Circuit and Electromagnetic System Design Notes

Note 64

30 October 2009

Use of the $H_{0,1}$ Mode in Circular Waveguide for Microwave Pulse Compression

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Abstract

This paper explores the use of the low-loss $H_{0,1}$ mode in circular waveguide for microwave pulse compression. Various problems concerning mode purity, and feeding in and extracting out the power are discussed.

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1. Introduction

In microwave pulse compression one limiting factor is the Q of the resonant cavity which is “filled” with energy from some lower power source, and then rapidly switched out at some much higher power. Of course, the pulse width of the output pulse is much less than the low-power charging time. This is consistent with energy conservation [5].

The power gain, G, that can be achieved has been estimated as [6]

$$
\begin{aligned}
G &\simeq \frac{1}{4\alpha \mathrm{L}} \\
\mathrm{L} &\equiv \text{length of waveguide cavity } (m) \\
\alpha &\equiv \text{exponential decay constant } (m^{-1})
\end{aligned}
\tag{1.1}
$$

Where the microwave decays as

$$
e^{-\alpha z} \tag{1.2}
$$

as the wave propagates a distance z along a waveguide. Here α depends on the waveguide cross-section shape and the surface resistance (skin effect) of the waveguide walls [16] as

$$
\begin{aligned}
R_s &= \frac{1}{\sigma \delta_s} = \sqrt{\frac{\omega \mu}{2\sigma}} \equiv \text{surface resistance} \\
\sigma &\equiv \text{wall (usually metal) conductivity} \\
\omega &= 2\pi f \equiv \text{radian frequency} \\
\mu &\equiv \text{permeability (usually } \mu_0) \\
\mu_0 &\equiv 4\pi \times 10^{-1} H / m
\end{aligned}
\tag{1.3}
$$

At 1 GHz copper has [4, 8]

$$
R_s \simeq 8.3 \ m\Omega \tag{1.4}
$$

for a typical number.

While α is proportional to L, it is also proportional to a^{-1} where a is some characteristic cross-section dimension [8, 16]. It also depends on the particular mode, and nearness to the cutoff frequency. There are also losses due to the waveguide-cavity end conductors, and apertures for feeding the power into and out of the cavity [7].

A recent paper [8] discusses how to increase the Q by making a rectangular waveguide with double the normal cross-section dimensions. Such is an overmoded cavity and care needs to be taken to not excite unwanted modes by use of symmetry and mode-suppression techniques.

In the present paper let us consider a special low-loss mode of a circular waveguide. This has found use in transmitting data long distances at the VLA (Very Large Array) near Socorro, New Mexico [12].

2. $H_{0,n}$ Modes

Our interest is in the $H_{0,1}$ mode (or $TE_{0,1}$ mode) in circular cylindrical waveguide due to its low loss and symmetry properties. The $H_{0,n}$ modes have the description [16]

$$
\begin{aligned}
H_z(\Psi,\phi) &= H_0\, J_0(k_c\, \Psi)e^{-j\beta z} \\
H_\Psi &= -H_0 \frac{j\beta}{k_c} J_0'(k_c\, \Psi)e^{-j\beta z} \\
H_\phi &= 0 \\
E_z(\Psi,\phi) &= 0 \ , \ E_\Psi = 0 \\
E_\phi(\Psi,\phi,z) &= H_0 \frac{j\omega\mu}{k_c} J_0'(k_c\, \Psi)e^{-j\beta z} \\
J_0'(k_c\, a) &= 0 \ , \ J_0'(\zeta) = -J_1(\zeta) \\
k_{c_{0,n}}\, a &= p_{0,m}' \equiv m\text{th root of } J_1 \\
J_0'\left(p_{0,m}'\right) &= 0 \\
p_{0,1}' &= 3.832 \ , \ p_{0,2}' = 7.016 \ , \ p_{0,3}' = 10.174 \\
\beta_{0,m} a &= \left[k^2 - k_{c_{0,m}}^2\right]^{1/2} a = \left[k^2 a^2 - p_{0,m}'^2\right]^{1/2} \\
k &= \frac{\omega}{\mathrm{v}} \ , \ \mathrm{v} = [\mu\varepsilon]^{-1/2} = c \text{ in free space} \\
Z_H &= Z_{TE} = -\frac{E_\phi}{H_\Psi} = Z_0 \frac{k}{\beta} \equiv \text{modal impedance} \\
Z_0 &= \left[\frac{\mu}{\varepsilon}\right]^{1/2} = \text{wave impedance of medium}
\end{aligned}
\tag{2.1}
$$

In the usual (Ψ, ϕ, z) cylindrical coordinates. Note the rotation symmetry properties of these modes. Three of the six field components are zero, and the remaining three are independent of ϕ.

These modes are significant for their low loss due to skin effect in the boundary conductors. The attenuation constant for an $H_{0,n}$ mode is [15]

$$\begin{aligned}
\alpha_{H_{0,n}}\left(\text{in } m^{-1}\right) &= \frac{R_s}{aZ_0}\left[\frac{f_c}{f}\right]^2\left[1-\left[\frac{f_c}{f}\right]^2\right]^{1/2} = \frac{R_s}{aZ_0}\left[\frac{\omega_c}{\omega}\right]^2\left[1-\left[\frac{\omega_c}{\omega}\right]^2\right]^{1/2} \\
&= \frac{R_s}{aZ_0}\left[\frac{k_{c_{0,m}}a}{ka}\right]^2 \quad \text{as} \quad \frac{\omega_c}{\omega} \to 0 \qquad (2.2) \\
&= \frac{R_s}{aZ_0}\left[\frac{p'_{0,m}}{ka}\right]^2 \quad \text{as} \quad \frac{\omega_c}{\omega} \to 0
\end{aligned}$$

From this we can see that for a very overmoded circular waveguide the attenuation becomes small proportional to a^{-2} for fixed frequency. Also from (2.1) we can see that the lowest attenuation is for $p'_{0,1}$, suggesting that the $H_{0,1}$ mode is best for this purpose. In principle one can achieve very low attenuations, thereby giving large Qs in (1.1). However, there are other problems to consider.

Maintaining two-dimensional rotation symmetry allows one to limit our attention to the $H_{0,n}$ modes. If one wishes to suppress the $n > 1$ modes one can use a cutoff waveguide filter as in [11]. In this device a large radius, *a*, is gradually shrunk (tapered) to a radius which only allows the $H_{0,1}$ mode to pass. The filter is then gradually expanded back to the larger radius. The taper should be sufficiently gradual to avoid mode conversion with the higher order modes [9]. One can extend this concept to launching and receiving this mode by such tapers at the ends of the waveguide.

It should be noted that there are various other modes that can propagate in a circular waveguide with lower cutoff frequencies than those of the $H_{0,n}$ modes. The non-$H_{0,n}$ modes can be avoided by constraining the currents in the wall to flow only in the ϕ direction. This has been accomplished to a good approximation by use of a helix-lined waveguide [10]. In this design the wire is aligned almost to $\vec{1}_\phi$, the patch angle of the helix being quite small.

3. Use of Symmetry Properties of $H_{0,n}$ Modes

The fields in (2.1 have special properties. Since E_z and E_Ψ are both zero one can insert conductors which do not conduct currents in the $\vec{1}_\phi$ direction. On a plane of constant z one can insert conductors as radial wires without interfering with any $H_{0,n}$ mode, for sufficiently thin wires as in Fig. 3.1. Furthermore, these can be extended as conducting sheets on planes of constant ϕ. Compare this to the use of such plates in rectangular waveguide [1, 3]. Such can be used as modal filters (suppressing other modes), and for dividing the $H_{0,n}$ modes into some number of sectoral (pie-wedge shaped) cross sections. Such could be used for example, to reroute the microwave power to some plane where the powers are recombined into a different waveguide cross section (e.g., rectangular). In the rerouting, the cross sections might be smoothly transitioned into other shapes.

Another related conductor configuration is that of small-radius z-directed wires. The conductors need not have significant radial (Ψ) extent.

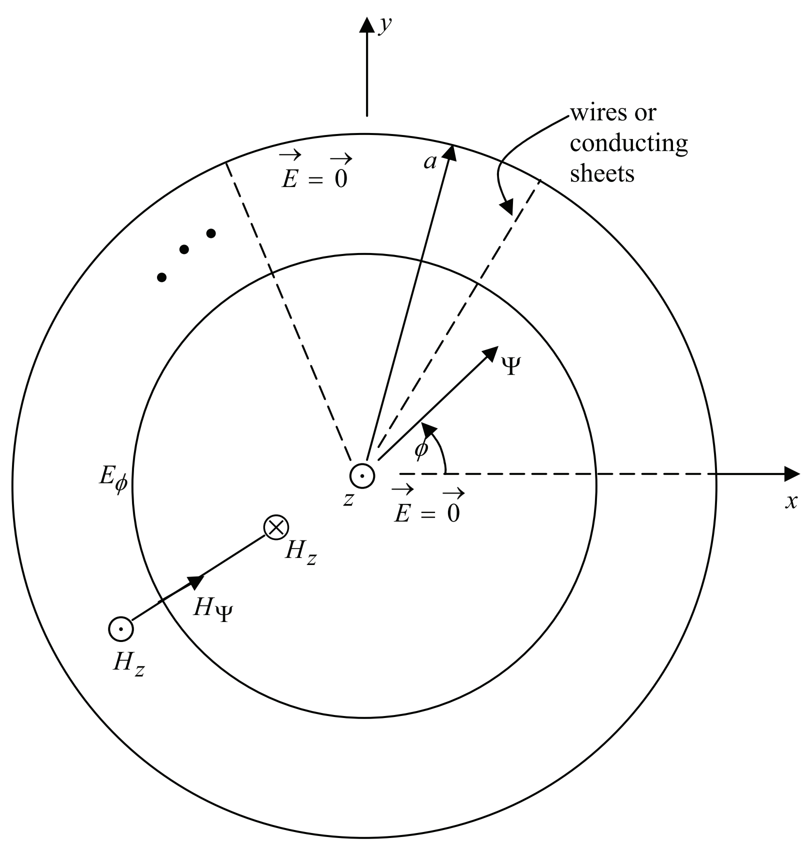

Fig. 3.1 Wires in Radial Direction, or Conducting Sheets on Planes of Constant ϕ for $H_{0,1}$ Mode

Note also the zero electric field on the z axis where a circular cylindrical conductor can be inserted giving a coaxial geometry, for which the $H_{0,1}$ mode is a higher order coaxial mode. To radiate such a mode one might borrow the COBRA concept [2, 14] to introduce appropriate different delays in different sectors to produce a circularly polarized wave radiated out of a circular conical horn antenna. The original COBRA concept uses radially (Ψ) oriented electric fields, but the principle here (ϕ oriented electric fields) is the same.

For the $H_{0,n}$ modes with $n > 1$ there are also nulls in E_ϕ corresponding to the additional roots $p'_{0,n'}$ for $1 < n' \leq n$. One could place ϕ-directed conductors at such places for $\Psi < a$ without interfering with the chosen higher-order mode. However, our interest is primarily in the $H_{0,1}$ mode.

4. Launching $H_{0,1}$ Mode from Low-Power Source

Making a cavity of length, L, out of such a waveguide we still need to couple microwave power into the $H_{0,1}$ mode. One can envision various ways to do this.

Figure 4.1 shows one way to approach this problem. Let some number N of uniformly spaced rectangular waveguides (in angle ϕ_0) with

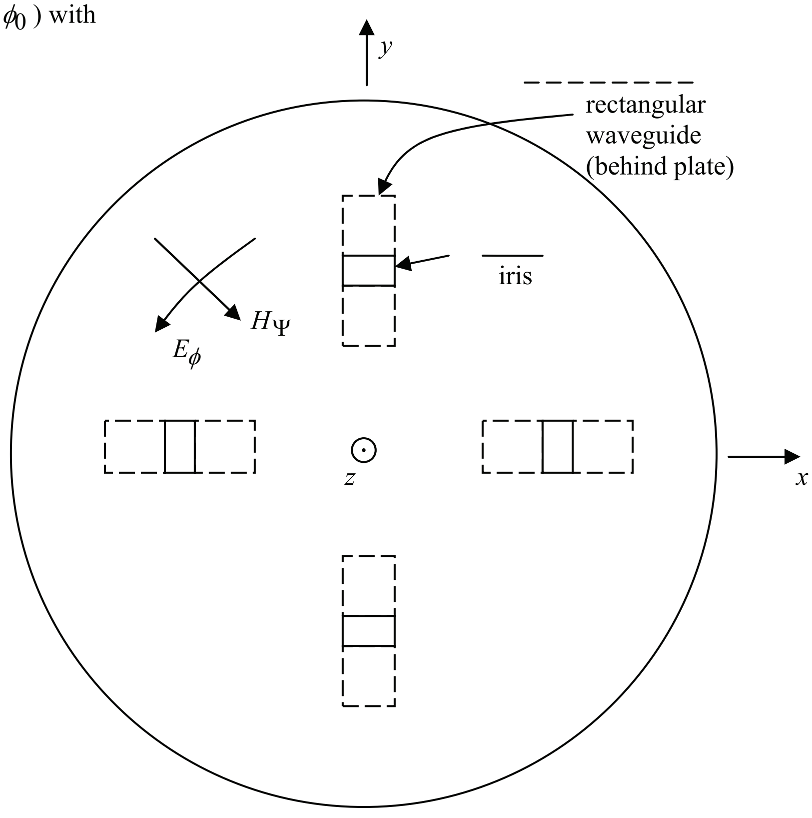

Fig. 4.1 Coupling from Rectangular Waveguides with Irises into $H_{0,1}$ Mode (Illustrated for N = 4).

$$\phi_0 = \frac{N}{2\pi} \tag{4.1}$$

feed power via identical irises (or irides) through a flat conducting plate ($\perp \vec{1}_z$) terminating the circular waveguide. This assures that modes with $\cos(\ell\phi)$ or $\sin(\ell\phi)$ variation for $\ell < N$ are not excited. Symmetry is important. For N as an integer power of 2, one can successively bifurcate a rectangular waveguide to make identical signals reach the N irises. For this purpose one can utilize symmetry planes (at $\pi/4$ and $3\pi/4$ in Fig. 4.1) to successively bifurcate rectangular waveguide as in [1]. Recalling the discussion concerning Fig. 3.1, we can think of the circular waveguide as comprising N sectoral (pie-wedge shaped) waveguides. Furthermore, we can radially position the irises for minimum coupling to the higher order $H_{0,n}$ modes.

There are also various kinds of couplers to this type of waveguide that have been used [10]. A special type of “sector coupler” is discussed in [13]. The various techniques need to be optimized to achieve the gain discussed in [6].

5. Extracting High-Power $H_{0,1}$ Modes

Extracting the power from the resonant cavity requires some kind of switch which, when open, reflects the power back. Then by closing the switch the energy should be dumped into the load in a minimum time, typically the round-trip transit time of the waveguide cavity (at the group velocity).

For this purpose, one approach is to transition into a rectangular waveguide from the circular waveguide. One approach to this is as indicated in Fig. 5.1. In this approach all the microwave power is divided into N sectoral waveguides. Each of these is in turn transitioned into a rectangular waveguide. Utilizing symmetry these N waveguides are recombined into a single rectangular waveguide. This can be accomplished by combining two waveguide at a time, provided N is some integer power of 2. The power is then switched out by standard techniques [5].

6. Concluding Remarks

The $H_{0,1}$ mode of a circular cylindrical waveguide is attractive for microwave pulse compression due to its low-loss characteristics. However, there are many problems to consider. Losses also occur at reflecting end plates, modal filters, mode launching, and mode extraction. These all need detailed consideration.

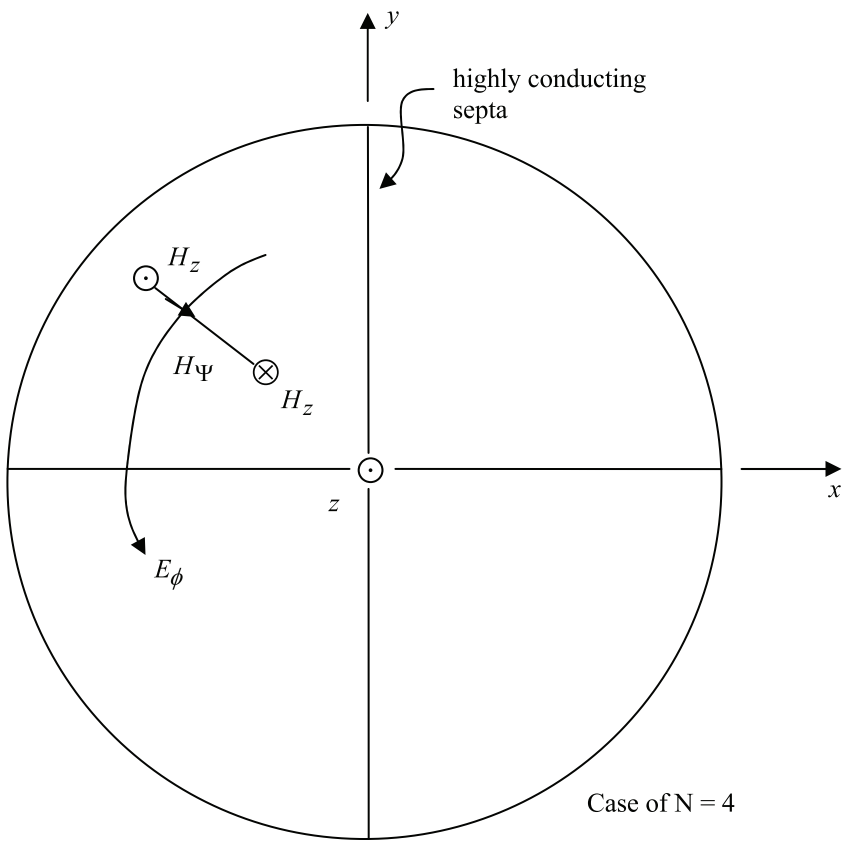

Fig. 5.1 Conducting Septa Partitioning the $H_{0,1}$ Mode into N Sectoral Waveguides.

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