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Coupling Ports in Waveguide Cavities For Multiplying Fields in Pulse-Compression Schemes

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Abstract

This paper gives some insight into how to pump up a resonant cavity for microwave pulse compression. Canonical examples are considered for the resonant modes in rectangular-waveguide cavities, and the appropriate location of coupling ports through which to feed the power into the cavity.

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1. Introduction

One potential way to make a high-power hypoband source is pulse compression [1]. In this approach some kind of resonant structure (e.g., cavity) with high Q_c is fed by a lower-power tuned source in such a way that a higher power oscillation is produced which is later switched quickly into a load (e.g., antenna) producing a high-power oscillation which exists for a shorter time than that required to pump up the oscillation. There are various factors which need to be optimized for this purpose [2].

The present paper is concerned with one aspect of this problem, viz., how to couple in the power from a lower-power source into an oscillation which builds up to a high power. As we shall see, it is not only a question of a high Q_c, but also a geometric question concerning where in the oscillating-mode pattern to feed in the power. For present illustration we consider a cavity formed from standard rectangular waveguide.

2. Half-Guide-Wavelength Cavity

Let us consider a “baby problem” as C. H. Papas would put it. This is a simple example which exhibits the essential physics of the problem. As in Fig. 2.1, let this be a rectangular-waveguide cavity of length d. The parameters of the lowest mode $(H_{1,0})$ are [3, 4]

$$\gamma = \left[\gamma_0^2 + \left[\frac{\pi}{a}\right]^2\right]^{1/2} \qquad \text{(propagation constant)}$$

$$\gamma_0 = \frac{s}{c} \;,\quad s = j\omega \;,\quad \gamma = jk$$

$$c = [\mu\varepsilon]^{-1/2} \;,\quad Z_0 = \left[\frac{\mu}{\varepsilon}\right]^{1/2}$$

$$\gamma = jk$$

$$k = \left[k_0^2 - \left[\frac{\pi}{a}\right]^2\right]^{1/2} \qquad \text{(propagation constant)}$$

$$k_0 = \frac{\omega}{c} \tag{2.1}$$

$$Z_w = Y_w^{-1} = \frac{k_0}{k} Z_0 \qquad \text{(modal impedance)}$$

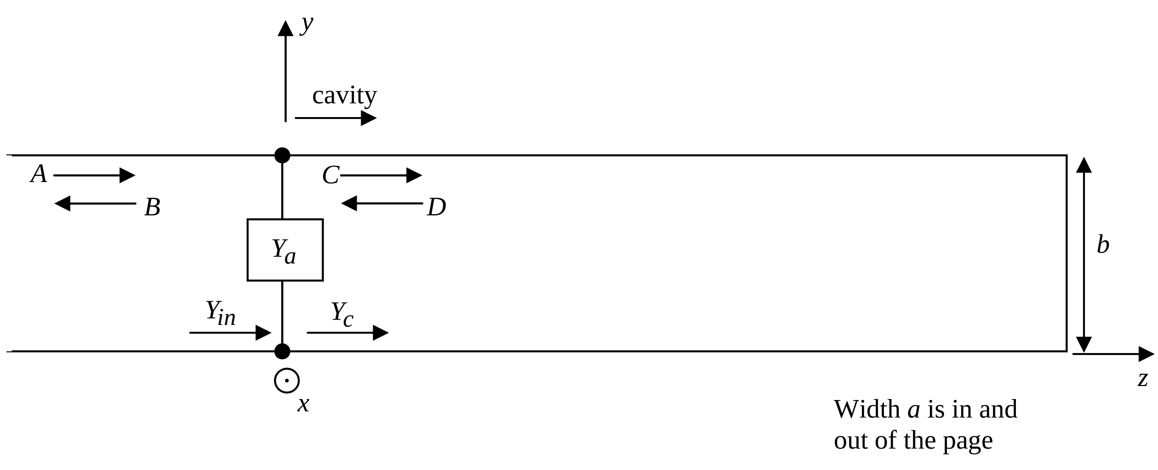


Fig. 2.1 Rectangular Waveguide Cavity

The waves to the left of the cavity ($z < 0$) are characterized by (for electric field)

$$A\ e^{-\gamma z} \equiv \text{incident wave}$$

$$B\, e^{\gamma z} \equiv \text{reflected wave} \tag{2.2}$$

To the right ($z > 0$, the cavity) we have

$$C\, e^{-\gamma z} \equiv \text{right-going wave}$$

$$D\, e^{\gamma z} \equiv \text{left-going wave} \tag{2.3}$$

Applying the boundary condition at z = d gives

$$D = -e^{-2\gamma d} C \tag{2.4}$$

As a first observation choose the frequency such that

$$kd = \pi \tag{2.5}$$

Then at $z = 0$ (the cavity aperture) we have

$$D = -C \tag{2.6}$$

giving zero electric field. Then on the $z = 0$ plane we can place any thin conductor (with or without a hole in it) with no effect, and have

$$B = -A$$

$$A = C \quad \text{(continuity of incident wave)} \tag{2.7}$$

With $C/A = 1$, there is *no* multiplication of fields in the cavity.

The reader can note that the above result also applies for d as any positive integer number of half wavelengths.

3. Resonance Condition Near Guide Half Wavelength

Let us now look more closely near the cavity resonance by expanding

$$kd = \pi + \Delta kd \tag{3.1}$$

We have the transverse fields of the form

$$\begin{aligned} E &= Ae^{-\gamma z} + Be^{\gamma z} \quad , \quad H = Y_w\left[Ae^{-\gamma z} - Be^{\gamma z}\right] \quad \text{for} \quad z < 0 \\ E &= Ce^{-\gamma z} + De^{\gamma z} \quad , \quad H = Y_w\left[Ce^{-\gamma z} - De^{\gamma z}\right] \quad \text{for} \quad z > 0 \end{aligned} \tag{3.2}$$

with the common factor $\sin(\pi x/a)$ suppressed. Looking into the cavity at $z = 0$ we have a wave admittance

$$\begin{aligned} Y_c &= Y_w \frac{1+e^{-2\gamma d}}{1-e^{-2\gamma d}} = Y_w \coth(\gamma d) \\ &= -Y_w\, j\cot(kd) = -Y_w\, j\cot(\Delta kd) \end{aligned} \tag{3.3}$$

The input admittance to the left of $z = 0$ is

$$Y_{in} = Y_c + Y_a \qquad \text{(parallel combination)} \qquad 3.4)$$

where

$$Y_a = \text{aperture admittance (imaginary for } s = j\omega \text{ since lossless)} \tag{3.5}$$

This admittance is treated in various places (e.g., [3, 4]). For a small centered hole of radius r_0 this is an inductance (small)

$$Y_a = \frac{1}{sL_a} \quad , \quad Y_w\,\omega L_a \simeq \frac{8\,kr_0^3}{3ab} \tag{3.6}$$

so that Y_a is negative imaginary. A capacitive obstacle gives a positive imaginary Y_a. For convenience we define a normalized admittance

$$\eta_a = Z_w Y_a \quad \text{(imaginary for } s = j\omega) \tag{3.7}$$

The boundary condition (electric field or voltage) at $z = 0$ is

$$A + B = C + D = C\left[1 - e^{-2\gamma d}\right] \tag{3.8}$$

Relating B to A through the reflection at $z = 0$ we have

$$\begin{aligned}
B &= \frac{Z_{in} - Z_w}{Z_{in} + Z_w} \quad A = \frac{1 - Z_w Y_{in}}{1 + Z_w Y_{in}} A \\
A + B &= A\,2\left[1 + Z_w Y_{in}\right]^{-1} = C\left[1 - e^{-2\gamma d}\right] \\
\frac{C}{A} &= 2\left[1 + Z_w Y_{in}\right]^{-1}\left[1 - e^{-2\gamma d}\right]^{-1} \\
&= 2\left[1 + \eta_a + Z_w Y_c\right]^{-1}\left[1 - e^{-2\gamma d}\right]^{-1} \\
&= 2\left[\left[1 - e^{-2\gamma d}\right]\left[1 + \eta_a\right] + \left[1 + e^{-2\gamma d}\right]\right]^{-1} \\
&= 2\left[1 + e^{-2\gamma d}\right]^{-1}\left[1 + \tanh(2\gamma d)\left[1 + \eta_a\right]\right]^{-1}
\end{aligned} \tag{3.9}$$

Expanding near $kd = \pi$ gives

$$\begin{aligned}
2\left[1 + e^{-j2kd}\right]^{-1} &= 2\left[2 - j2kd + \mathrm{O}\left(\left[\Delta kd\right]^2\right)\right]^{-1} \\
&= 1 + j\Delta kd + \mathrm{O}\left(\left[\Delta kd\right]^2\right) \\
\mathrm{X} &= 1 + j\tan\left(\Delta kd\right)\left[1 + \eta_a\right] \\
\left|\mathrm{X}\right|^2 &= \left[1 + j\tan\left(\Delta kd\right)\left[1 + \eta_a\right]\right]\left[1 - j\tan\left(\Delta kd\right)\left[1 + \eta_a\right]\right] \\
&= 1 + j\tan^2\left(\Delta kd\right)\left[1 + \left|\eta_a\right|^2\right] + 2\tan\left(\Delta kd\right)\left|\eta_a\right|
\end{aligned} \tag{3.10}$$

For resonance this denominator is minimized at

$$\begin{aligned}
0 &= 2\tan\left(\Delta kd\right)\left[1 + \left|\eta_a\right|^2\right] + 2\left|\eta_a\right| \\
\tan\left(\Delta kd\right) &= -\frac{\left|\eta_a\right|}{1 + \left|\eta_a\right|^2} \simeq \Delta kd
\end{aligned} \tag{3.11}$$

This change in resonance frequency is maximized at

$$|\eta_a| = 1 \quad , \quad \tan(\Delta kd) = 1 \quad , \quad \Delta kd = \frac{\pi}{4} \tag{3.12}$$

which is a large frequency shift and a large inductance. For high-Q resonance much larger $|\eta_a|$ are desirable.

Substituting in (3.10) we find

$$|X|^2_{\min} = \left[1+|\eta_a|^2\right]^{-1} \tag{3.13}$$

which tends to zero (for infinite cavity Q_c) as $|\eta_a| \to \infty$ (small inductance) and $\Delta kd \to 0$.

The Q of the loaded cavity is (for a sharp resonance) given by

$$Q_a = \frac{\omega_1}{2\Delta\omega} \quad , \quad \omega_2 = \omega_1 \pm \Delta\omega \tag{3.14}$$

with $\Delta\omega$ taken as the change in frequency from ω_1 (the peak of the resonance) to $2^{-1/2}$ (0.707) of the peak value. So set $|X|^2$ to 2 times the minimum value to give

$$\begin{aligned}
&2\left[1+|\eta_a|^2\right]^{-1} = 1+\tan(\Delta kd)\left[1+|\eta_a|^2\right] + 2\tan(\Delta kd)|\eta_a| \\
&\tan(\Delta kd) + \frac{2|\eta_a|}{1+|\eta_a|^2}\tan(\Delta kd) + \frac{|\eta_a|^2-1}{\left[1+|\eta_a|^2\right]^2} = 0 \\
&\tan(\Delta kd) = -\frac{|\eta_a|}{1+|\eta_a|^2} \pm \left[\frac{|\eta_a|^2}{\left[1+|\eta_a|^2\right]^2} - \frac{|\eta_a|^2-1}{\left[1+|\eta_a|^2\right]^2}\right]^{1/2} \\
&\qquad = -\frac{|\eta_a|}{1+|\eta_a|^2} \pm \frac{1}{1+|\eta_a|^2}
\end{aligned} \tag{3.15}$$

The first term gives the resonance frequency as in (3.11). The second term gives (small Δkd)

$$\frac{d}{c}\Delta\omega \simeq \left[1+|\eta_a|^2\right]^{-1} \tag{3.16}$$

From this the Q is

$$Q_a = \frac{\omega_1 d}{c}\frac{c}{2d\Delta\omega} \simeq \frac{\pi}{2}\left[1+|\eta_a|^2\right] \tag{3.17}$$

which becomes large for small aperture inductance (large $|\eta_a|$). Then we can estimate from (3.13)

$$\left|\frac{\mathrm{C}}{\mathrm{A}}\right|^2 \simeq 1+|\eta_a|^2 \simeq \frac{2}{\pi}Q_a \tag{3.18}$$

as the potential power multiplication.

From the usual formula for adding Qs we have

$$Q^{-1} = Q_a^{-1} + Q_c^{-1} \tag{3.19}$$

So the above result assumes a cavity Q_c larger than Q_a, and Q_c gives a limit on the attainable Q.

4. Positioning Coupling Ports

From the foregoing we can see that a small coupling hole into a resonant cavity can lead to a high Q with accompanying amplification of the electromagnetic fields. This then leads to the question of optimization of the location and shape of this coupling port.

Considering the simple example in Section 2, we have the situation in which the electric field at the aperture is zero and the tangential magnetic field matches on both sides, independent of the aperture shape. One might look for similar situations to see whether some improvement can be made in some sense.

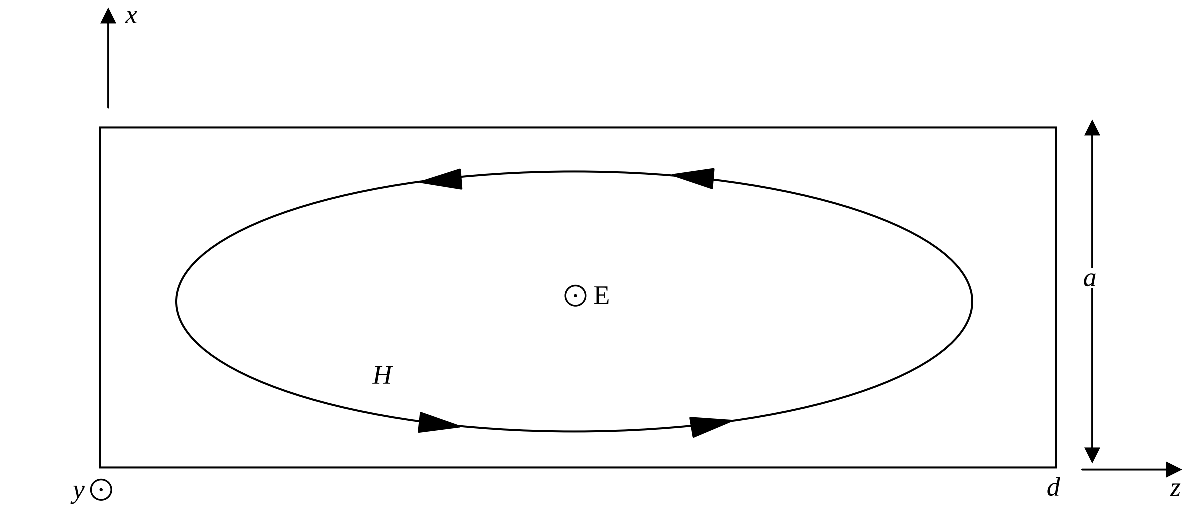
For this purpose consider the half-wavelength cavity for its spatial field distribution. As we know, the narrow side walls, as well as the end closures at $z = 0,d$ have no normal electric field in the $H_{1,0}$ mode. Look, however at the tangential magnetic field on the side walls. It varies as $\sin(\pi z/d)$. Suppose now that for $kd = \pi$ we place a magnetic field H_z on the exterior of the cavity at $z = d/2$, the center of the side wall. Call this location ① in Fig. 4.1 with magnetic field H_1. This will be matched by the same magnetic field on the inside of the cavity at this position. With the boundary conditions satisfied over all the cavity boundary, this gives the correct value for the internal fields. There is, however, no amplification of the magnetic field.

Now consider a position ② near $z = 0$ (the end of the side wall). Call the tangential magnetic field there H_2. If we match H_2 to the external magnetic field as before, then H_1 inside the guide will be larger by the ratio

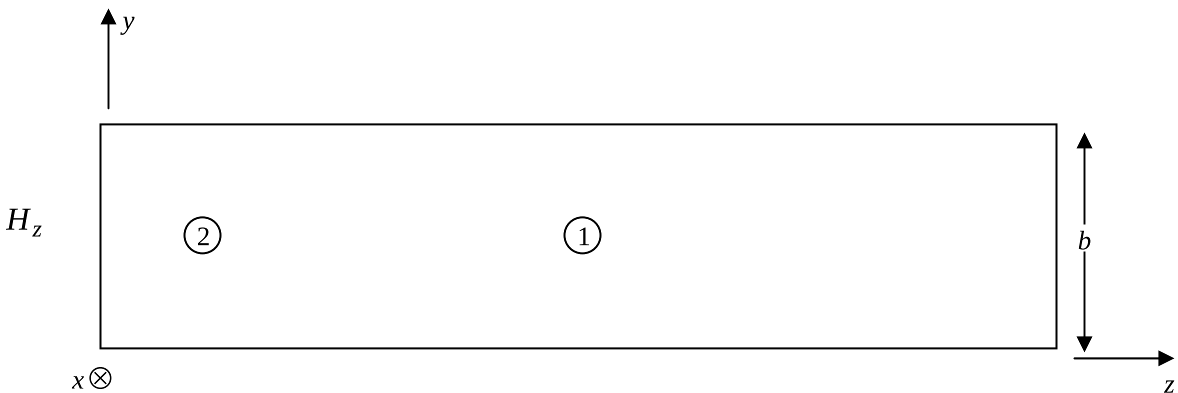
$$f_h = \frac{H_1}{H_2} = \sin^{-1}(\pi z/d) \tag{4.1}$$

Here we have assumed that the size of the coupling hole (in the z direction) is small compared to z so that there is negligible variation of the magnetic field over the port at ②. Again the cavity boundary conditions are matched for $kd = \pi$.

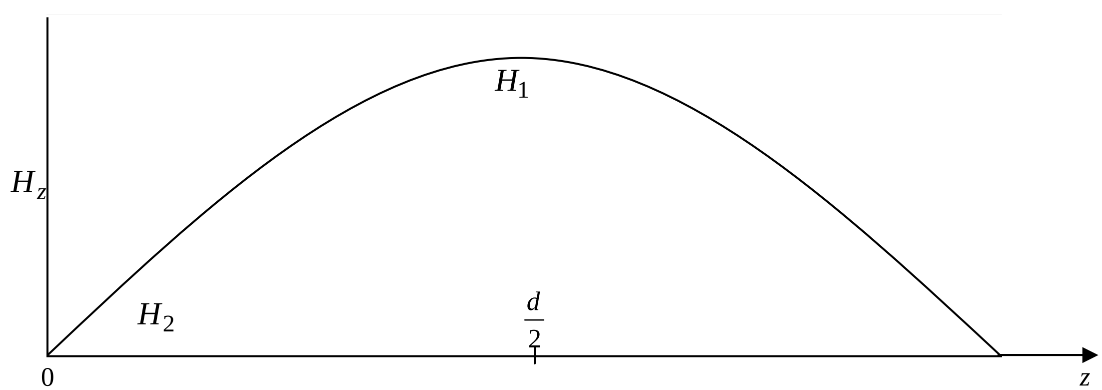
Of course, like in Section 3, if we vary the frequency near $kd = \pi$, we can achieve a resonance condition with even larger field amplification. Note that the coupling at ② implies that the fields leak out through the port more slowly when the excitation is removed. This is another way to say that the Q_a is increased by moving the port from ① to ②, even with the same size coupling hole. Said another way, the coupling hole at ② can be larger for the same Q_a. Again, the cavity Q must be larger than Q_a if this Q is to be achieved.



A. View normal to broad wall



B. View normal to side (narrow) wall with possible coupling ports



C. Tangential magnetic field on side wall

Fig. 4.1 Electromagnetic Fields in Half-Wavelength Rectangular Waveguide Cavity

As a simple illustration, Fig. 4.2 shows a possible way to feed the cavity from a rectangular waveguide. In this case H_{ext} is twice the incident magnetic field (when $kd = \pi$). This is, in turn, matched to H_1 or H_2 depending on the position of the coupling waveguide. This is only one possible configuration. One can conceive of many more.

While a rectangular waveguide is chosen for our example cavity, many other shapes are possible, and higher order modes (many wavelengths across inside) are also possible. The important point is that the fields vary considerably over the cavity walls, and one can choose the coupling location and type (electric or magnetic) for field amplification.

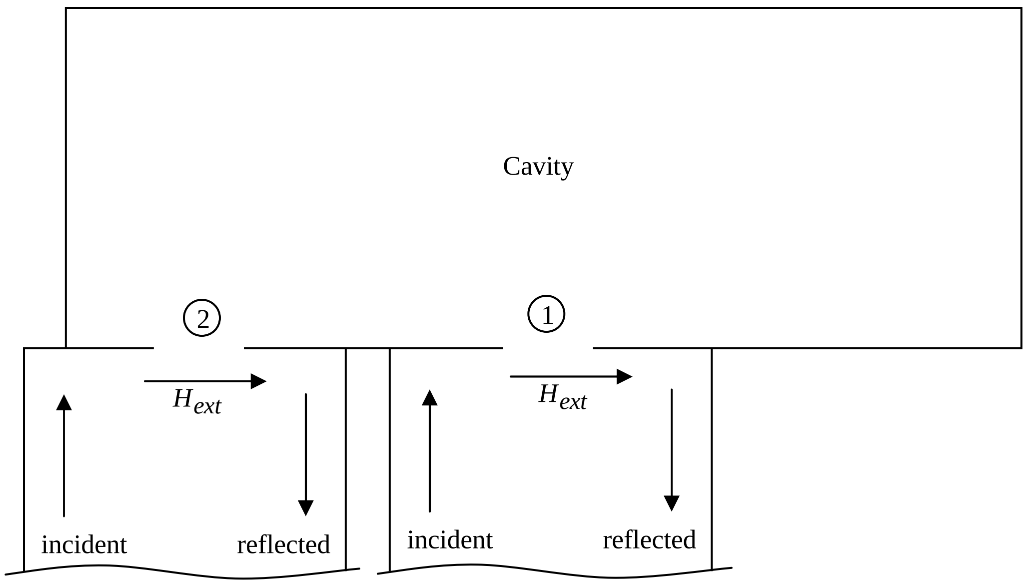


Fig. 4.2 Waveguide-Fed Cavity

5. Inclusion of Coupling Port in Symmetrical Waveguide Cavity System

While this paper is not concerned with the details of rapidly switching out the amplified fields into a load, one must incorporate the input port into such systems. As discussed in [1] symmetry is an important concept in such systems. The action of the switch is to destroy the symmetry in the oscillating system in such a way as to allow the energy to rapidly exit the cavity into the desired load.

Considering now waveguide cavities, the magic tee is one way of feeding in antisymmetric fields at the Δ port, while feeding out fields through the Σ port [3, 4]. A switch in the waveguide converts the antisymmetric resonance into a quasi-symmetric field pattern. A significant question concerns feeding into the cavity in a high-*Q* manner. Consider the configuration in Fig. 5.1. The incident field in the Δ port and the fields in the waveguide cavity are antisymmetric with zero E_t (tangential electric field) and maximum H_t (tangential magnetic field) on the symmetry plane. This makes the *E* field a null at the center of the Σ port, leading to no excitation of the $H_{1,0}$ mode in the output waveguide.

This leaves the question of how to build up the resonance by feeding in power through the Δ port. If the waveguide feeds in directly with no small coupling hole or iris the Q will be quite low. As illustrated in Fig. 5.1, one could place such an iris directly in the broad wall of the cavity symmetrically on the symmetry plane (position ①). This gives a situation similar to that in Sections 2 and 3. One can also move the iris to position ②, away from the junction as a means of tuning to the resonance. We have assumed that each arm of the resonant cavity is a positive integer of $\lambda/2$ in length.

There are many other possible ways of feeding such a cavity. One can insert a small coupling hole at various positions along the waveguide cavity. This introduces a small asymmetry if the coupling hole is small. This can be tuned out by adjusting the lengths of one or both cavity arms. Similar compensation is needed for the switch presence (before it fires). Fig. 5.2 shows an example of coupling to the magnetic field at location ③ on the narrow wall near the symmetry plane (or some other null of the longitudinal component of the magnetic field). This is similar to location ② in Section 4.

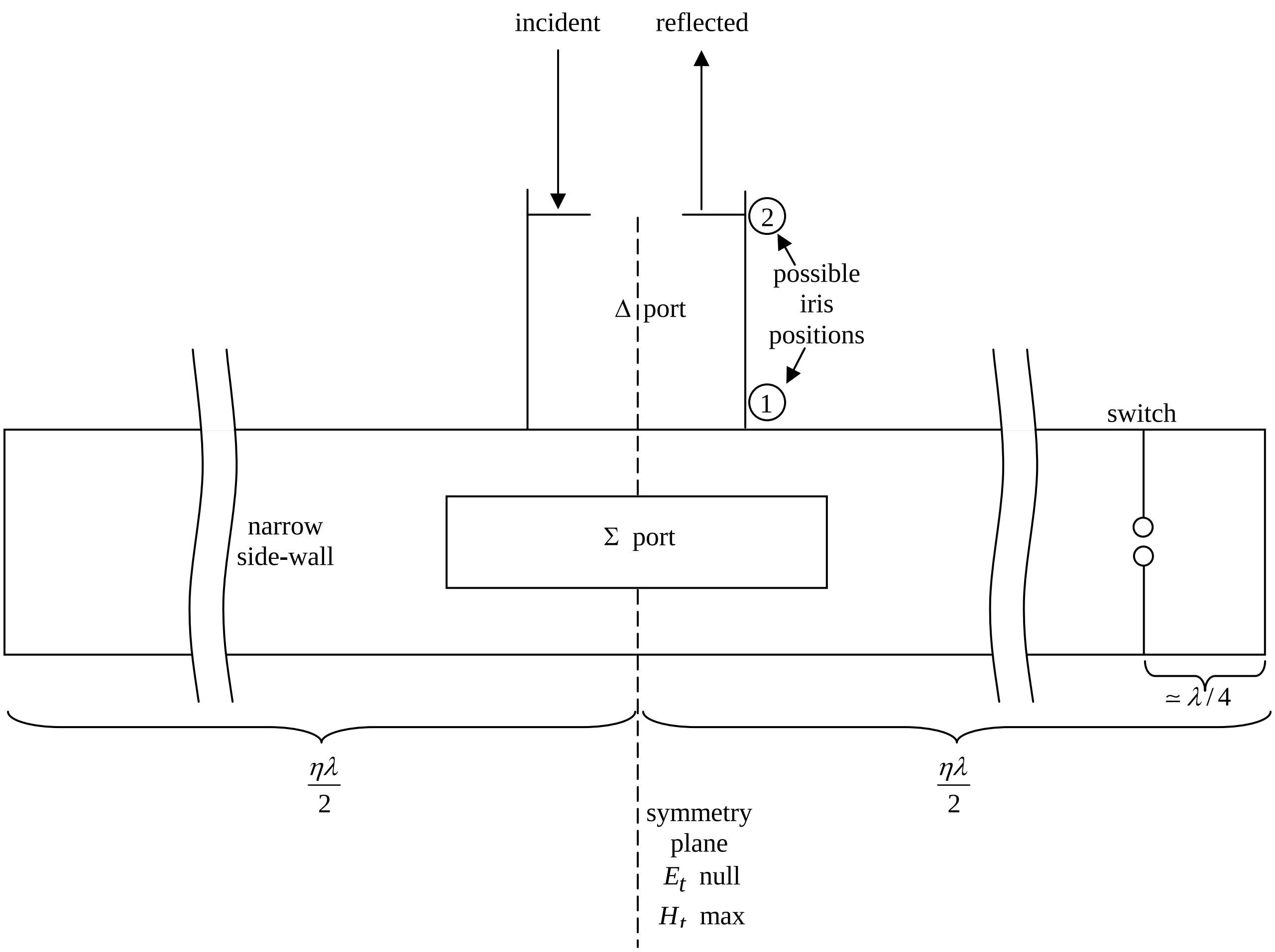


Fig. 5.1 Feeding a Magic Tee

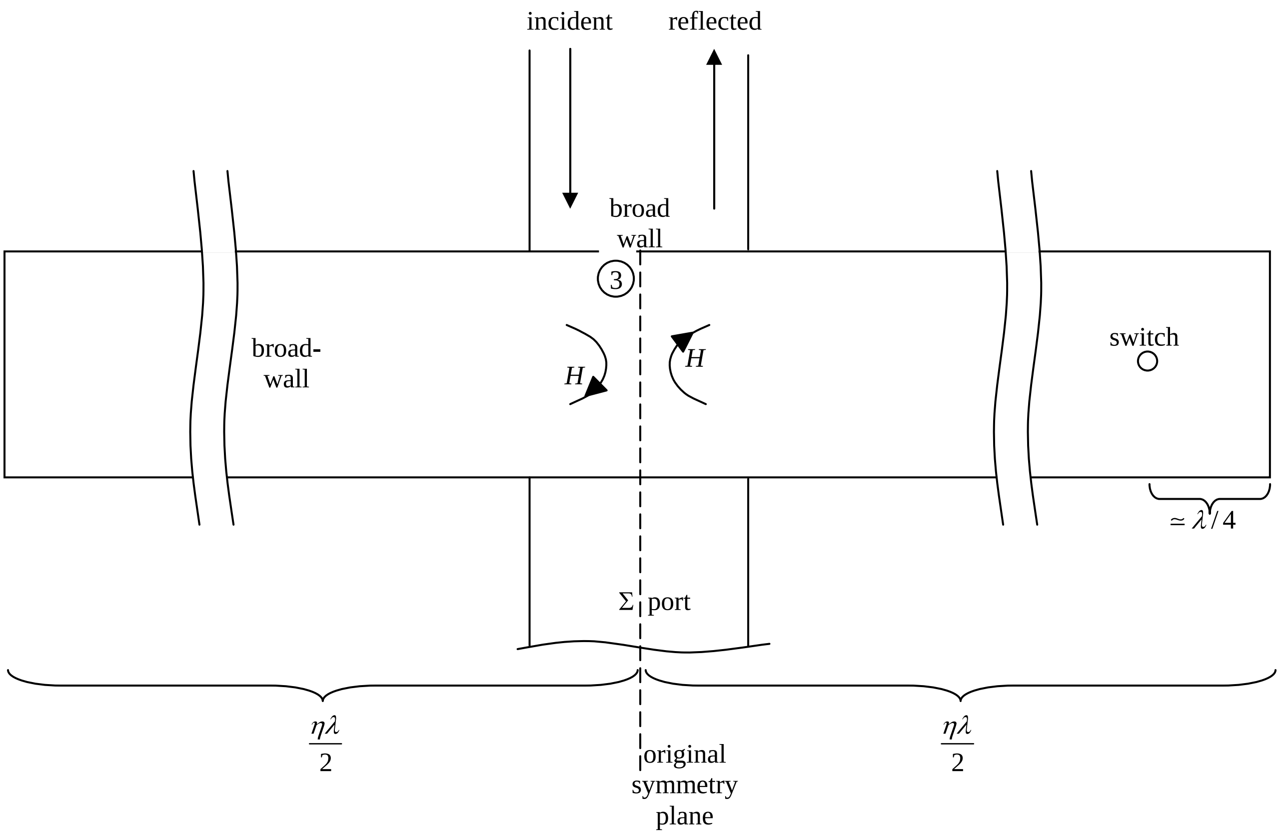


Fig. 5.2 Feeding Near the Symmetry Plane

6. Concluding Remarks

This only indicates the many possibilities for pumping up the power in a resonant cavity for a pulse compression scheme. Certain advantages are found for placing the input coupling port near an appropriate null of the resonant mode. This gives an initial field amplification based on field ratios in the cavity. It also contributes to the required high Q, and allows for a larger coupling port.

References

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