

**Bioelectric Notes**

Note 6

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**Modified Subnanosecond Sample Holder**

Carl E. Baum and Serhat Altunc  
Department of Electrical and Computer Engineering  
University of New Mexico  
Albuquerque, NM 87131

Karl H. Schoenbach and Shu Xiao  
Frank Reidy Research Center for Bioelectrics  
Old Dominion University  
Norfolk, VA 23510

Abstract

*Previous considerations have concerned the design of a sample holder shaped as a finite-length circular cylinder. The present paper considers an extension to a cylindrical shell. This gives some advantages in matching the pulse into the biological sample (solution), provided the conductivity can be raised.*

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## 1. Introduction

A recent paper [1] discusses techniques for exposing biological samples with very short pulses (~ 100 ps or so) in a coaxial sample holder. The switch with charged capacitor is at one end with the sample at the other end. The pulse is so short that the wave-propagation properties of the sample holder and sample configuration are important.

Using a saline solution with

$$\epsilon_r = \frac{\epsilon_s}{\epsilon_0} \cong 81 \quad (\text{relative dielectric constant})$$

$$\sigma_s = 0.3 \text{ S/m} \quad (\text{saline solution conductivity})$$
(1.1)

a cylindrical test sample centered on the body-of-rotation axis had some problems matching the resistance of the sample to the coax characteristic impedance,  $Z_c$ . Noting the relaxation time of the sample medium as

$$T = \frac{\epsilon_s}{\sigma_s} \cong 2.3 \text{ ns}$$
(1.2)

one observes that in the times of interest the medium looks more like a capacitor than a resistor.

Instead of a fat (large diameter ~ cm) sample, one might make a very narrow (but still a few cm long) sample, to allow the field to penetrate the sample in times of interest. However, this gives a resistance much larger than  $Z_c$ .

The present paper investigates another sample geometry based on a cylindrical shell.

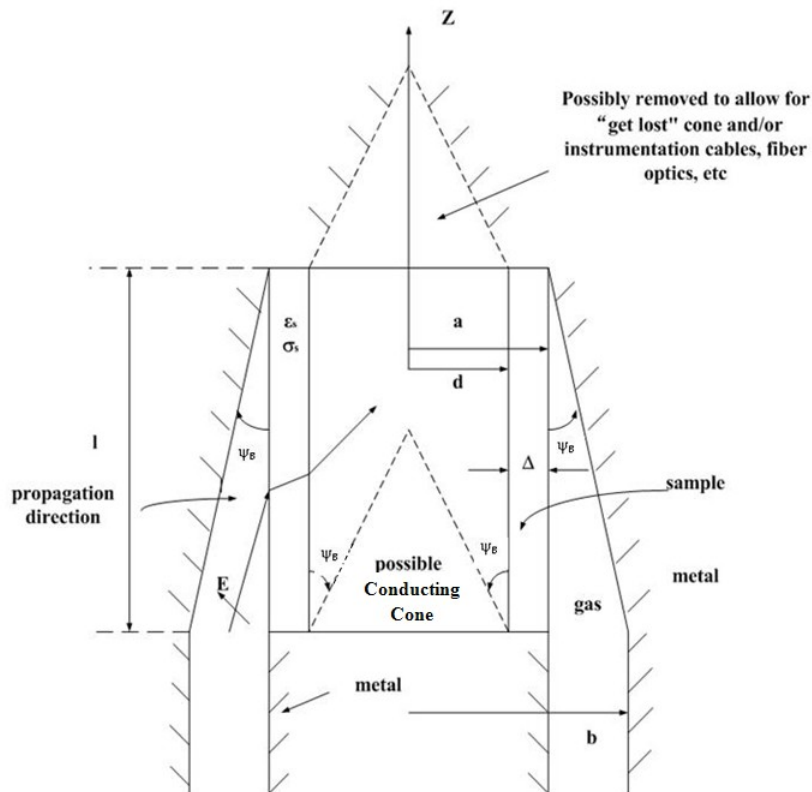


Fig. 2.1 Cylindrical-shell test sample.

## 2. New Test-Sample Geometry

Consider another variation of the test-sample geometry. As in Fig. 2.1 this consists of a cylindrical shell with

$$\begin{aligned}
 a &= \text{outer radius} \\
 d &= \text{inner radius} \\
 \Delta &= a - d = \text{shell thickness} \\
 l &= \text{shell length} \\
 b &= \text{coax outer radius.}
 \end{aligned} \tag{2.1}$$

This can be compared to the cylindrical sample in [2, (Fig. 3.1)]. The Brewster angle for matching E (or TM) waves into the sample is

$$\psi_B = \arctan\left(\varepsilon_r^{-\frac{1}{2}}\right) \cong 6.3^\circ \tag{2.2}$$

Note that for small  $(b - a)$  compared to  $a$  one can make a plane-wave approximation for guiding the wave into the cylindrical-shell sample. Keeping the Brewster angle  $\psi_B$ , the sample length is

$$\begin{aligned}
 l &= [b - a] \tau_0 t(\psi_B) = [b - a] \varepsilon_r^{\frac{1}{2}} \\
 &\cong 9[b - a] \cong 9 \eta
 \end{aligned} \tag{2.3}$$

As discussed in [1], the characteristic impedance of the coax taper from its full value to zero at the end of the sample has

$$\begin{aligned}
 \frac{dZ_c}{dZ} &= -R' \cong -\frac{Z_c}{l} \\
 R' &= [\sigma_s \pi [a^2 - d^2]]^{-1} \\
 &= [\sigma_s \pi [a + d] \Delta]^{-1} \cong [\sigma_s 2\pi a \Delta]^{-1} \\
 &\equiv \text{resistance per unit length of "terminator"}
 \end{aligned} \tag{2.4}$$

If

$$b - a \ll a \tag{2.5}$$

then we have approximately

$$\begin{aligned}
 Z_c &= \frac{Z_0}{2\pi} \ln\left(\frac{b}{a}\right) = \frac{Z_0}{2\pi} \ln\left(1 + \frac{\eta}{a}\right) \\
 &\cong \frac{Z_0}{2\pi} \frac{\eta}{a} \cong 60 \frac{\eta}{a}
 \end{aligned} \tag{2.6}$$

$Z_0 \cong 377 \Omega \equiv$  wave impedance of free space

From the foregoing we have

$$Z_c \cong \frac{Z_0 \eta}{2\pi a} \cong 60 \frac{\eta}{a} \cong 9\eta[\sigma_s \pi[2a - \Delta]\Delta]^{-1}$$

$$60\sigma_s \pi[2a - \Delta]\Delta \cong 9a \quad (2.7)$$

$$\left[2 - \frac{\Delta}{a}\right]\Delta \cong \frac{3}{20} [\pi\sigma_s]^{-1}$$

This quadratic equation has the solution

$$\Delta = \frac{a}{2} \left[ 2 \pm \left[ 4 - 4 \left[ \frac{2}{a} \frac{3}{20\pi\sigma_s} \right]^{\frac{1}{2}} \right] \right]^{\frac{1}{2}}$$

$$= a \left[ 1 \pm \left[ 1 - \frac{3}{10\pi a \sigma_s} \right]^{\frac{1}{2}} \right] \quad (2.8)$$

Looking for small  $\Delta$  (compared to  $a$ ) gives

$$\Delta = a \left[ 1 - \left[ 1 - \frac{3}{10\pi a \sigma_s} \right]^{\frac{1}{2}} \right] \quad (2.9)$$

For a real solution we require

$$\sigma_s a \geq \frac{3}{10\pi} \cong 0.095 \text{ (Siemens)} \quad (2.10)$$

For a conductivity as in (1.1) then we have

$$a \geq .31 \text{ m} \quad (\text{or } 12 \text{ cm}) \quad (2.11)$$

This is a fairly large radius. At this limiting value we have

$$\Delta = a \quad (2.12)$$

or a solid cylindrical sample. At this thickness, with a 100 ps pulse (spatial width in water  $\cong .33$  cm) the wave will pass through the sample (large) creating a non-uniform field distribution in the sample. Of course, one could increase  $a$  yet further (over the (2.11) value). The asymptotic form (of (2.9)) for large  $a$  is

$$\Delta \cong \frac{3}{20\pi a \sigma_s} \quad (2.13)$$

Increasing  $a$  would decrease  $\Delta$ , but to make  $\Delta$  smaller than the spatial pulse width would make a quite large.

### 3. Increasing the Sample Conductivity

One way to decrease  $a$  is to increase  $\sigma_s$ , say to

$$\sigma_s \cong 4 \text{ S/m} \quad (\text{sea water}) \quad (3.1)$$

In this case the relaxation time is

$$\tau = \frac{\epsilon_s}{\sigma_s} \cong 179 \text{ ps} \quad (3.2)$$

which is approaching our intended pulse width.

From (2.10) we now require for a real solution

$$a \geq \frac{3}{10\pi\sigma_s} \cong .024 \text{ m} \quad (\text{or } 2.4 \text{ cm}) \quad (3.3)$$

This is a more reasonable number. This is still larger than the spatial pulse width in water ( $\cong 3.3 \text{ mm}$ ). So we might look to a larger  $a$ . From (2.13) we have for large  $a$

$$\Delta \cong \frac{3}{20\pi a \sigma_s} \cong \frac{.012}{a} \quad (3.4)$$

So we might have, as an example,

$$a = .2 \text{ m}$$

$$\Delta \cong .06 \text{ m} \quad (\text{from (4.4)}) \quad (3.5)$$

$$\Delta \cong .061 \text{ m} \quad \cong 6.1 \text{ cm} \quad (\text{from (2.9)})$$

This is more practical than the previous case but is still rather large for  $a$ .

We would like to make  $\Delta$  correspond to one or fewer pulse widths in water. This makes  $a$  even larger. For

$$\Delta \cong 3 \text{ mm} \quad (3.6)$$

then we have

$$a \cong 4 \text{ m} \quad (3.7)$$

This is way too large. To go further, we might increase  $\sigma_s$  beyond that of sea water.

### 4. Concluding Remarks

A cylindrical shell might then be a better model for the sample geometry. However, to get the shell thickness down to the pulse width (in water), requires increasing the sample conductivity while still maintaining a small sample diameter. The discussion here allows trading the various parameters for each other. However, the negative effect of high conductivity solution on biological cells limits the conductivity to approximately twice that of the conductivity of a physiological solution. For mammalian cells this is approximately 1 S/m. For solutions with conductivities in excess of this value, the imbalance in osmotic pressure leads to shrinking of the cells and cell death in times of minutes or less.

## References

1. C.E. Baum, "Subnanosecond Sample Holder," Measurement Note 60, November 2003.
2. D.V. Giri and C.E. Baum, "Design Guidelines for Flat-Plate Conical Guided Wave EMP Simulators With Distributed Terminators," Sensor and Simulation Note 402, October 1996.