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SOLUTION OF PEAKING EQUATION FOR FINITE STORAGE CAPACITOR SIZE

by

John L. Harrison
Maxwell Laboratories, Inc.

ABSTRACT

The peaking circuit is used extensively in NEM simulators. It has been shown by Lupton[1] that the circuit can be designed to give a step function response when the initial store is a battery. This paper extends the analysis for the case where this store is a capacitor, and shows that a pure exponentially decaying pulse can be generated when the source inductance is the only circuit inductance. Expressions for determining the capacitance of the peaking capacitor and switching times are developed.

SOLUTION OF PEAKING EQUATION FOR FINITE STORAGE CAPACITOR SIZE

The peaking circuit considered in this paper is shown in Figure 1. This circuit has been used extensively in pulse power applications to provide a fast rising output wave from a storage capacitor having a relatively high inductance.

The circuit consists of the capacitor C_1 in series with its inductance L, which is switched into a low inductance peaking capacitor C_2 by switch S_1. The circuit is subsequently switched into a resistive load, R, by switch S_2.

The circuit will, in practice, have some inductance in its output loop. In this analysis, this output inductance is assumed to be small enough to be neglected.

The required output from this circuit is usually a pure exponentially decaying pulse. This analysis obtains a solution for the relationship between the parameters of the circuit, and the switching times of switches S_1 and S_2 that will provide a pure exponential decay.

BACKGROUND

This circuit has been analyzed by Lupton[1] for the case when $C_1 \rightarrow \infty$, and has been used in transfer circuits when $C_1 = C_2$ and $L \rightarrow \infty$. The solution obtained in this paper spans the gap between these extremes.

ANALYSIS

The characteristic equation in Laplacian notation for the output current and voltage can be obtained from the loop equations.

$$-\frac{V_{C1}(0)}{p} + \frac{i_1}{C_1 p} + Lpi_1 - Li_1(0) + Ri_2 = 0 \qquad (1)$$

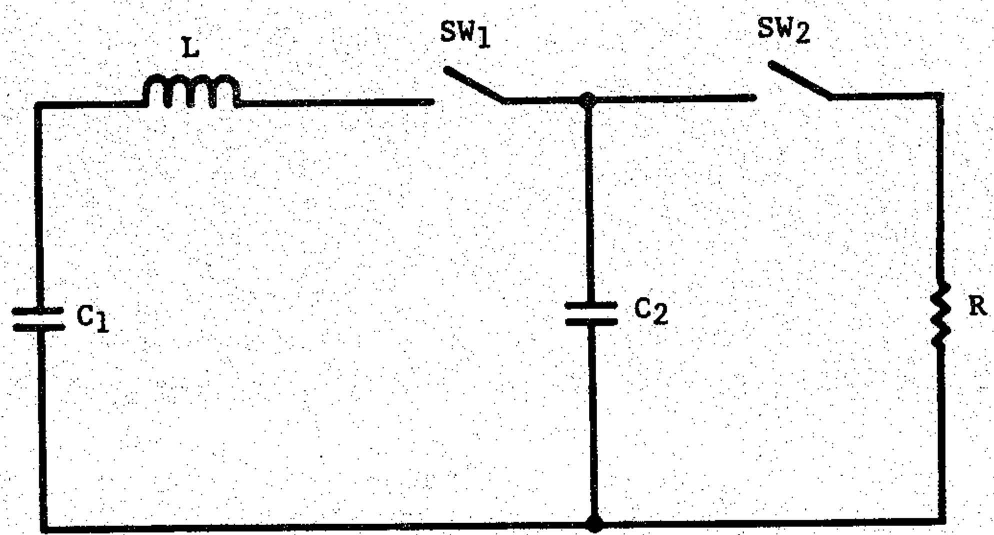

Figure 1. Schematic Diagram of the peaking circuit.

$$\frac{i_1 - i_2}{C_2 p} + \frac{V_{C2}(0)}{p} - Ri_2 = 0 \quad . \tag{2}$$

Solving for i_2 gives

$$i_2 = \frac{1}{LC_2R} \frac{V_{C1}(0) + Lpi_1(0) + V_{C2}(0)\left\{\frac{C_2}{C_1} + LC_2p^2\right\}}{p^3 + \frac{p^2}{RC_2} + \frac{(C_1 + C_2)p}{LC_1C_2} + \frac{1}{RLC_1C_2}} \tag{3}$$

$$V_\ell = Ri_2 \tag{4}$$

If SW_1 closes at a time $t = 0$ and SW_2 closes at a time

$$t = t_0 \text{ and } \omega_0 = \sqrt{\frac{C_1 + C_2}{LC_1C_2}}$$

(the resonance of the first loop), then the values for the initial conditions, $i_2(0)$, $V_{C1}(0)$ and $V_{C2}(0)$ are

$$i_1(0) = V_0 \sqrt{\frac{C_1C_2}{L(C_1 + C_2)}} \sin \omega_0 t_0$$

$$V_{C1}(0) = V_0 \left\{1 - \frac{1}{\omega_0} \sqrt{\frac{C_2}{LC_1(C_1 + C_2)}} (1 - \cos \omega_0 t_0)\right\}$$

$$V_{C2}(0) = V_0 \times \frac{1}{\omega_0} \sqrt{\frac{C_1}{LC_2(C_1 + C_2)}} (1 - \cos \omega_0 t_0) \quad .$$

Substituting these values in i_2 from Equation (3), gives a value of V_ℓ , Equation (4) of

$$V_\ell = V_0 \cdot \frac{\frac{C_1}{C_1 + C_2} p^2(1 - \cos \omega_0 t_0) + \frac{C_1}{C_1 + C_2} p\omega_0 \sin \omega_0 t_0 + \frac{1}{LC_2}}{p^3 + \frac{p^2}{RC_2} + \frac{(C_1 + C_2)p}{LC_1C_2} + \frac{1}{RLC_1C_2}} \quad . \tag{5}$$

The characteristic equation for an exponentially decaying pulse is

$$V_\ell = \frac{\alpha V_0}{p + \beta} \quad .$$

Equating the energy dissipated in the load with the energy initially stored in C_1 we find that

$$\beta = \frac{\alpha^2}{RC_1} \quad .$$

Computer results[2] indicate that an exponential decay can be achieved if $\alpha = 1$. If this is the value for an exponentially decaying output pulse, the following two requirements must be met:

$$(1) \quad \frac{C_1}{C_1 + C_2} (1 - \cos \omega_0 t_0) = 1 \quad , \tag{6}$$

whence

$$\sin \omega_0 t_0 = \sqrt{\frac{C_1^2 - C_1^2}{C_1^2}} \quad . \tag{7}$$

$$(2) \quad p + \frac{1}{RC_1} \tag{8}$$

must be a divisor of the denominator of Equation (5) and that the quotient of this division by congruent to the numerator of Equation (5).

Dividing the denominator of Equation (5) by the relationship of 8 gives a remainder of

$$\frac{C_1C_2R^2 + C_1^2R^2 + LC_2 - LC_1}{R^3LC_1^3C_2} - \frac{1}{RLC_1C_2} = 0 \quad .$$

The equality being required if $p + 1/RC$ is a factor of the denominator,

$$\therefore R^2 = \frac{L(C_1 - C_2)}{C_1C_2} \tag{9}$$

the quotient of the division is

$$p^2 + \frac{p}{R}\left(\frac{C_1 - C_2}{C_1C_2}\right) + \frac{1}{L}\left(\frac{C_1 + C_2}{C_1C_2}\right) + \frac{1}{R^2C_1}\left(\frac{C_2 - C_1}{C_1C_2}\right) \tag{10}$$

We can now substitute the values of Equations (6) and (7) and for ω_0 in the numerator of Equation (5) and for R of Equation (9) in the quotient given in (10) to show that the numerator and quotient are congruent. They are.

CONCLUSION

Thus, it has been shown that Equation (5) can be reduced to the form

$$V_\ell = \frac{V_0}{p + \frac{1}{RC_1}}$$

if

$$R^2 = \frac{L(C_1 - C_2)}{C_1 C_2} \quad ,$$

or

$$C_2 = \frac{LC_1}{R^2 C_1 + L}$$

$$= \frac{Z_0{}^2}{R^2 + Z_0{}^2}\, C_1$$

where $Z_0 = \sqrt{\frac{L}{C_1}}$

and

$$t_0 = \frac{1}{\omega_0} \cos^{-1} \left(-\frac{C_2}{C_1}\right) \quad .$$

It has not been shown that these are unique values for obtaining the exponential form. There may, therefore, be other relationships that will provide a pure exponentially decaying pulse from this circuit.

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REFERENCES

(1) Waveform Distortion from Peaking Circuit Switch Jitter, William H. Lupton NRL Memorandum Report 1829, November 1967 and Note 1 "Pulsed Electrical Power Circuit and Electromagnetic System Design Notes." AFWL-TR-73-166, April 1973.

(2) Final Report for TORUS Generator Study, Maxwell Laboratories, Incorporated Report MLR-101, 21 October 1970.