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Sequential Quarter-Wave Transmission-Line Transformers

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Abstract

This paper considers a sequence (or cascade) of quarter-wave transmission-line transformers for application at discrete frequencies. The same gain can be achieved, but with a voltage increase in smaller increments. This can be advantageous in certain applications where high-voltage insulation is a concern.

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1. Introduction

A recent paper [4] considers a quarter-wave transmission-line transformer for improving the matching of a switched oscillator [3] to an antenna with an input impedance in the 100 Ω range [1]. There it was observed that a significant increase could be made in the peak voltage (or equivalently, peak power) delivered to the antenna. The stored energy in the switched oscillator is, of course, then delivered in a shorter time while still approximately maintaining the basic oscillator frequency.

In stepping up the voltage delivered to the antenna we have the associated problem of insulating the antenna input so as to withstand the higher voltage. Perhaps it would be better to step up the voltage in a more gradual way so that as one progressed away from the reflector focus and the feed arms became farther apart the voltage could be allowed to be larger without breakdown. As we shall see, this is accompanied by an increase in the transmission-line characteristic impedance as one progresses away from the switched oscillator.

2. Sequence of Quarter-Wave Transformers

Consider the sequence described in Fig. 2.1. Here we have a sequence of N + 1 transmission lines with characteristic impedances Z_{c_n} for $n = 0, 1, 2, \ldots, N$. Here 0 labels the switched oscillator and N labels the load, which is taken as a constant resistance corresponding to a TEM lossless transmission line which may also approximate the impedance of certain antennas.

Beginning from some small impedance Z_{c_0} (perhaps a few ohms) the transmission-line sections have characteristic impedances increasing as $n \to N$, i.e.

$$Z_{c_{n-1}} < Z_{c_n} < Z_{c_{n-1}} \tag{2.1}$$

As discussed in [1, 4] and elsewhere, a quarter-wave transformer has a transit time t_0 related to a frequency

$$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{4\,t_0} \tag{2.2}$$

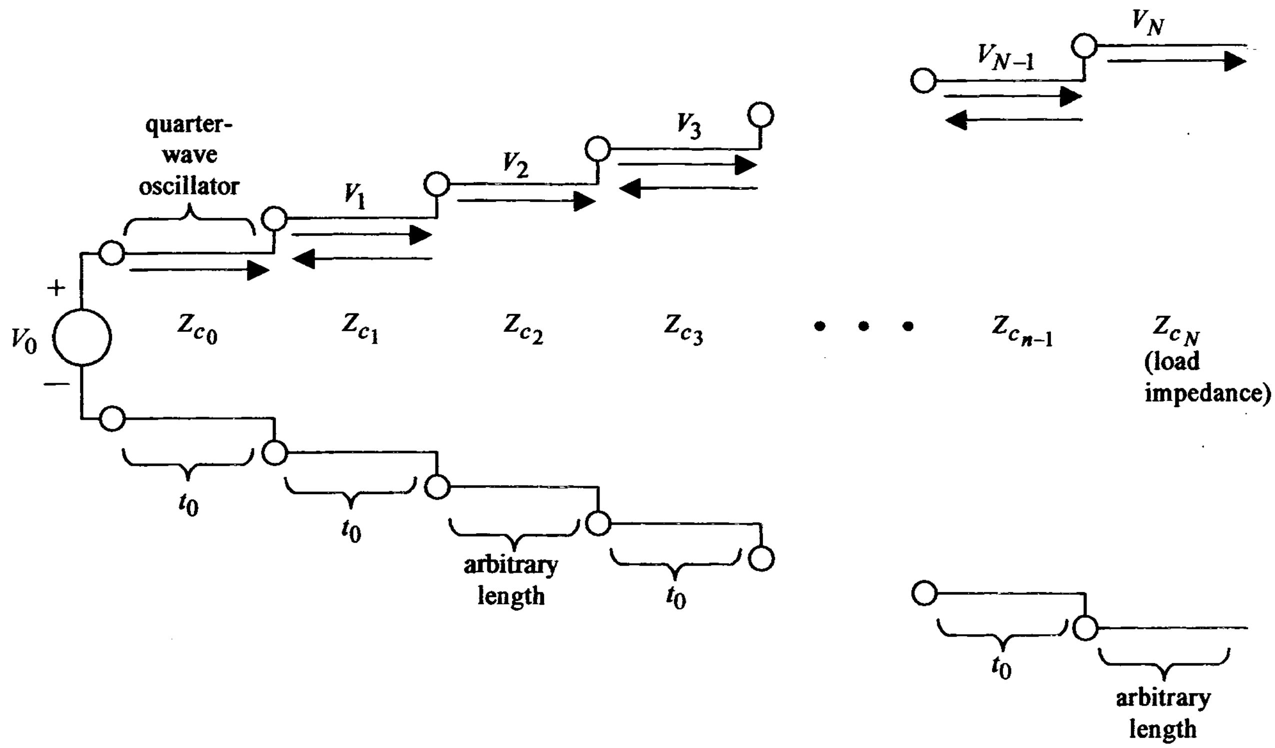
Note that t_0 is the transit time in the switched oscillator as well, being quarter-wave resonant.

The first quarter-wave transformer has $n = 1$. At frequency f_0 it transmits all the power to a load Z_{c_2} with

$$\begin{aligned} Z_{c_1} &= \left[Z_{c_0}\, Z_{c_2}\right]^{1/2} \\ \frac{V_2}{V_0} &= \left[\frac{Z_{c_2}}{Z_{c_0}}\right]^{1/2} \end{aligned} \tag{2.3}$$

Requiring that there be no reflection on the $n = 2$ transmission line, the length (transit time) of this line is *arbitrary* (including the possibility of zero length).

Carrying the argument forward, let odd n correspond to quarter-wave transformers and even n to transmission lines with the wave propagating in only one direction (forward) with arbitrary length. So (2.3) generalizes to



N = even

Fig. 2.1 Sequence of Quarter-Wave Transmission-Line Transformers.

$$Z_{c_n} = \left[Z_{c_{n-1}} Z_{c_{n+1}} \right]^{1/2} \quad \text{for } n \text{ odd}$$
$$\frac{V_{n+1}}{V_{n-1}} = \left[\frac{Z_{c_{n+1}}}{Z_{c_{n-1}}} \right]^{1/2} \quad \text{for } n \text{ odd} \tag{2.4}$$

Cascading these results we have

$$\frac{V_N}{V_N} = \frac{V_2}{V_0} = \frac{V_4}{V_2} \cdots \frac{V_N}{V_{N-2}} = \left[\frac{Z_{c_N}}{Z_{c_0}} \right] \quad \text{for } N \text{ even} \tag{2.5}$$

corresponding to total power transmission at frequency f_0 (and odd multiples) through the cascaded transformers.

While the foregoing allows both increase and decrease in Z_{c_n} as one progresses through the sequence, a monotonically increasing Z_{c_n} as in (2.1) gives

$$V_{n-1} < V_{n+1} \quad \text{for } n \text{ odd} \tag{2.6}$$

in the transmission lines with singly directed waves. In the transformer sections the situation is more complicated. At f_0 there is as smooth increase from V_{n-1} to V_{n+1} through the transformer. At odd multiples of f_0 the voltage increases and decreases through the transformer.

3. Transient Gain

By transient gain, we mean the step up of the leading edge of a pulse (such as a step function) as it propagates through the transformers. In [4] we found that the transient gain through the first transformer is

$$T_1 = 4\left[1 + \left[\frac{Z_{c_0}}{Z_{c_0}}\right]^{1/2}\right]^{-2} \tag{3.1}$$

This generalizes to

$$\begin{aligned} T_1 &= 4\left[1 + \left[\frac{Z_{c_{n-1}}}{Z_{c_{n+10}}}\right]^{1/2}\right]^{-2} \quad \text{for } n \text{ odd} \\ T &= T_1\ T_3 \cdots T_{N-1} = \prod_{n=1}^{N-1,2} T_n \\ \ell n(T) &= \sum_{n=1}^{N-1,2} \ell n(T_n) \end{aligned} \tag{3.2}$$

Consider now what happens for large N. The incremental T_n became

$$\begin{aligned} \Delta Z_{c_{n+1}} &\equiv Z_{c_{n+1}} - Z_{c_{n-1}} \quad \text{for n odd} \\ T_n &= \left[\frac{1}{2} + \frac{1}{2}\left[\frac{Z_{c_{n+1}}}{Z_{c_{n-1}}}\right]^{1/2}\right]^{-2} \\ &= \left[\frac{1}{2} + \frac{1}{2}\left[1 + \frac{\Delta Z_{c_{n+1}}}{Z_{c_{n-1}}}\right]^{-1/2}\right]^{-2} \\ &= \left[1 - \frac{1}{4}\frac{\Delta Z_{c_{n+1}}}{Z_{c_{n-1}}} + O\left(\left[\frac{\Delta Z_{c_{n+1}}}{Z_{c_{n-1}}}\right]^2\right)\right]^{-2} \\ &= 1 + \frac{1}{2}\frac{\Delta Z_{c_{n+1}}}{Z_{c_{n-1}}} + O\left(\left[\frac{\Delta Z_{c_{n+1}}}{Z_{c_{n-1}}}\right]^2\right) \\ \ell n(T_n) &= \frac{1}{2}\frac{\Delta Z_{c_{n+1}}}{Z_{c_{n-1}}} + O\left(\left[\frac{\Delta Z_{c_{n+1}}}{Z_{c_{n-1}}}\right]^2\right) \end{aligned} \tag{3.3}$$

The sum goes to an integral in the limit as

$$\ell n(T) = \lim_{N\to\infty} \sum_{m=2}^{N,2} \left[\frac{1}{2} \frac{\Delta Z_{c_m}}{Z_{c_m}} + O\left(\left[\frac{\Delta Z_{c_m}}{Z_{c_m}} \right]^2 \right) \right]$$

$$= \frac{1}{2} \int_{Z_{c0}}^{Z_L} \frac{dZ_c}{Z_c} = \frac{1}{2} \ell n\left(\frac{Z_L}{Z_{c0}} \right) \quad \text{s } N \to \infty \tag{3.4}$$

$$T = \left[\frac{Z_L}{Z_{c0}} \right]^{1/2} \quad \text{as } N \to \infty$$

$$Z_L = Z_N = \text{load impedance}$$

$$= \text{constant for all choices of } N$$

This result is the same as the high-frequency response of a transmission-line pulse transformer [2]. Here, of course, our attention is focused on single frequency applications, for which we obtain the same result in a limiting sense.

4. Concluding Remarks

A sequence (cascade) of quarter-wave transmission-line transformers can then have some advantages over a single transformer. The voltage is allowed to more gradually increase through the sequence, potentially giving better insulation characteristics in some applications. In the limit of a large number of cascaded transformers the ensemble gives a version for discrete frequencies comparable to a continuously varying transmission-line characteristic impedance as used for a pulse transformer.

References

1. C. E. Baum, "Antennas for the Switched-Oscillator", Sensor and Simulation Note 455, March 2001.

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3. C. E. Baum, "Switched Oscillators", Circuit and Electromagnetic System Design Note 45, September 2000.

4. C. E. Baum, "A Transmission-Line Transformer for Matching the Switched Oscillator to a Higher-Impedance Resistive Load", Circuit and Electromagnetic System Design Note 46, August 2002.