Circuit and Electromagnetic System Design Notes

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The Lumped-Element Switched Oscillator

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Abstract

This paper discusses an alternate way to build a switched oscillator. The transmission-line oscillator is replaced by a lumped inductance and a lumped capacitance. This is particularly appropriate for low-frequency applications.

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1. Introduction

The switched oscillator has proved to be a useful source for high-power mesoband electromagnetic radiators [5]. In this type of source [4] one has a charged length of transmission line of low characteristic impedance Z_c and transit time t_r which has a closing switch at the opposite end of the oscillator from the (high impedance) antenna load (Fig. 1.1). This can come in single-ended (coaxial) forms or differential forms [1, 2]. Often a blocking capacitor is required between the switched oscillator and the antenna so that the antenna is not raised to a high potential (voltage) while the oscillator is charged to some high voltage V_0.

Upon the discharge of the closing switch a wave propagates back and forth in the quarter-wave transmission line, setting up an oscillation at frequency

$$f_0 = \frac{1}{4t_r} \tag{1.1}$$

For the case of a resistive antenna load Z_a with small damping we have the number of cycles N for the amplitude of the oscillation to decay to e^{-1} as [4]

$$\begin{aligned} N &= \frac{1}{4}\frac{Z_a}{Z_c} \\ Q &= \pi N \text{ (quality factor)} \end{aligned} \tag{1.2}$$

As we go lower in frequency f_0, the length of the oscillator becomes large (particularly in the case of a gas dielectric). One could fold the high-voltage transmission line, with the accompanying mechanical problems. Here we suggest an alternate solution based on a lumped-element switched oscillator.

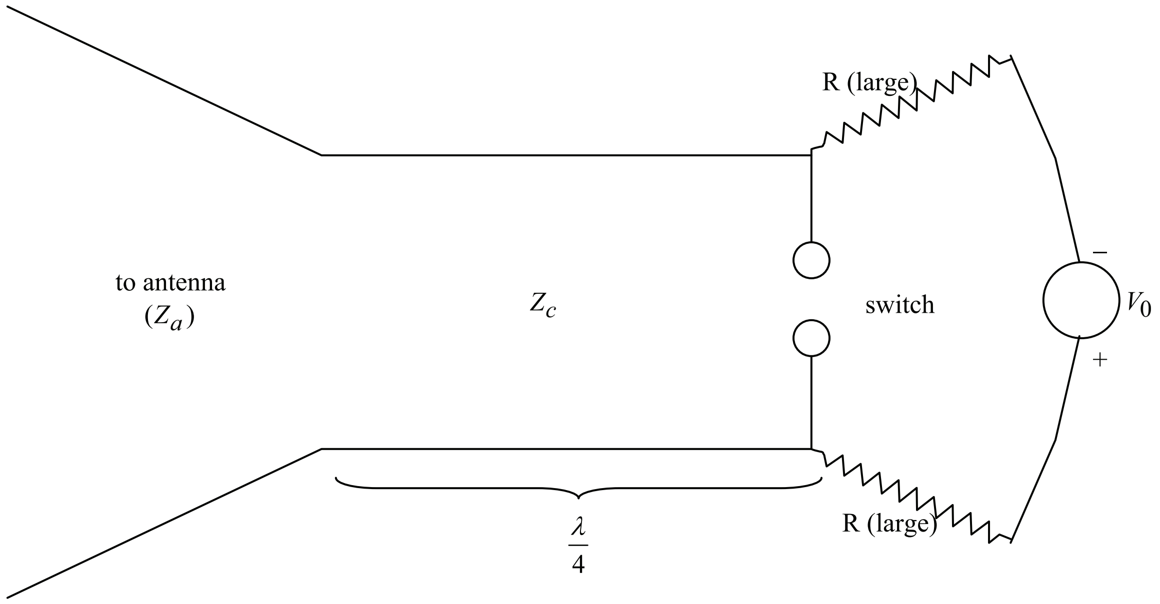

Fig. 1.1 Switched oscillator

2. Transition to Lumped Elements

As is well known, one can make a lumped-element transmission line as indicated in Fig. 2.1. A simple way to view this is by considering a TEM transmission line with

$$
\begin{aligned}
&\mathrm{L}' = \mu f_g \equiv \text{per-unit-length inductance} \\
&\mathrm{C}' = \varepsilon f_g^{-1} \equiv \text{per-unit-length capacitance} \\
&f_g \equiv \text{geometric factor (dimensionless)} \\
&Z_c = \left[\frac{L'}{C'}\right]^{1/2} \equiv \text{characteristic impedance} \\
&\mathrm{v} = [\mu\varepsilon]^{-1/2} = [L'C']^{-1/2} = \text{propagation speed} \\
&\mu \equiv \text{permeability} \\
&\varepsilon \equiv \text{permittivity}
\end{aligned}
\tag{2.1}
$$

Then we can take an incremental length Δz giving a set of incremental inductances and capacitances as in Fig. 2.1.

One can construct such a lumped-element transmission line from inductors and capacitors. However, it has high-frequency limitations as the wavelength λ decreases toward Δz. One could make a switched oscillator this way, if desired. However, this leads to another way to build a switched oscillator based on a single section from Fig. 2.1.

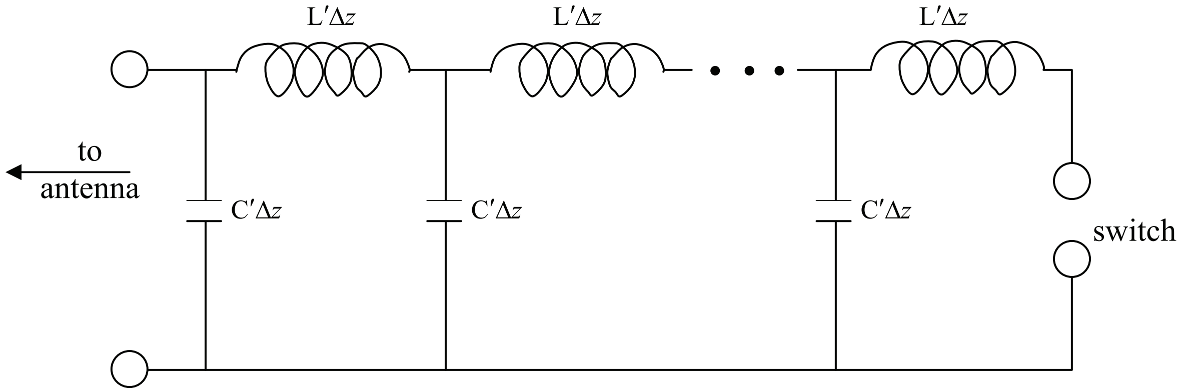

Fig. 2.1 Lumped-Element Approximation of Transmission Line

3. Characteristics of the L C Switched Oscillator

In Fig. 3.1 we have the basic L C switched oscillator. A capacitance C is charged (slowly) through some large resistance to a potential V_0 with a stored energy

$$U_0 = \frac{1}{2}\,\mathrm{C}\,\mathrm{V}_0^2 \tag{3.1}$$

There is typically a blocking capacitor C_b (large compared to C) to isolate the antenna from the static potential to which the oscillator is being charged, while presenting a (very) low impedance to the oscillatory signal.

With a large antenna impedance the oscillator operates at a frequency

$$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi}\left[\mathrm{L\,C}\right]^{-1/2} \tag{3.2}$$

Assuming an open circuit at the antenna the voltage waveform there is

$$\begin{aligned} \tilde{V}_a(s) &= \frac{\dfrac{1}{\mathrm{sC}}}{\dfrac{1}{\mathrm{sC}} + \mathrm{sL}}\,\frac{V_0}{s} = \frac{V_0}{s}\,\frac{1}{1 + s^2\mathrm{L\,C}} \\ V_a(t) &= V_0\left[1 - \cos(\omega_0 t)\right]u(t) \end{aligned} \tag{3.3}$$

for a step switch closure.

If we assume a resistive impedance R for the antenna, then we have

$$\begin{aligned} \tilde{V}_a(s) &= \frac{\left[\dfrac{1}{R} + \mathrm{sC}\right]^{-1}}{\left[\dfrac{1}{R} + \mathrm{sC}\right]^{-1} + \mathrm{sL}}\,\frac{V_0}{s} \\ &= \frac{V_0}{s}\left[1 + \left[\frac{1}{R} + \mathrm{sC}\right]\mathrm{sL}\right]^{-1} = \frac{V_0}{s}\left[s^2\mathrm{LC} + \frac{\mathrm{sL}}{\mathrm{R}} + 1\right]^{-1} \\ &= \frac{V_0}{s}\,\frac{1}{\mathrm{LC}}\left[\left[s + \frac{1}{2\mathrm{RC}}\right]^2 + \frac{1}{\mathrm{LC}} - \frac{1}{4\mathrm{R}^2\mathrm{C}^2}\right]^{-1} \end{aligned}$$

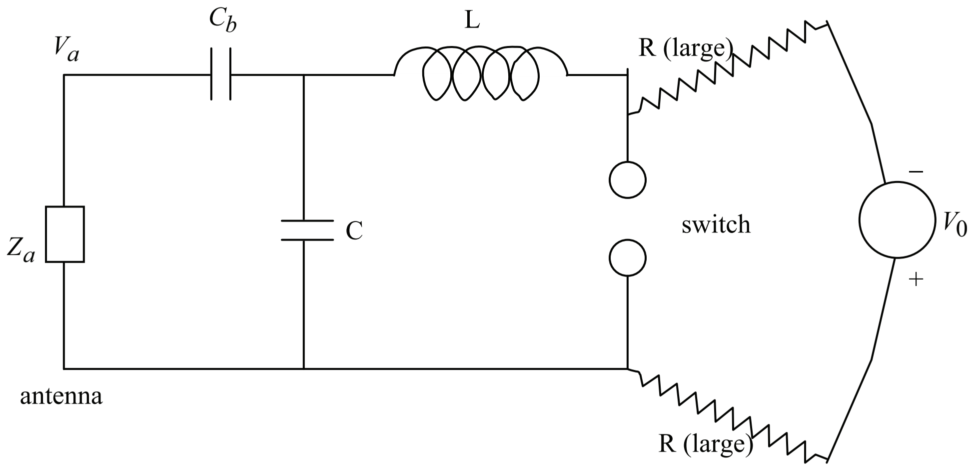

Fig. 3.1 Switched LC Oscillator

$$= \frac{V_0}{s}\,\omega_0^2\left[\left[s+\frac{1}{2\mathrm{RC}}\right]^2+\omega_1^2\right]^{-1}$$

$$\omega_1 \equiv \left[\frac{1}{\mathrm{LC}}-\frac{1}{4\mathrm{R}^2\mathrm{C}^2}\right]^{1/2} = \left[\omega_0^2-\frac{1}{4\mathrm{R}^2\mathrm{C}^2}\right]^{1/2} \tag{3.2}$$

where we have assumed

$$\omega_0^2 > \frac{1}{4\mathrm{R}^2\mathrm{C}^2} \tag{3.3}$$

so that the oscillator is still resonant.

In time domain we have from standard tables [6]

$$\tilde{V}_a(s) = V_0\omega_0^2\left[s\left[s+\frac{1}{2\mathrm{RC}}+j\omega_1\right]\right]^{-1}\left[s\left[s+\frac{1}{2\mathrm{RC}}-j\omega_1\right]\right]^{-1}$$

$$V_a(t) = V_0\left[1-e^{-\frac{t}{2\mathrm{RC}}}\left[\cos(\omega_1 t)+\frac{1}{2\mathrm{RC}\omega_1}\sin(\omega_1 t)\right]\right]u(t) \tag{3.4}$$

which reduces to (2.3) as $R \to \infty$. Here we see the oscillation decaying with time constant 2RC. Again, N being the number of cycles for the oscillation to decay to e^{-1}, we have

$$N = \frac{Q}{\pi} = \frac{\omega_1 RC}{\pi} \tag{3.5}$$

Large N is consistent with the resonant condition in (3.3).

Here we see that the transient voltage swings initially from 0 to approximately $2V_0$ (after a half cycle). For some transient-pulse applications this can be regarded as a voltage doubling device with $1/(2f_0)$ as the charging time. In principle, one can even use this as a first stage in a multiple-stage system. Each successive stage with an inductor, capacitor, and closing switch would have its charging time much shorter than the previous stage.

From the point of view of feeding a resonant antenna system, it is the oscillation magnitude of V_0 (+ and –) that is of interest. In a previous paper [3 (Section 4)] it is shown that the traditional transmission-line switched oscillator, while doubling the transient voltage, raises the amplitude at the dominant resonance to about $(4/\pi)V_0$, or an oscillation of about $1.27V_0$, a little more than the present case.

4. Concluding Remarks

Thus we have another way to build a switched oscillator. Being based on lumped inductance and capacitance, one is not limited by an overly large quarter-wave resonator, particularly at low frequencies. This then broadens the category of switched oscillators.

References

1. C. E. Baum, “Differential Switched Oscillators and Associated Antennas”, Sensor and Simulation Note 457, June 2001.

2. C. E. Baum, “Differential Switched Oscillators and Associated Antennas, Part 2”, Sensor and Simulation Note 484, November 2003.

3. C. E. Baum, “Combined Electric and Magnetic Dipoles for Mesoband Radiation”, Sensor and Simulation Note 523, August 2007.

4. C. E. Baum, “Switched Oscillators”, Circuit and Electromagnetic System Design Note 45, September 2000.

5. W. D. Prather, C. E. Baum, F. J. Torres, F. Sabath, and D. Nitsch, “Survey of Worldwide High-Power Wideband Capabilities”, IEEE Trans. EMC, 2004, pp. 335-344.

6. M. A. Abromowitz and I. A. Stegun, *Handbook of Mathematical Functions*, U. S. Gov’t Printing Office, 1964.