the transport of heat out of the sample is an important consideration. A decrease in the pulse length of the applied voltage produces an increase in the thermal breakdown strength.

When the pulse length of the applied voltage is sufficiently short (or the conductivity of the material is low), one finds a different kind of phenomena (i.e., intrinsic breakdown) governing the breakdown process. This kind of breakdown can be characterized by a breakdown field strength, since the process is relatively insensitive to the geometrical arrangement of the electrodes (for uniform fields) or the thickness of the sample. The values of the field strength for intrinsic breakdown are, in general, higher than for thermal breakdown. A decrease in pulse length in the 10^{-6} to 10^{-8} second region has little effect on the value of the intrinsic breakdown field strength.

In this report, we are primarily interested in the intrinsic breakdown process. Many of the theoretical explanations for the intrinsic breakdown process predict sharply defined values of breakdown field strength (i.e., no statistical spread) and a complete independence of the geometrical arrangement of the electrodes, the sample thickness, etc. (Reference 3). However, other breakdown theories predict that the process is inherently statistical in nature and that instead of a well-defined breakdown field strength one should talk about the probability of breakdown as a function of field strength (References 4 and 5). Recently, there has been more evidence of the statistical nature of the breakdown process (References 6 and 7) as well as more data on the well-known thickness variation of the mean breakdown strength of thin films (Reference 8).

J. C. Martin (Reference 9) has pointed out that a real statistical spread in the measured values of intrinsic breakdown field strength (not measurement errors) implies that the mean value of the breakdown field strength would decrease with increasing volume of the sample.

The reasoning behind this conclusion becomes apparent when one plots the probability of survival of a given sample against the applied field strength (solid curve in Figure 1). If one placed ten of these samples in parallel or in series, then presumably, breakdown of the combination would occur when the sample with lowest breakdown strength failed. The probability of survival for the combination can be calculated by taking the product of the probabilities of the individual samples.

The dotted curve in Figure 1 shows the result of raising the probability for a single sample to the tenth power. The mean breakdown field is shifted to a lower value.

From the empirical data presented in Reference 9, Martin draws two important conclusions about the volume dependence of the breakdown field strength. The first is the relation for the mean breakdown field strength

$$\overline{F} = k/(v)^{1/m} \tag{1}$$

where v is the volume of the sample, and k and m are constants for a given insulator.

The second conclusion is that the probability distribution of the normalized breakdown strength, $F/\overline{F}$, is independent of the volume. In particular, this volume independence means that one should
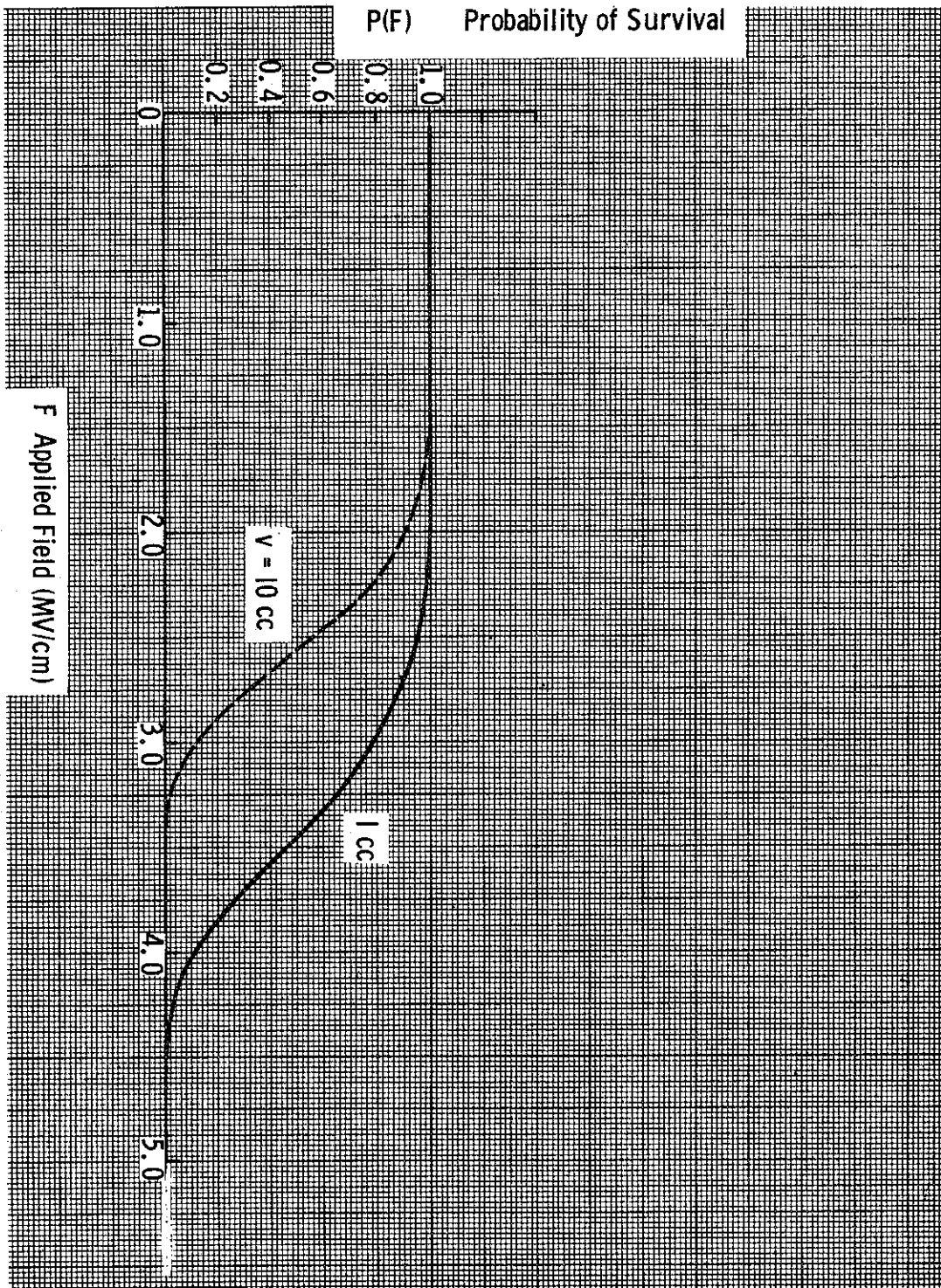
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Figure 1.   Probability of Survival Versus Applied Field Strength

be able to calculate the standard deviation, σ, for the normalized distribution, and then the standard deviation for the probability distribution corresponding to any volume is given simply by

$$\sigma' = \sigma\overline{F} \tag{2}$$

where σ is independent of the volume (see Appendix 1).

I. Smith (Reference 10) points out that one can obtain an analytical expression for a probability distribution with the two properties cited above by the following argument: Let $P\,(F/\overline{F})$ be defined as probability that a sample with mean breakdown field $\overline{F}$ will survive an applied field F. $P\,(F/\overline{F})$ is assumed to be independent of the volume. Suppose that $\overline{F}_1$ is the mean breakdown field for a volume v_1. If the volume were increased to a new volume $v_2 = cv_1$ with breakdown strength $\overline{F}_2$ then

$$\left[P\,(F/\overline{F}_1)\right]^C = P\,(F/\overline{F}_2)$$

because of the way in which probabilities are combined. From Equation 1

$$\overline{F}_2 = k/v_2^{1/m} = k/(C^{1/m}\,v_1^{1/m}) = \overline{F}_1/C^{1/m}$$

so that

$$\left[P\,(F/\overline{F}_1)\right]^C = P\,(C^{1/m}\,F/\overline{F}_1) \tag{3}$$

One function which has the property expressed in Equation 3 is

$$P\,(F/\overline{F}) = \exp\left[-a\,(F/\overline{F})^m\right] \tag{4}$$

In the next section, we will derive expressions for the constant a and the standard deviation σ for the function in Equation 4.

It turns out that σ is a function of m only. This result allows one to calculate the mean breakdown field strength for any volume once the mean breakdown field $\overline{F}$ and the standard deviation $\sigma' = \sigma\overline{F}$ have been measured for a given sample.

2. <u>Standard Deviation for the Normalized Probability Distribution</u>

Define: $f = F/\overline{F}$ (5)

$$P\ (f) = e^{-af^m}$$

$P\ (f)$ = Probability of surviving a normalized field f

$p\ (f)\ df$ = Probability that breakdown will occur in an internal (f, f + df)

then

$$P\ (f + \Delta f) = P\ (f) - p\ (f)\ \Delta f$$

$$\frac{dP}{df} = -\ p\ (f)$$

The probability function is properly normalized

$$\int_0^\infty p\ (f)\ df = -\ \int_0^\infty \frac{dP}{df}\ df = -\ \exp\ (-af^m)\Big]_0^\infty = 1.0$$

The mean value of the normalized field strength is

$$\overline{f} = \int_0^\infty f\ p\ (f)\ df = -\ \int_0^\infty f\ \frac{dP}{df}\ df \qquad (6)$$

One can integrate this by parts

$$\int_0^\infty f \frac{dP}{df} df = fP\Big]_0^\infty - \int_0^\infty P \, df$$

where

$$fP\Big]_0^\infty = 0$$

Since by L'Hospital's rule

$$\lim_{f \to \infty} fP = \lim_{f \to \infty} \frac{f}{\exp(af^m)} = \lim_{f \to \infty} \frac{1}{amf^{m-1} \exp(af^m)} = 0$$

Also using Equation A2 in Appendix 2

$$\int_0^\infty P \, df = \int_0^\infty \exp(-af^m) \, df = \frac{\Gamma(1/m + 1)}{a^{(1/m)}} \qquad (7)$$

where Γ is the usual gamma function defined by

$$\Gamma(n) = \int_0^\infty x^{n-1} e^{-x} dx \qquad n > 0$$

From the definition of Equation 5, $\bar{f} = 1$ (see Appendix 1). If this is combined with Equations 6 and 7 one gets the result that

$$a = \left[\Gamma(1/m + 1)\right]^m \qquad (8)$$

The standard deviation for the normalized distribution is given by

$$\sigma^2 = -\int_0^\infty (f - \bar{f})^2 \frac{dP}{df} df \qquad (9)$$

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To integrate by parts, let

$$u = (f - \bar{f})^2 \qquad\qquad dv = \frac{dP}{df}\, df$$

$$du = 2\,(f - \bar{f})\, df \qquad\qquad v = P$$

$$\int_0^\infty (f-\bar{f})^2\, \frac{dP}{df}\, df = (f-\bar{f})^2\, \exp(-af^m)\Big]_0^\infty - 2\int_0^\infty (f-\bar{f})\, \exp(-af^m)\, df \quad (10)$$

Applying L'Hospital's rule as before

$$(f-f)^2\, \exp\,(-af^m)\Big]_0^\infty = -\bar{f}^2 = -\,1.0 \tag{11}$$

Also using Equations A2 and A3 of Appendix 2

$$\int_0^\infty (f-\bar{f})\, \exp(-af^m)\, df = \int_0^\infty f\, \exp\,(-af^m)\, df - \bar{f} \int_0^\infty \exp\,(-af^m)\, df$$

$$= \frac{\Gamma(2/m + 1)}{2\, a^{(2/m)}} - \bar{f}\, \frac{\Gamma(1/m + 1)}{a^{(1/m)}} \tag{12}$$

Combining Equations 9, 10, 11, 12, and remembering that $\bar{f} = 1.0$

$$\sigma^2 = \frac{a^{(2/m)} + \Gamma\,(2/m + 1) - 2\, a^{(1/m)}\, \Gamma\,(1/m + 1)}{a^{(2/m)}}$$

and substituting for a the expression in Equation 8 gives

$$\sigma = \left\{ \frac{\Gamma(2/m + 1)}{\left[\Gamma(1/m + 1)\right]^2} - 1.0 \right\}^{\frac{1}{2}} \tag{13}$$

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The quotient in Equation 13 is very close to 1.0 for large values of m, so that a considerable amount of accuracy is required to calculate the values of σ. The double precision FORTRAN library program for the gamma function was used with an IBM 360/44 computer to evaluate σ as a function of m. The results are plotted in Figure 2.

3.   A Probability Function for Non-Uniform Fields

All of the foregoing ideas about intrinsic electrical breakdown are intended specifically for the condition that the electric field throughout the sample is uniform. One should be cautious about applying the same formalism for non-uniform fields. There are definite polarity effects associated with some electrode shapes that give non-uniform fields and it is not clear how this polarity effect can be explained in terms of a volume-dependent breakdown process. However, it is interesting to apply the previously developed theory to dielectric samples in which the electric field is not uniform. A comparison with experimental data might be very useful for determining the applicability of the theory of Sections 1 and 2.

Let v be the volume of a dielectric with a non-uniform applied field. Consider a small element of volume Δv_i located in v. Δv_i is assumed to be sufficiently small so that the electric field is approximately uniform in Δv_i. The mean breakdown field in Δv_i is (from Equation 1)

$$\overline{F}_i = k/(\Delta v_i)^{(1/m)} \tag{14}$$

The probability of survival, P_i, for the element Δv_i when a field, F_i, is applied is (from Equation 4)

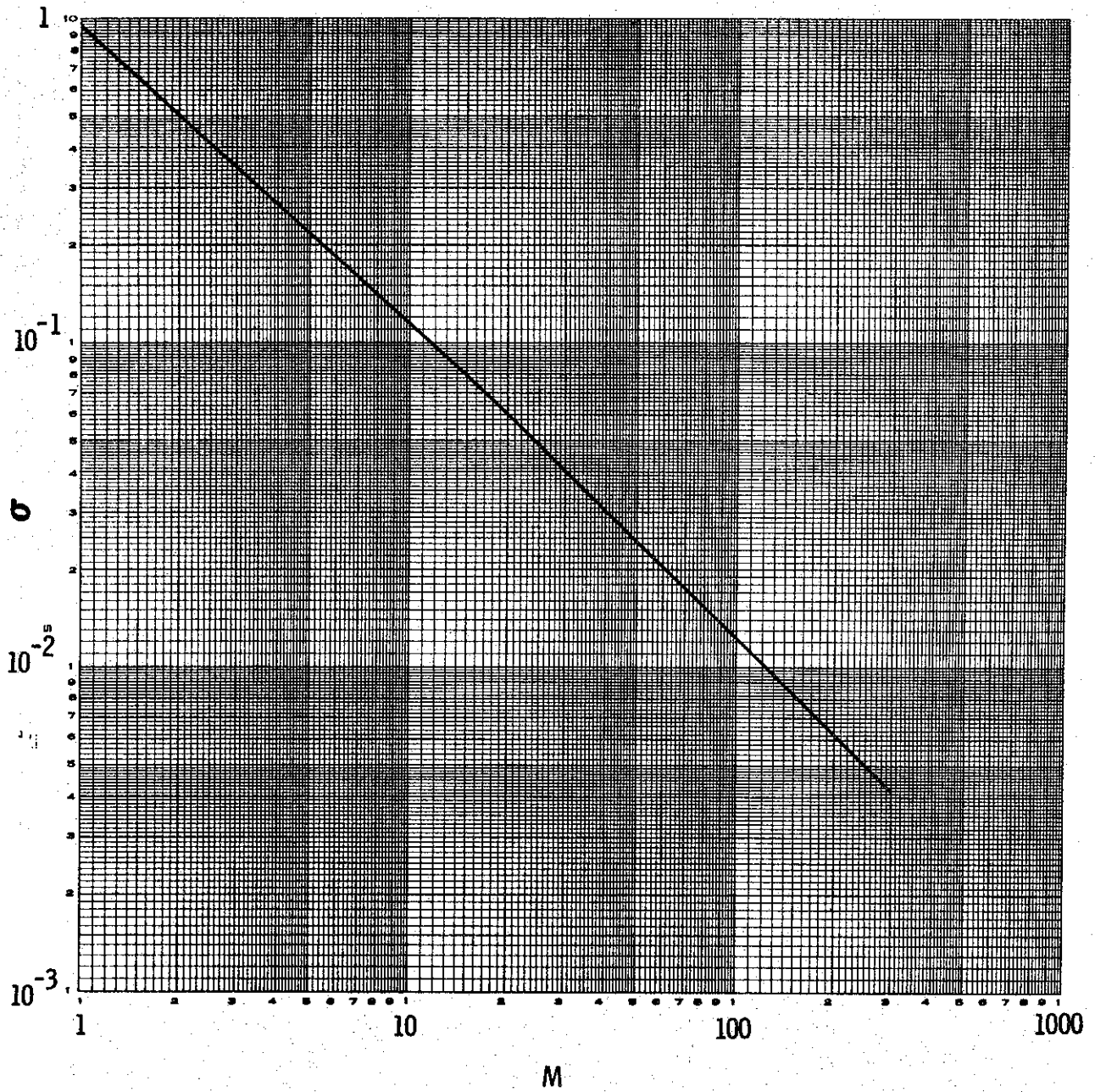$$P_i = \exp\left[-a\ (F_i/\overline{F}_i)^m\right] \tag{15}$$

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Figure 2. Standard Deviation Versus M

Combining Equations 14 and 15 and substituting for a from Equation 8 gives

$$P_i = \exp\left\{-\Delta v_i \left[\frac{\Gamma(1/m + 1)\ Fi}{k}\right]^m\right\} \qquad (16)$$

If U is the potential difference applied to the volume, v, then the probability of survival P(U,v) for the whole sample is

$$P(U,v) = \prod_{i=1}^{n} P_i = \exp\left\{-\sum_{i=1}^{n} \Delta v_i \left[\frac{\Gamma(1/m + 1)\ Fi}{k}\right]^m\right\}$$

where

$$v = \sum_{i=1}^{n} \Delta v_i$$

Taking the limit as the maximum of the elements, Δv_i goes to zero

$$P(U,v) = \exp\left\{-\left[\frac{\Gamma(1/m + 1)}{k}\right]^m \int_v F^m\, d\tau\right\}$$

It is possible to define an equivalent volume v' by letting

$$\int_v F^m\, d\tau = F_o^m \int_v (F/F_o)^m\, d\tau = F_o^m v'$$

where

$$v' = \int_v (F/F_o)^m\, d\tau$$

and F_o is the maximum field strength in v.

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The probability of survival can be expressed in terms of F_o and v' as

$$P = \exp \left\{ - v' \left[\frac{\Gamma(1/m + 1)}{k} F_o \right]^m \right\} \tag{17}$$

From Equation 16, one can see that the probability of survival in Equation 17 is the same as if a uniform field F_o were applied over the equivalent volume v'. As an example, consider the breakdown problem between coaxial cylinders. The inner and outer radii are r_1 and r_2, respectively, and the axial length is Zo.

The electric field at any point between the conductors is given by

$$F = \frac{U}{r \ln (r_2/r_1)}$$

where U is the potential difference between the conductors.

The maximum field is

$$F_o = \frac{U}{r_1 \ln (r_2/r_1)}$$

The equivalent volume v' is

$$v' = \int_V (F/F_o)^m \, d\tau = \int_V (r_1/r)^m \, d\tau =$$

$$2\pi \, Zo \int_{r_1}^{r_2} (r_1/r)^m \, r \, dr$$

$$v' = 2\pi \, r_1 \, Zo \left(\frac{r_1}{m-2} \right) \left[1 - (r_1/r_2)^{(m-2)} \right] \tag{18}$$

12          $m > 2$

The equivalent volume for this case is just the area of the inner cylinder multiplied by a fraction of the radius of the inner cylinder. For $r_2/r_1 = 2.0$ and $m = 8$.

$$v' = (2\pi\, r_1\, Zo)\, (.164\, r_1)$$

The expression in square brackets in Equation 18 is usually negligible for interesting values of m, so that the equivalent volume for coaxial cylinders is approximately the area of the inner cylinder multiplied by $r_1/(m-2)$.

In a similar way one can obtain the expression for the equivalent volume for concentric spherical electrodes (inner radius r_1, outer radius r_2)

$$v' = 4\pi\, r_1^2 \left(\frac{r_1}{2m-3}\right) \left[1 - \left(\frac{r_1}{r_2}\right)^{(2m-3)}\right]$$

REFERENCES

1. J. J. O'Dwyer, <u>The Theory of Dielectric Breakdown of Solids</u>, Oxford University Press, New York 1964.

2. S. Whitehead, <u>Dielectric Breakdown of Solids</u>, Oxford University Press, New York, 1951.

3. H. Frohlich, J. H. Simpson, "Intrinsic Dielectric Breakdown in Solids," <u>Advances in Electronics</u>, Vol. II, Academic Press, New York, 1950.

4. F. Seitz, "On the Theory of Electron Multiplication in Crystals," <u>Phys. Rev.</u>, Vol. 76, No. 9, 1376-1393, November 1949.

5. C. Zener, "A Theory of the Electrical Breakdown of Solid Dielectrics," <u>Proc. Roy. Soc.</u>, 145, pp. 523-529, 1934.

6. N. Klein, "Electrical Breakdown in Thin Dielectric Films," <u>J. Electrochem, Soc.</u>, Vol. 116, No. 7, 963-972, July 1969.

7. N. Klein, E. Burstein, "Electrical Pulse Breakdown of Silicon Oxide Films," <u>J. Appl. Phys.</u>, Vol. 40, No. 7, 2278-2740, June 1969.

8. W. S. Nicol, "Thickness Variation of Breakdown Field Strength in Plasma Oxidized Aluminum Films," <u>Proceedings of the IEEE</u>, Vol. 56, No. 1, 109-110, January 1968.

9. J. C. Martin, <u>Volume Effect of the Pulse Breakdown of Plastics</u>, Report SSWA/JCM/6511/A, A.W.R.E., Aldermaston, U.K., November 1965.

10. Ian Smith, Internal Report, Physics International, 1967.

11. H. W. Reddick, F. H. Miller, Advanced Mathematics for Engineers, John Wiley & Sons, Inc., New York, 1947.

APPENDIX 1

This section is concerned with the formal relation between the probability function for the field strength F and the probability function for the normalized field strength f

$$f = F/\overline{F}$$

$$P(F) = P\left[F(f)\right]$$

$$\frac{dP}{df} = \frac{dP}{dF}\frac{dF}{df} = \overline{F}\frac{dP}{dF}$$

$$\overline{F} = -\int_0^\infty F\frac{dP}{dF}dF = -\int_0^\infty \overline{F} f (1/\overline{F})\frac{dP}{df}\overline{F} df = \overline{F}\,\overline{f}$$

so that $\overline{f} = 1.0$

also

$$(\sigma')^2 = -\int_0^\infty (F-\overline{F})^2\frac{dP}{dF}dF$$

$$= -\overline{F}^2\int_0^\infty (F/\overline{F}-1)^2 (1/\overline{F})\frac{dP}{df}\overline{F} df$$

$$= -\overline{F}^2\int_0^\infty (f-\overline{f})^2\frac{dP}{df} df$$

$$(\sigma')^2 = \sigma^2\,\overline{F}^2$$

APPENDIX 2

To evaluate the integral

$$\int_0^\infty x^q \exp(-a\,x^m)\,dx$$

let

$$y = a\,x^m$$

$$dy = am\,x^{m-1}\,dx = am\,(y/a)^{(1-1/m)}\,dx$$

$$= a^{(1/m)}\,m\,y^{(1-1/m)}\,dx$$

then

$$\int_0^\infty x^q \exp(-ax^m)\,dx = \int_0^\infty \frac{(y/a)^{(q/m)}\,\exp(-y)}{a^{(1/m)}\,m\,y^{(1-1/m)}}\,dy$$

$$= \frac{1}{a^{(q/m + 1/m)}\,m}\int_0^\infty y^{(q/m + 1/m - 1)}\,\exp(-y)\,dy$$

$$= \left[\frac{q+1}{m}\,\Gamma\left(\frac{q+1}{m}\right)\right] \Big/ \left[(q+1)\,a^{(q/m + 1/m)}\right] \tag{A1}$$

where the gamma function is defined by

$$\Gamma(n) = \int_0^\infty z^{n-1}\,e^{-z}\,dz$$

$$n > 0$$

and has the useful property (Reference 11)

$$n\Gamma(n) = \Gamma(n+1)$$

From Equation A1 with q = o

$$\int_0^\infty \exp(-a\ x^m)\ dx = \frac{(1/m)\ \Gamma\ (1/m)}{a^{(1/m)}} = \frac{\Gamma(1/m + 1)}{a^{(1/m)}} \qquad (A2)$$

From Equation A1 with q = 1

$$\int_0^\infty x\ \exp(-a\ x^m)\ dx = \frac{(2/m)\ \Gamma\ (2/m)}{2\ a^{(2/m)}} = \frac{\Gamma(2/m + 1)}{2\ a^{(2/m)}} \qquad (A3)$$