BAS

Sensor Notes

Note 2

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Inductive Techniques Using Uniform Transmission-Line Structures for Measuring Eulerian Position in Lagrangian-Flow Shock Waves in High-Density Media

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Abstract

This note considers the problem of electromagnetic measurement of position of an elementary volume in a shock-induced medium flow. The case of one coordinate longitudinal to the flow is treated giving a one-dimensional problem. A transmission-line structure is used parallel to the flow and it is found that it should be short circuited at the end which first receives the shock; this minimizes the effects of conductivity and permittivity of the ambient medium, and gives a basically inductive sensor. The effects of transverse expansion (or contraction) of the medium is minimized by various techniques, including symmetry. Sensor designs involving self and mutual inductances are also considered, and some of the possible RF signal discrimination techniques which might be integrated with such a sensor are discussed. Various design problems are pointed out as areas for future research.

I. Introduction

One of the important variables in a shock wave is the position of some elementary portion of the material medium as a function of time. This "particle position" is in general not simply related to other variables such as pressure (or more generally stress), not only because of the spatially distributed nature of the shock environment, but especially due to the nonlinear nature of the shock environment at high transient pressures. As the position (in general a three vector) of the elementary sample of mass (the Eulerian coordinates) moves, one can also interpret this phenomenon in terms of particle velocity. However, one should be careful to distinguish between first, the position of a sample of the medium as a function of time, and second, the velocity of the medium as a function of time at a fixed position in some stationary rest frame (usually stationary with respect to the initial conditions of the medium). This note is concerned with measuring the position in the first sense in that something which moves with the medium is tracked by electrical means. The derivative of this type of quantity with respect to time is also a velocity, but the velocity of a moving elementary mass (or "particle") in the medium.

It is important that the measurement of interest be not significantly perturbed by other medium parameters, particularly by those parameters which may be also changing in the shock environment. To the extent that these other parameters are uncertain, then the results of the measurement are correspondingly uncertain. In this note the technique of concern electrically locates the position by scattering electromagnetic fields off a conducting object moving with the medium. Since in this technique the sensor consists primarily of the surrounding medium and this medium transmits the electromagnetic fields (incident plus scattered) it is important that the electrical properties of the medium (including changes under high-pressure conditions) do not significantly affect the sensor performance. Basically this requires that the sensor be insensitive to the electrical parameters of the medium (permittivity ε and conductivity σ), but can use the magnetic properties of the medium (permeability μ) since we assume

$$\mu \simeq \mu_o \quad \text{(permeability of free space)} \tag{1.1}$$

even under the high pressure conditions. In general we allow

$$\begin{aligned} \overleftrightarrow{\varepsilon} &= \overleftrightarrow{\varepsilon}(\vec{r},t) \quad \text{(dyadic permittivity)} \\ \overleftrightarrow{\sigma} &= \overleftrightarrow{\sigma}(\vec{r},t) \quad \text{(dyadic conductivity)} \end{aligned} \tag{1.2}$$

i.e., dyadic functions of space $\vec{r} = (x,y,z)$ and time t for the electrical conditions.

Another requirement concerns the conductors and other materials (other than the ambient medium itself) that are used to construct the sensor. These should be of sufficiently small mass and volume so as not to disturb the flow of the ambient medium in the shock conditions.

This note addresses some of the design techniques for transmission-line structures operated in self-inductance and mutual-inductance configurations for measuring the particle position (or corresponding velocity) in an ambient medium of sufficient density. Beyond these considerations there are many detailed design questions concerning materials, fabrication, associated electronic equipment, etc. This note attempts to discuss some of the basic physical techniques involved and the associated design implications.

II. Transmission-Line Structure in Electromagnetically Uniform Ambient Medium

Consider schematically a transmission line of length $\ell(t)$ which varies slowly as a function of t. (See fig. 2.1.) Changes are associated with the deformation of the medium for which the associated shock velocity (velocities) v_s are small compared to the speed of light in vacuum c or in the medium $\tilde{v}$. The transmission line has a characteristic impedance $\tilde{Z}_c$ where

$$\tilde{Z}_c = \tilde{Z}_c(z,s) = f_g(z)\ \tilde{Z}(z,s)$$

$$f_g(z) \equiv \text{dimensionless geometrical factor}$$

$$z \equiv \text{coordinate along transmission line}$$

$$\tilde{Z}(z,s) = \left[\frac{\tilde{\sigma}(z,s) + s\tilde{\varepsilon}(z,s)}{s\mu_o}\right]^{1/2}$$

$$\equiv \text{wave impedance of ambient medium}$$

$$\tilde{\sigma}(z,s) \equiv \text{conductivity of ambient medium (assumed a scalar)}$$

$$\tilde{\varepsilon}(z,s) \equiv \text{permittivity of ambient medium (assumed a scalar)}$$

$$\mu_o \equiv 4\pi \times 10^{-7}\ \text{H/m}$$

$$\equiv \text{permeability of free space}$$

$$= \text{permeability of ambient medium} \qquad (2.1)$$

In another form this is

$$\tilde{Z}_c(z,s) = \left[\frac{sL'(z)}{\tilde{G}'(z,s) + s\tilde{C}'(z,s)}\right]^{1/2}$$

$$L'(z) = \mu_o f_g(z) \equiv \text{inductance per unit length}$$

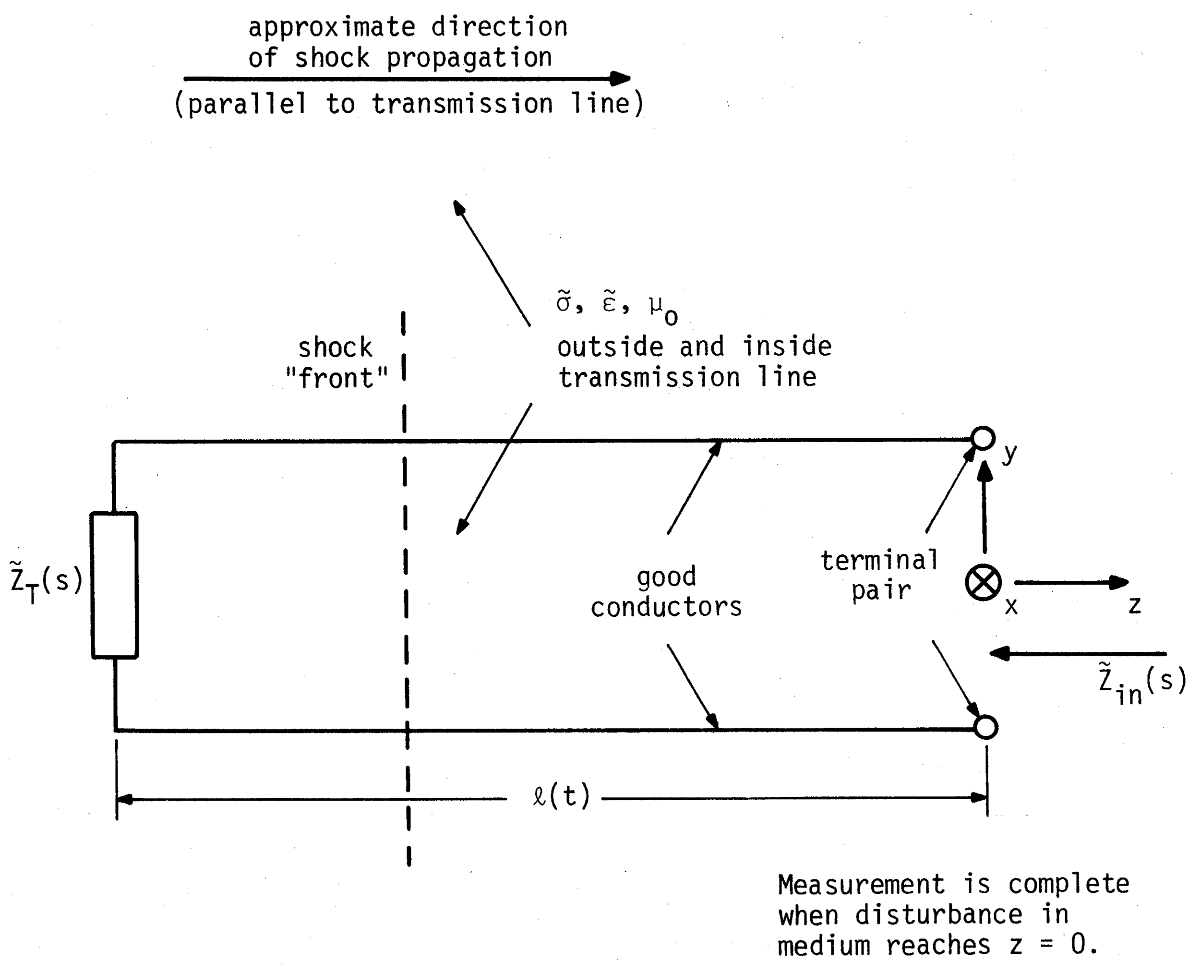


Figure 2.1. Transmission Line with Axis Parallel to Shock Propagation in Ambient Medium

$$\tilde{C}'(z,s) = \frac{\tilde{\varepsilon}(z,s)}{f_g(z)} \equiv \text{capacitance per unit length}$$

$$\tilde{G}'(z,s) = \frac{\tilde{\sigma}(z,s)}{f_g(z)} \equiv \text{conductance per unit length} \qquad (2.2)$$

In terms of these same parameters the propagation constant is

$$\tilde{\gamma}(z,s) = [s\mu_o(\tilde{\sigma}(z,s) + s\tilde{\varepsilon}(z,s))]^{1/2}$$

$$= [sL'(z)(\tilde{G}'(z,s) + s\tilde{C}'(z,s))]^{1/2} \qquad (2.3)$$

Note that the electromagnetic properties of the medium that are changed under the shock conditions can in general be a function of both position z and complex frequency s; they may also be functions of time t because of the changes induced in the medium, but these changes are assumed slow compared to times required for the electromagnetic processes to achieve a steady state. Where required, the electromagnetic influence of these changes in the ambient medium can be introduced as perturbations.

Here the complex frequency or two-sided-Laplace-transform variable is

$$s = \Omega + j\omega \qquad (2.4)$$

where $s = j\omega$ is used for CW considerations with the usual frequency given by

$$f = \frac{\omega}{2\pi} \qquad (2.5)$$

Note also the termination impedance $\tilde{Z}_T(s)$ for our transmission line and the desired quantity, the input impedance $\tilde{Z}_{in}(s)$. A tilde ~ above a quantity is used to indicate the two-sided Laplace transform as

$$\tilde{f}(s) \equiv \int_{-\infty}^{\infty} f(t)e^{-st}\,dt$$

$$f(t) = \frac{1}{2\pi j} \int_{\Omega_o - j\infty}^{\Omega_o + j\infty} \tilde{f}(s) e^{st} \, ds \tag{2.6}$$

with Ω_o in the strip of convergence.

Note that the wave impedance and propagation constant of the medium can be combined to give

$$\begin{aligned} \tilde{\gamma}(z,s)\, \tilde{Z}(z,s) &= s\mu_o \\ \frac{\tilde{\gamma}(z,s)}{\tilde{Z}(z,s)} &= \tilde{\sigma}(z,s) + s\tilde{\varepsilon}(z,s) \end{aligned} \tag{2.7}$$

and similarly using the characteristic impedance

$$\begin{aligned} \tilde{\gamma}(z,s)\, \tilde{Z}_c(z,s) &= sL'(z) \\ \frac{\tilde{\gamma}(z,s)}{\tilde{Z}_c(z,s)} &= \tilde{G}'(z,s) + s\tilde{C}'(z,s) \end{aligned} \tag{2.8}$$

These relations show that a certain form of combination of the propagation constant and wave impedance or characteristic impedance is independent of the conductivity and permittivity of the medium.

Let us first assume that the electromagnetic parameters are not functions of position z along the direction of shock propagation. The reflection coefficient (for voltage) at $z = -\ell$ is

$$\tilde{\Gamma}_T(s) = \frac{\tilde{Z}_T(s) - \tilde{Z}_c(s)}{\tilde{Z}_T(s) + \tilde{Z}_c(s)} \tag{2.9}$$

giving an input impedance at $z = 0$ as

$$\tilde{Z}_{in}(s) = \tilde{Z}_c(s) \frac{1 + \tilde{\Gamma}_T(s)\, e^{-2\tilde{\gamma}(s)\ell}}{1 - \tilde{\Gamma}_T(s)\, e^{-2\tilde{\gamma}(s)\ell}} \tag{2.10}$$

As this expression stands one would expect the input impedance to depend on $\tilde{\sigma}$ and $\tilde{\varepsilon}$ through $\tilde{\gamma}$ and $\tilde{Z}$. For ℓ comparable to or large compared to radian wavelengths ($|\tilde{\gamma}\ell| >> 1$) this dependence is indeed the case because the termination and medium characteristics are combined in a complicated way. Time-domain considerations indicate that for early times or high frequencies a causal reflection coefficient $\tilde{\Gamma}_T$ cannot cancel the properties (also causal) of the ambient medium as contained in the delay associated with the propagation constant $\tilde{\gamma}$.

Consider the case then that the frequency is sufficiently small such that $|\tilde{\gamma}\ell| << 1$. Substituting from (2.9) in (2.10), rearranging terms, and expanding the exponentials in a power series about $\tilde{\gamma}\ell = 0$ gives

$$\tilde{Z}_{in}(s) = \tilde{Z}_c(s)\,\frac{\tilde{Z}_T(s)\left[e^{\tilde{\gamma}(s)\ell} + e^{-\tilde{\gamma}(s)\ell}\right] + \tilde{Z}_c(s)\left[e^{\tilde{\gamma}(s)\ell} - e^{-\tilde{\gamma}(s)\ell}\right]}{\tilde{Z}_T(s)\left[e^{\tilde{\gamma}(s)\ell} - e^{-\tilde{\gamma}(s)\ell}\right] + \tilde{Z}_c(s)\left[e^{\tilde{\gamma}(s)\ell} + e^{-\tilde{\gamma}(s)\ell}\right]}$$

$$= \tilde{Z}_c(s)\,\frac{\tilde{Z}_T(s)\left[2 + (\tilde{\gamma}(s)\ell)^2 + O((\tilde{\gamma}(s)\ell)^4)\right] + \tilde{Z}_c(s)\left[2\tilde{\gamma}(s)\ell + \frac{1}{3}(\tilde{\gamma}(s)\ell)^3 + O((\tilde{\gamma}(s)\ell)^5)\right]}{\tilde{Z}_T(s)\left[2\tilde{\gamma}(s)\ell + \frac{1}{3}(\tilde{\gamma}(s)\ell)^3 + O((\tilde{\gamma}(s)\ell)^5)\right] + \tilde{Z}_c(s)\left[2 + (\tilde{\gamma}(s)\ell)^2 + O((\tilde{\gamma}(s)\ell)^4)\right]}$$

$$= \frac{\tilde{Z}_T(s)\left[2 + (\tilde{\gamma}(s)\ell)^2 + O((\tilde{\gamma}(s)\ell)^4)\right] + sL'\ell\left[2 + \frac{1}{3}(\tilde{\gamma}(s)\ell)^2 + O((\tilde{\gamma}(s)\ell)^4)\right]}{\tilde{Z}_T(s)\left[\tilde{G}'(s) + s\tilde{C}'(s)\right]\ell\left[2 + \frac{1}{3}(\tilde{\gamma}(s)\ell)^2 + O((\tilde{\gamma}(s)\ell)^4)\right] + \left[2 + (\tilde{\gamma}(s)\ell)^2 + O((\tilde{\gamma}(s)\ell)^4\right]} \qquad (2.11)$$

From this result note that for $|\tilde{\gamma}(s)\ell| << 1$ the resulting denominator function is dependent on $\tilde{G}'(s) + s\tilde{C}'(s)$ to zeroeth order for constant $\tilde{Z}_T(s) \neq 0$ as $s \to 0$. Letting $\tilde{G}'(s)$ be a constant (or bounded by a constant) as $s \to 0$ then $\tilde{Z}_T(s) \to 0$ as $s \to 0$ will remove this dependence on $\tilde{G}'$ (and $\tilde{C}'$) and hence make the input impedance independent of $\tilde{\sigma}(s)$ and $\tilde{\varepsilon}(s)$ to zeroeth order for low frequencies. Note that $\tilde{Z}_T(s)$ could be an inductance as sL_T to meet this constraint. A short circuit is a very small inductance approximately and so we set

$$\tilde{Z}_T(s) \equiv 0 \tag{2.12}$$

giving

$$\tilde{Z}_{in}(s) = sL \frac{1 + \frac{1}{6}(\tilde{\gamma}(s)\ell)^2 + O((\tilde{\gamma}(s)\ell)^4)}{1 + \frac{1}{2}(\tilde{\gamma}(s)\ell)^2 + O((\tilde{\gamma}(s)\ell)^4)}$$

$$L \equiv L'\ell \equiv \text{transmission-line inductance} \tag{2.13}$$

This says that a shorted transmission line (with lossless conductors) placed in the ambient medium and electrically exposed to the medium has an inductive impedance at low frequencies, this low-frequency impedance being independent of both the conductivity and permittivity of the medium.

Exploring the implications of (2.13) further the numerator and denominator series expansions are combined as

$$\begin{aligned}\tilde{Z}_{in}(s) &= sL\left[1 - \frac{1}{3}(\tilde{\gamma}(s)\ell)^2 + O((\tilde{\gamma}(s)\ell)^4)\right] \\ &= sL\left[1 - \frac{1}{3}\, s\mu_o\ell(\tilde{\sigma}(s) + s\tilde{\varepsilon}(s))\ell + O((\tilde{\gamma}(s)\ell)^4)\right] \\ &= sL\left[1 - \frac{1}{3}\, sL(\tilde{G}'(s)\ell + s\tilde{C}'(s)\ell) + O((\tilde{\gamma}(s)\ell)^4)\right]\end{aligned} \tag{2.14}$$

This gives a correction term indicating how much the conductivity and permittivity can change the input impedance. Rewriting (2.14) as

$$\tilde{Z}_{in}(s) = sL\left[1 + \tilde{\Delta}(s) + O((\tilde{\gamma}(s)\ell)^4)\right]$$

$$= s\mu_o \ell f_g\left[1 + \tilde{\Delta}(s) + O((\tilde{\gamma}(s)\ell)^4)\right] \qquad (2.15)$$

the first order error $\tilde{\Delta}$ is

$$\tilde{\Delta}(s) = -\frac{1}{3} s\mu_o \ell(\tilde{\sigma}(s) + s\tilde{\varepsilon}(s))\ell$$

$$= -\frac{1}{3} sL(\tilde{G}'(s)\ell + s\tilde{C}'(s)\ell) \qquad (2.16)$$

One criterion for an accurate measurement is then that $|\tilde{\Delta}| << 1$. Since at low frequencies $\tilde{\Delta}(s) \to 0$ as $s \to 0$ this criterion is consistent with our prior considerations which argued toward a low-frequency measurement. Now we have some estimate of "how low is low." As an example one might let

$$\sigma \equiv 10^{-2} \text{ S/m}$$

$$\varepsilon = 10\varepsilon_o \simeq .885 \times 10^{-10} \text{ F/m} \qquad (2.17)$$

corresponding to not atypical soil or concrete (neglecting frequency dependence). Then choosing frequencies with

$$s \equiv j\omega$$

$$\omega \equiv 2\pi f \qquad (2.18)$$

we have some examples as

$$f \equiv 1 \text{ MHz}$$

$$j\omega\varepsilon \simeq j.56 \times 10^{-3} \text{ S/m}$$

$$\sigma = 10^{-2} \text{ S/m}$$

$$|\sigma + j\omega\varepsilon| \simeq 10^{-2} \text{ S/m}$$

$$|j\omega\mu_o| \simeq 7.9 \ \Omega/\text{m}$$

$$|\tilde{\Delta}(s)| \simeq \frac{1}{3}(7.9)(10^{-2})\ \ell^2 \simeq 2.6 \times 10^{-2}\ \ell^2 \tag{2.19}$$

and

$$f \equiv 10 \text{ MHz}$$

$$j\omega\varepsilon \simeq j.56 \times 10^{-2} \text{ S/m}$$

$$\sigma = 10^{-2}$$

$$|\sigma + j\omega\varepsilon| \simeq 1.2 \times 10^{-2} \text{ S/m}$$

$$|j\omega\mu_o| \simeq 79 \ \Omega/\text{m}$$

$$|\tilde{\Delta}(s)| \simeq \frac{1}{3}(79)(1.2 \times 10^{-2})\ \ell^2 \simeq .32\ \ell^2 \tag{2.20}$$

with ℓ, of course, in meters. For 1 MHz with $\ell \simeq 1$ m the effect of conductivity and permittivity is quite small. For 10 MHz one should keep ℓ somewhat smaller, say 30 cm, for comparably small errors.

Under high-pressure shock conditions the medium may have somewhat different conductivity. Increased conductivity lowers the operating frequency f, the length ℓ, or both. It is important to know or at least bound the conductivity of the medium under the applicable shock conditions, but such is beyond the scope of this note and perhaps requires some special experiments. In any event sufficiently small f and ℓ can make $|\tilde{\Delta}|$ small compared to one.

An interesting application of (2.15) is contained in $\mathrm{Im}(\tilde{\Delta}(s))$ (i.e., the imaginery part of $\tilde{\Delta}(s)$) which is in principle measurable. In a measurement of $\tilde{Z}_{in}(s)$ consider, for $s = j\omega$, the result

$$\arg(\tilde{Z}_{in}(j\omega)) = \arg(j\omega L) + \arg\left[1 + \tilde{\Delta}(j\omega) + O((\tilde{\gamma}(j\omega)\ell)^4)\right]$$

$$= \frac{\pi}{2} + \arctan\left[\frac{\mathrm{Im}(\tilde{\Delta}(j\omega)) + O((\tilde{\gamma}(j\omega)\ell)^4)}{1 + \mathrm{Re}(\tilde{\Delta}(j\omega)) + O((\tilde{\gamma}(j\omega)\ell)^4)}\right]$$

$$= \frac{\pi}{2} + \tilde{\delta}(j\omega)$$

$$\tilde{\delta}(j\ ) \simeq \mathrm{Im}(\tilde{\Delta}(j\omega))$$

$$= -\frac{1}{3}\,\mathrm{Im}\left[j\omega L(\tilde{G}'(j\omega) + j\omega\tilde{C}'(j\omega)\right]$$

$$\text{for } |\tilde{\Delta}(j\omega)| \to 0 \qquad (2.21)$$

For $\tilde{\sigma}$ real and $\tilde{\varepsilon}$ real

$$\tilde{\delta}(j\omega) \simeq -\frac{1}{3}\,\omega L\tilde{G}'(j\omega)$$

$$= -\frac{1}{3}\,\omega(\mu_o\ell)(\tilde{\sigma}(j\omega)\ell) \qquad (2.22)$$

This result indicates that, while σ must be small to have a valid measurement, a measurement of the input impedance which includes phase can have a self-checking characteristic in that the deviation of the phase from $\pi/2$ is an indication of the perturbation of the measurement by the conductivity of the ambient medium. By measurement at more than one frequency in the same sensor one may even determine how valid is the measurement at the lowest frequency, thereby giving the experiment another self-checking property.

The reader may note that the assumption of perfectly conducting transmission-line conductors (including shorted termination) is not exact in reality. In some cases the conductivity of certain metals may not be sufficiently large. To account for this one may introduce a longitudinal impedance per unit length $\tilde{Z}'_\ell(z,s)$ associated with the transmission-line conductors; this adds to $sL'(z)$ in the per-unit-length model of the transmission line. Neglecting skin depth one might think of this extra term as a resistance per unit length $R'(z)$ giving a resistance R around the conductor path, as seen at the transmission-line port where $\tilde{Z}_{in}(s)$ is measured, as

$$R = R_T + \int_0^\ell R'(z)\, dz \tag{2.23}$$

where, for R' assumed independent of z,

$$R = R_T + R'\ell \tag{2.24}$$

Here R_T is the corresponding assumed resistive impedance of the "shorted" termination.

If the conductivity σ of the ambient medium gets too large, then R can affect the experiment, even though R be small, since some significant part of the current which ideally flows in the termination now flows through the ambient medium. Basically the requirement is

$$R << \left[\left| \tilde{G}'(s) + s\tilde{C}'(s) \right| \ell \right]^{-1} \tag{2.25}$$

or more generally

$$\left| \tilde{Z}_T(s) + \int_0^\ell \tilde{Z}'_\ell(z,s)\, dz \right| << \left[\left| \tilde{G}'(s) + s\tilde{C}'(s) \right| \ell \right]^{-1} \tag{2.26}$$

This constraint says that ℓ should not be too large and/or the cross-section dimensions of the conductors (and their conductivities) should not be too small. Note that the conductivities of the transmission-line conductors may be also affected under high-pressure shock conditions.

A more accurate version of the conductor-conductivity effects can be obtained by noting that the transmission-line model can be modified to include $\tilde{Z}'_\ell(z,s)$ in the longitudinal impedance per unit length as

$$
\begin{aligned}
\tilde{Z}'(z,s) &= sL'(z) + \tilde{Z}'_\ell(z,s) \\
\tilde{Y}'(z,s) &= s\tilde{C}'(z,s) + \tilde{G}'(z,s) \\
\tilde{\gamma}(z,s) &= [\tilde{Z}'(z,s)\ \tilde{Y}'(z,s)]^{1/2} \\
\tilde{Z}_c(z,s) &= \left[\frac{\tilde{Z}'(z,s)}{\tilde{Y}'(z,s)}\right]^{1/2}
\end{aligned}
\tag{2.27}
$$

In this form $\tilde{Z}'_\ell(z,s)$ enters as a direct correction to $sL'(z)$, such as found in (2.11), again assuming $\tilde{\gamma}\ell$ small. For an inductive impedance to be dominant, then $\tilde{Z}'_\ell$ must also be small compared to sL' (even during the high-pressure shock conditions).

III. Longitudinal Variation of Conductivity and Dielectric Constant in Ambient Medium

With the transmission-line conductors assumed approximately parallel to the direction of flow of the medium, and assumed to be flowing with the medium, there may be conductivity and permittivity variations along the transmission line. Figure 3.1 shows the case of a transmission-line with short-circuit termination and two regions with assumed different conductivities (and perhaps different permittivities).

The results of the previous section can be used to calculate the input impedance in two steps. Consider first the impedance at the shock front which is used to approximately distinguish the two regions. From (2.15) we write

$$\tilde{Z}_1(s) = sL_1\left[1 + \tilde{\Delta}_1(s) + O((\tilde{\gamma}_1(s)\ell_1)^4)\right]$$
$$L_1 = L'\ell_1 = \mu_o f_g \ell_1 \tag{3.1}$$

where corresponding to (2.16) we have

$$\tilde{\Delta}_1(s) = -\frac{1}{3}\, s\mu_o \ell_1(\tilde{\sigma}_1(s) + s\tilde{\varepsilon}_1(s))\, \ell_1$$
$$= -\frac{1}{3}\, sL_1(\tilde{G}_1(s)\ell_1 + s\tilde{C}_1(s)\ell_1) \tag{3.2}$$

For these expressions to be useful we assume that $|\tilde{\Delta}_1| \ll 1$. However, since $\ell_1 < \ell$, then we can allow $\tilde{\sigma}_1$ to be somewhat higher than the unperturbed conductivity $\tilde{\sigma}_2$ (with similar comments applying to the permittivity) since $\tilde{\Delta}_1$ is proportional to $\tilde{\sigma}_1\ell_1^2$ for fixed frequency. From this one can conclude that for large $\tilde{\sigma}_1$ there still exists some small ℓ_1 and therefore small time interval past shock passage of the shorted termination for which $\tilde{Z}_1$ is approximately purely inductive. In some cases this time interval might be too small to be of interest if $\tilde{\sigma}_1$ is too large; the operating frequency constrains minimum times of interest. Of course DC operation is also possible, but that is another subject.

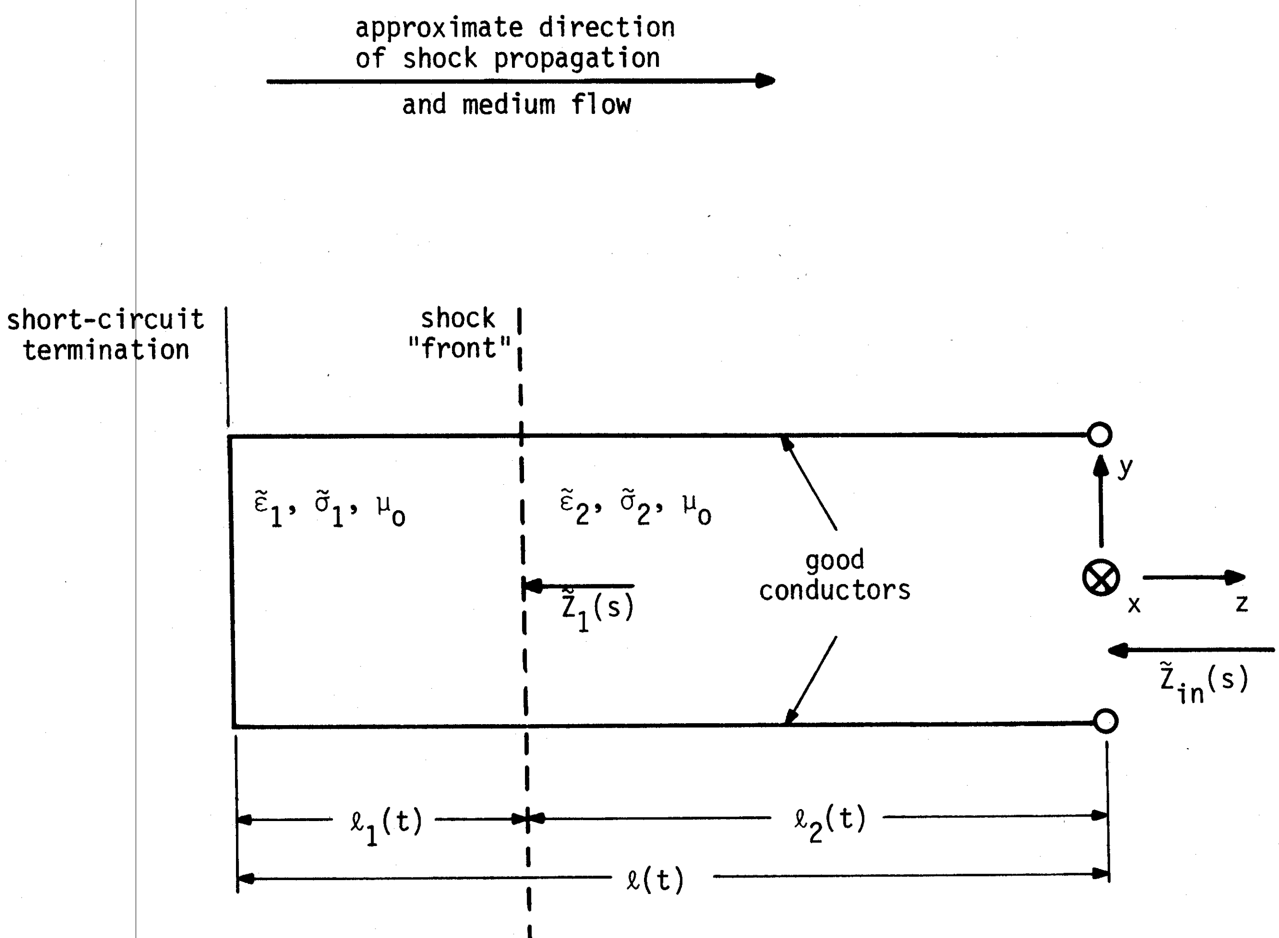


Figure 3.1. Two Parts of Transmission Line with Different Conductivities and Permittivities

Carrying the analysis further the impedance $\tilde{Z}_1$ can be considered a termination of the second part of the transmission line of length ℓ_2. From (2.11) and common L' for both regions we can write

$$\tilde{Z}_{in}(s) = \frac{\tilde{Z}_1(s)\left[2 + (\tilde{\gamma}_2(s)\ell_2)^2 + O((\tilde{\gamma}_2(s)\ell_2)^4)\right] + sL'\ell_2\left[2 + \frac{1}{3}(\tilde{\gamma}_2(s)\ell_2)^2 + O((\tilde{\gamma}_2(s)\ell_2)^4)\right]}{\tilde{Z}_1(s)\left[\tilde{G}_2'(s) + s\tilde{C}_2'(s)\right]\ell_2\left[2 + \frac{1}{3}(\tilde{\gamma}_2(s)\ell_2)^2 + O((\tilde{\gamma}_2(s)\ell_2)^4)\right) + \left[2 + (\tilde{\gamma}_2(s)\ell_2)^2 + O((\tilde{\gamma}(s)\ell)^4)\right]} \tag{3.3}$$

Suppose now that we assume (noting $\ell_2 \leq \ell$)

$$\begin{aligned} |\tilde{\gamma}_2(s)\ell| &<< 1 \\ |s\mu_o\left[\tilde{\sigma}_2(s) + s\tilde{\varepsilon}_2(s)\right]|\ \ell^2 &<< 1 \end{aligned} \tag{3.4}$$

and (noting $\ell_1 < \ell$)

$$\begin{aligned} |\tilde{Z}_1(s)\left[\tilde{G}_2'(s) + s\tilde{C}_2'(s)\right]\ell| &\simeq |sL_1\left[\tilde{G}_2'(s) + s\tilde{C}_2'(s)\right]\ell| \\ &= |s\mu_o\ell_1\left[\tilde{\sigma}_2'(s) + s\tilde{\varepsilon}_2'(s)\right]\ell| << 1 \end{aligned} \tag{3.5}$$

The first restriction requires $\tilde{\sigma}_2$ and $\tilde{\varepsilon}_2$ to be sufficiently small that the length of line be electrically small. Combined with the previous restrictions concerning small $\tilde{Z}_1(s)$ the second restriction follows. Then neglecting $\tilde{\gamma}_2\ell_2$ terms as well as the $\tilde{Z}_1(s)(\tilde{G}_2' + s\tilde{C}_2')\ell_2$ term we have

$$\tilde{Z}_{in}(s) \simeq \tilde{Z}_1(s) + sL'\ell_2$$

$$= sL'\ell_1\left[1 + \tilde{\Delta}_1(s) + O((\tilde{\gamma}_1(s)\ell_1)^4)\right] + sL'\ell_2$$

$$= sL'\ell + sL'\ell_1 \left[\tilde{\Delta}_1(s) + O((\tilde{\gamma}_1(s)\ell_1)^4)\right]$$

$$\simeq sL'\ell + sL'\ell_1\tilde{\Delta}_1(s)$$

$$= sL'\ell\left[1 + \frac{\ell_1}{\ell}\tilde{\Delta}_1(s)\right]$$

$$= s\mu_o\ell f_g\left[1 + \frac{\ell_1}{\ell}\tilde{\Delta}_1(s)\right] \tag{3.6}$$

$$\ell \equiv \ell_1 + \ell_2$$

so that our relative error is $\tilde{\Delta}_1(s)$ (as in (3.2)) reduced by the ratio ℓ_1/ℓ. However the error relative to $\tilde{Z}_1(s)$ is just $\tilde{\Delta}_1$ and this may be more meaningful since the length change associated with the motion of the medium is more closely related to ℓ_1 than ℓ.

IV. Implications of Transverse Flow of Ambient Medium

Up till now we have considered the effects of changes in parameters as a function of the longitudinal coordinate z. However, the medium can flow in transverse directions as well. In an expanding flow there is a dilation or enlargement of the transverse dimensions of some portion of the ambient medium, and hence of the transmission-line transverse dimensions as well. In cases of contracting flow (as in a nozzle or an implosion) there is a corresponding decrease of the transmission-line cross-section dimensions.

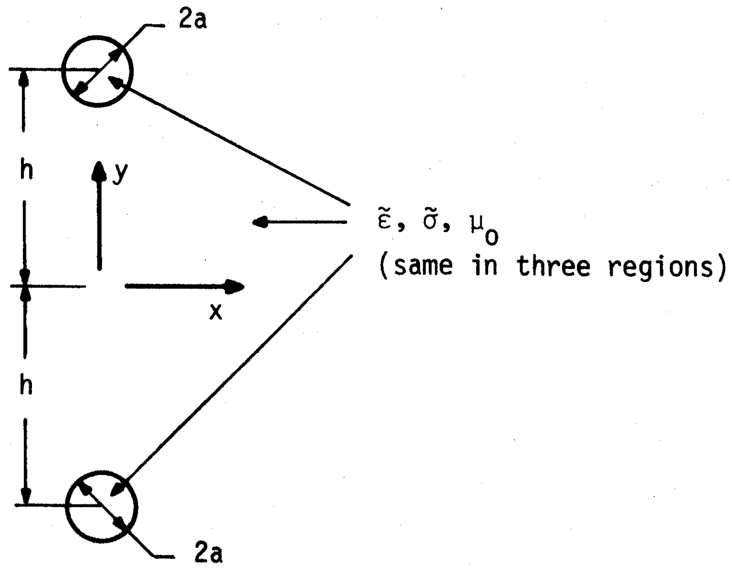
Figure 4.1 shows two transmission-line cross sections as examples. Note that in each case the conductors are thin shells with ambient medium on both sides of the highly conducting sheets so that the sensor flows with the medium.

As a special case let us assume that the expansion (or contraction) of the medium in the x and y directions is the same. Stated another way, if $x \to x'$ and $y \to y'$ under the medium flow then we require

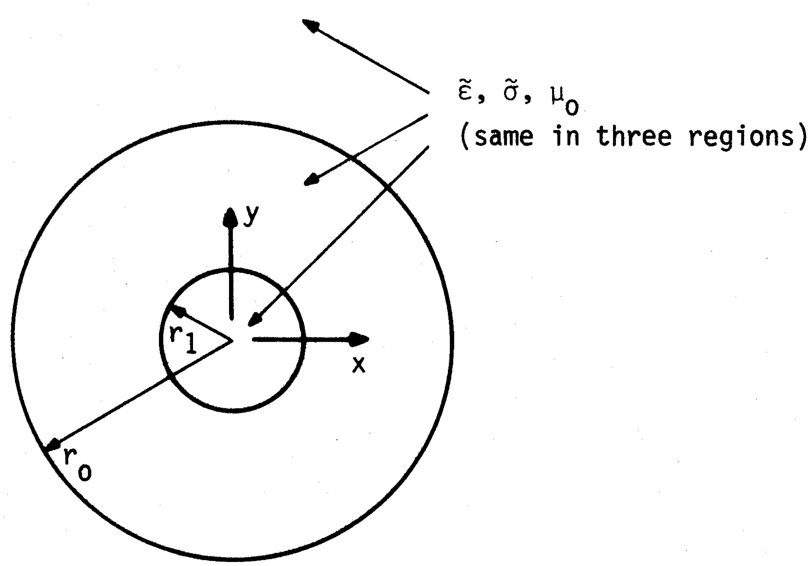
$$\frac{x'}{x} = \frac{y'}{y} \quad \text{for all } (x,y) \tag{4.1}$$

Since we are only concerned with the local behavior of the medium (near the transmission line) then we choose the origin for the cross-section coordinates $(x,y) = (0,0)$ somewhere near the transmission line, typically at the center of the cross section if such is definable by two (or more) symmetry planes through such a center, or by a symmetry axis.

For the case of isotropic transverse expansion (or contraction) our problem simplifies considerably. Specifically the dimensionless geometrical factor f_g of the transmission-line structure remains <u>unchanged</u>. This implies that the inductance per unit length L' remains unchanged. Summarizing, then we have



A. Two Identical Tubes



B. Circular Coaxial Cylindrical Conductors

Figure 4.1. Transmission-Line Cross Sections

$$f_g \text{ independent of } z$$

$$L' = \mu_o f_g \text{ independent of } z \tag{4.2}$$

This result is elementary, being based on scaling properties of electromagnetic problems in two dimensions; the fundamental f_g term is always observed to be a function of only dimensionless ratios of the cross-section characteristic dimensions. This applies to both examples in figure 4.1; the two tubes have f_g a function of only a/h, and the coax has f_g only a function of r_1/r_o; since such ratios are unchanged, then f_g is unchanged.

For the important case of the coax we have

$$f_g = \frac{1}{2\pi} \ln\left(\frac{r_o}{r_1}\right) = \frac{1}{2\pi} \ln\left(\frac{r_o'}{r_1'}\right) \tag{4.3}$$

This is perhaps the simplest case, but also a most important case for our consideration, due to its high symmetry as well as its electromagnetic shielding properties. Note here that r_o (or r_o') is the inner radius of the outer conductor, while r_1 (or r_1') is the outer radius of the inner conductor. The conductors are assumed thin so as not to interfere with the flow of the ambient medium, but not so thin as to introduce a significant longitudinal impedance (as compared to some fraction of sL) associated with the conductors.

Relaxing somewhat our assumptions concerning the flow of the ambient medium transverse to the z axis, let it no longer be isotropic but still somewhat restricted. Consider unit vectors corresponding to the unperturbed coordinates (x,y,z) (Eulerian position) with relations

$$\vec{1}_x \times \vec{1}_y = \vec{1}_z$$

$$\vec{1}_y \times \vec{1}_z = \vec{1}_x$$

$$\vec{1}_z \times \vec{1}_x = \vec{1}_y$$
$$\vec{r} = x\vec{1}_x + y\vec{1}_y + z\vec{1}_z \qquad (4.4)$$

giving a right-handed cartesian coordinate system with orthogonal unit vectors. For the perturbed coordinates (x',y',z') corresponding to the changed Eulerian position of the same incremental part of the medium we have

$$\vec{r}\,' = x'\vec{1}_{x'} + y'\vec{1}_{y'} + z'\vec{1}_{z'} \qquad (4.5)$$

Let us assume there exists a choice of our original coordinates such that (at least locally)

$$\vec{1}_{x'} = \vec{1}_x \ , \quad \vec{1}_{y'} = \vec{1}_y \ , \quad \vec{1}_{z'} = \vec{1}_z \qquad (4.6)$$

i.e., the coordinate directions are unchanged, and hence still orthogonal. This assumption can be relaxed somewhat if we adopt a differential-geometry approach [1] and allow (x',y',z') to be an orthogonal curvilinear coordinate system. In this case we would have (assuming right-handed (x',y',z'))

$$\vec{1}_{x'}(\vec{r}\,') \times \vec{1}_{y'}(\vec{r}\,') = \vec{1}_{z'}(\vec{r}\,')$$
$$\vec{1}_{y'}(\vec{r}\,') \times \vec{1}_{z'}(\vec{r}\,') = \vec{1}_{x'}(\vec{r}\,') \qquad (4.7)$$
$$\vec{1}_{z'}(\vec{r}\,') \times \vec{1}_{x'}(\vec{r}\,') = \vec{1}_{y'}(\vec{r}\,')$$

i.e., the coordinates are locally orthogonal but each unit vector (direction of increasing coordinate) is a function of the coordinates $\vec{r}\,'$ (and hence of $\vec{r}$). Note in particular that this differential-geometry approach also allows rotations while preserving local coordinate orthogonality.

Since uniform expansion in both x and y directions (while remaining orthogonal to z, the sensor longitudinal coordinate) leaves the transmission-line geometrical factor f_g unchanged, then let us consider the case that the medium expansion is different in these two coordinate directions. Let the x' and y' coordinates (and hence x and y) be chosen to diagonalize this medium expansion and thereby remain orthogonal under expansion in the above sense.

Consider an expansion ratio (for small (x',y'))

$$\xi \equiv \frac{y'}{y} \frac{x}{x'} \tag{4.8}$$

or in difference (incremental) form

$$\xi \equiv \frac{\Delta y'}{\Delta y} \frac{\Delta x}{\Delta x'} \tag{4.9}$$

where $\Delta \vec{r}\,'$ is the difference in two positions (nearby) in the medium under expansion. Let

$$\begin{aligned} \xi_o &\equiv \text{expansion ratio before expansion} \\ &\equiv 1 \end{aligned} \tag{4.10}$$

Similarly let the transmission-line geometrical factor be

$$\begin{aligned} f_{g_o} &\equiv \text{geometrical factor before expansion} \\ f_g &\equiv \text{geometrical factor during expansion} \end{aligned} \tag{4.11}$$

Appropriate changes are

$$\begin{aligned} \Delta\xi &\equiv \xi - \xi_o \equiv \xi - 1 \\ \Delta f_g &\equiv f_g - f_{g_o} \end{aligned} \tag{4.12}$$

To see some of the effects of this anisotropic expansion consider a simple example in figure 4.2A consisting of two plates of width 2a and spacing 2b with a >> b. The geometrical factor is

$$f_g \simeq \begin{cases} \dfrac{b}{a} & \text{before expansion} \\ \dfrac{b'}{a'} & \text{during expansion} \end{cases}$$

a' proportional to x'

b' proportional to y'

$$\frac{b'}{a'} = \frac{y'b}{y}\,\frac{x}{x'a} = \xi\,\frac{b}{a} \qquad (4.13)$$

Hence if x' expands more than y', then f_g decreases; if y' expands more than x', then f_g increases. For this case then we have

$$f_g \simeq \xi\,\frac{b}{a}\ , \qquad f_{g_o} \simeq \frac{b}{a}$$

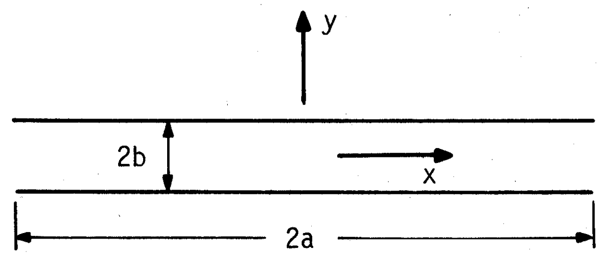
$$\Delta f_g \simeq (\xi - 1)\,\frac{b}{a}$$

$$\frac{\Delta f_g}{f_{g_o}} \simeq \xi - 1 \qquad (4.14)$$

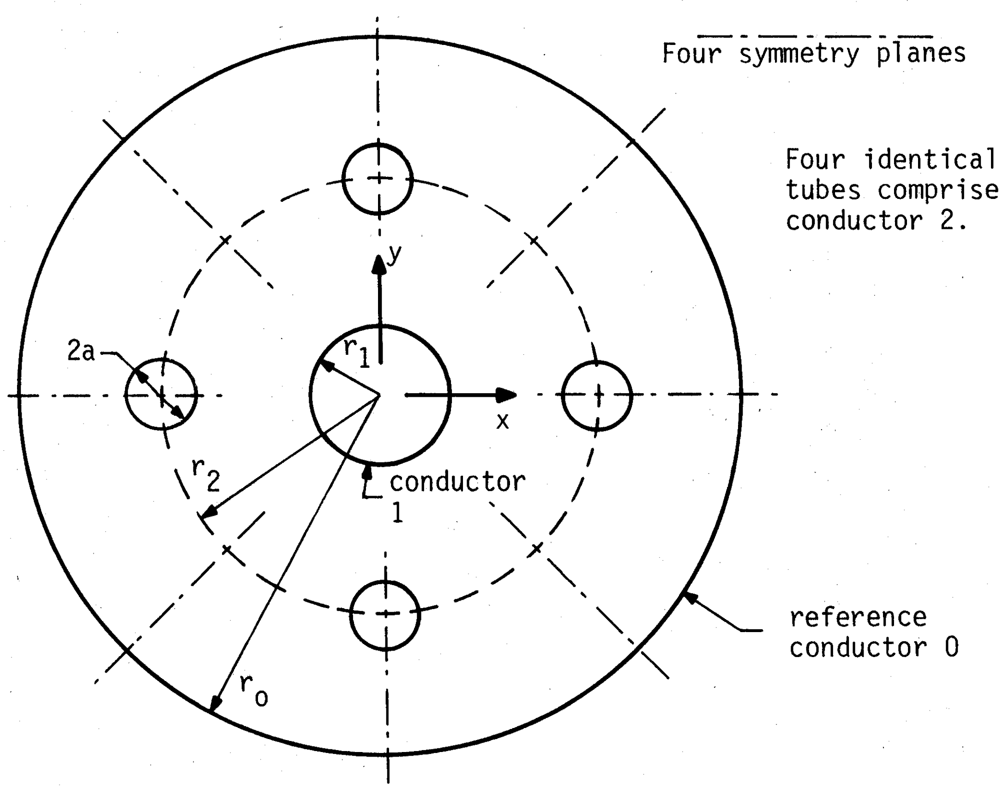
Thus the relative change of the geometrical factor is first order in $\xi - 1$ for the case of wide, closely-spaced parallel plates.

To reduce this problem of the effects of anisotropic transverse expansion on f_g one may invoke symmetry. If the transmission-line cross section is the same with respect to both x and y axes then x' expanding more than y' must produce the same effect on f_g as y' expanding more than x'. In both cases f_g will change in the same way because the symmetry makes the geometrical changes have the same effect. Quantitatively then for small $\Delta\xi$

$$\frac{\Delta f_g}{f_{g_o}} = \text{even function of } \Delta\xi \qquad (4.15)$$



A. Wide, Closely Spaced Parallel Plates



B. Circular Coax with Four Intermediate Symmetrical Tubes

Figure 4.2. Transmission-Line Cross Sections Related to Transverse Anisotropic Medium Flow

If Δf_g is an analytic function of $\Delta\xi$, then

$$\frac{\Delta f_g}{f_{g_o}} = O((\Delta\xi)^2)$$

$$= O((\xi - 1)^2) \qquad (4.16)$$

For small $\Delta\xi$ (i.e., small anisotropy of transverse medium flow) this makes changes in f_g small (by at least one order) compared to cases without this special symmetry, e.g., the parallel-plate result in (4.14).

An alternate approach to the influence of symmetry on expansion ratio uses a logarithmic expansion ratio as

$$\zeta \equiv \text{logarithmic expansion ratio}$$

$$\equiv \ell n(\xi)$$

$$= \ell n\left[\frac{\Delta y'}{\Delta y}\,\frac{\Delta x}{\Delta x'}\right] = \ell n\left[\frac{\Delta y'}{\Delta y}\right] - \ell n\left[\frac{\Delta x'}{\Delta x}\right] \qquad (4.17)$$

with

$$\zeta_o \equiv \text{logarithmic expansion ratio before expansion}$$

$$\equiv 0 \qquad (4.18)$$

This form clearly illustrates the symmetry with respect to changes in x' and y' as a sign reversal for changes in ζ. In terms of this logarithmic expansion ratio and for the transmission-line cross section being the same with respect to both x and y (including electrical connections) then we have

$$\frac{\Delta f_g}{f_{g_o}} = \text{even function of } \zeta \qquad (4.19)$$

which is valid even for large $|\zeta|$. This case then has for small ζ (assuming f_g an analytic function of ζ)

$$\frac{\Delta f_g}{f_g} = O(\zeta^2) \tag{4.20}$$

The type of symmetry discussed here is a rotation symmetry in two dimensions (the (x,y) plane). Having the same geometry with respect to both x and y axes (for all choices of orthogonal x and y axes) means invariance with respect to a rotation by $\pi/2$. This in turn implies a set of rotations: $\pm\pi/2$, $\pm\pi$, $\pm 3\pi/2$, and 0 or $\pm 2\pi$ (identity). This defines a finite uniaxial rotation group in two dimensions labelled C_4 where C_n is a uniaxial symmetry group (a cyclic group) related to a single n-fold rotation axis (rotation by $2\pi/n$) [2]. Note that in addition to the geometrical symmetry of the conductors electrical symmetry is also required in the currents, potentials, etc.; conductors in symmetrical positions must be driven and loaded with the same symmetry. Of course higher order uniaxial groups such as C_{4m} ($m = 2,3,4,\ldots$) contain C_4 as a commutative subgroup and are therefore consistent with C_4 symmetry. In the limit we have the two-dimensional pure-rotation group C_∞ [2].

The constraint of C_4 symmetry still allows a wide variety of cross-section shapes. Some of these, such as portions of spirals, may not be interesting for our present purposes. Typically we will include a plane of symmetry through the z axis. However the C_4 symmetry then implies there must be another plane of symmetry through the z axis and perpendicular to the first symmetry plane (rotation by $\pi/2$). Combining reflection (symmetry planes) and rotations by $\pi/2$ one obtains further symmetries including inversion and the location of two more symmetry planes through the z axis at angles of $\pi/4$ with respect to the first symmetry plane. Note that this is the same symmetry as possessed by a square (and other figures) in the (x,y) plane. The corresponding symmetry group is labeled $C_{4,v}$ and is of order 8 (8 group elements) [2]. Higher order symmetries such as $C_{4m,v}$ ($m = 2,3,4,\ldots$) contain $C_{4,v}$ as a subgroup. In the limit as $m \to \infty$ we have $C_{\infty,v}$. This symmetry is possessed by the coaxial geometry of figure 4.1B.

Another example of this $C_{4,v}$ symmetry is illustrated in figure 4.2B. Here the coaxial cylinders of radius r_1 and r_o certainly meet the above symmetry constraints; a circular coax such as this part (or as in figure 4.1B) has even higher order symmetry. In addition four tubes (hollow wires filled with ambient medium perhaps) each of radius a are shown with centers on radius r_2 with $r_1 < r_2 - a < r_2 + a < r_o$, and with even angular spacing of $\pi/2$. This gives a case of four symmetry planes as discussed above. Note that the four tubes at r_2 must have the same transmission-line currents (including direction with respect to $\vec{1}_z$,) and the same potentials for this symmetry to hold. This is obtained by connecting the four tubes together through equal impedances to a common terminal at a common z in each instance. Note that this geometry is suitable for a mutual-inductance type of sensor since there are three accessible terminals (two ports) at $z = 0$; at $z = -\ell$ all three sets of conductors would be shorted together with a flat conducting disk (as in figure 3.1). Note also triaxial geometry (three coaxial circular cylinders) also meets the above symmetry requirement and could be useful for a mutual inductance sensor. In addition an outer conducting cylinder is useful as a shield to keep out environmental electromagnetic noise (such as from EMP or from other instrumentation).

Other techniques can be also used to make f_g insensitive to changes in the ratio of cross-section dimensions. As in the case of a coax f_g depends on a logarithm of the ratio of dimensions. This observation applies to any case of a thin strip, wire, etc. spaced a distance large compared to the conductor cross-section dimensions from another conductor. If f_g is of the form

$$f_g = A \ \ln(\frac{a}{b})$$

$$\frac{a}{b} >> 1 \tag{4.21}$$

then small relative changes in a/b give even smaller changes in f_g as

$$\begin{aligned} f_g + \Delta f_g &= A \ \ln\left(\frac{a}{b} + \Delta(\frac{a}{b})\right) \\ &= A\left\{\ln(\frac{a}{b}) + \ln\left[1 + \frac{b}{a} \Delta(\frac{a}{b})\right]\right\} \\ &\simeq A\left\{\ln(\frac{a}{b}) + \frac{b}{a} \Delta(\frac{a}{b})\right\} \\ f_g\left[1 + \frac{\Delta f_g}{f_g}\right] &\simeq A \ \ln(\frac{a}{b})\left\{1 + \frac{\frac{b}{a} \Delta(\frac{a}{b})}{\ln(\frac{a}{b})}\right\} \\ &\simeq A \ \ln(\frac{a}{b})\left\{1 + \frac{1}{\ln(\frac{a}{b})} \left[\frac{\Delta a}{a} - \frac{\Delta b}{b}\right]\right\} \end{aligned} \tag{4.22}$$

so that the relative change in f_g is reduced by an additional factor of $\ln(a/b)$. This result illustrates that the logarithm is a slowly varying function of its argument. The introduction of appropriate logarithmic factors in the design of f_g can then make f_g less sensitive to the transverse medium flow. This can be used in conjunction with the symmetry discussed above.

V. Extension to Mutual Inductances

It is important to note that while our discussion has centered on the simpler case of a single inductance which varies under the shock conditions, the same general results apply for mutual inductances (or more general mutual impedances) between conductors in transmission-line geometries as well. Consider a multiconductor transmission line as in figure 5.1; this example has two conductors plus a reference conductor for voltage reference (zero volts). As usual, assume the medium flow is parallel to the transmission line (z axis). Let the ambient medium have electromagnetic parameters that are independent of z and are assumed linear and scalar; as usual the permeability is assumed μ_o.

Consider an N-conductor-plus-reference transmission line. Take an incremental length Δz which has a uniform permittivity, conductivity, and permeability. This is the well-known degenerate case of (perfect) transmission-line conductors in a uniform medium for which the N transmission-line modes of propagation are all TEM and the propagation matrix $(\tilde{\gamma}_{n,m}(s))$ has N identical eigenvalues, all $\tilde{\gamma}(s)$, as in (2.3). Next let the ratios of cross-section dimensions be unchanged under medium expansion, or by use of symmetry as discussed in the previous section let the effects of such changes be reduced to some acceptably small level. Then we have N propagating modes all with the same propagation constant

$$\tilde{\gamma}(z,s) = [s\mu_o(\tilde{\sigma}(z,s) + s\tilde{\varepsilon}(z,s))]^{1/2} \tag{5.1}$$

and a characteristic-impedance matrix

$$\left(\tilde{Z}_{c_{n,m}}(z,s)\right) = \tilde{Z}(z,s)(f_{g_{n,m}})$$

$$n,m = 1,2,\ldots,N$$

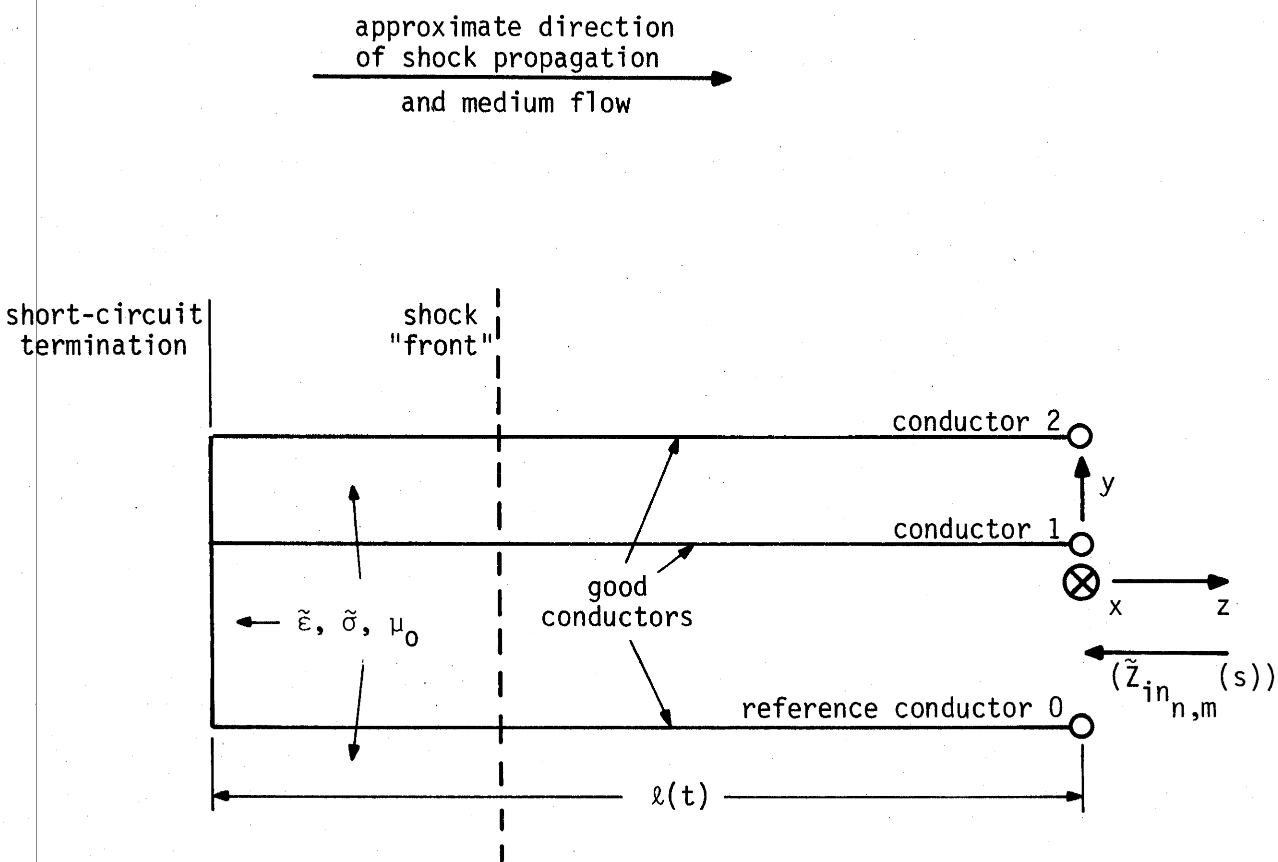


Figure 5.1. Multiconductor Transmission Line with Axis Parallel to Flow in Ambient Medium: Case of Two Conductors Plus Reference

$$\tilde{Z}(z,s) = \left[\frac{\tilde{\sigma}(z,s) + s\tilde{\varepsilon}(z,s)}{s\mu_o}\right]^{1/2} \tag{5.2}$$

$(f_{g_{n,m}}) \equiv$ dimensionless geometrical-factor matrix

The input impedance then has the same results as in sections II and III for shorted termination (now all conductors being shorted together at $z = -\ell$) except that f_g is replaced by $(f_{g_{n,m}})$. See, for example (2.15) and (3.6). Here we have

$$\left(\tilde{Z}_{in_{n,m}}(s)\right) = \tilde{Z}_{in_o}(s)(f_{g_{n,m}}) \tag{5.3}$$

where $\tilde{Z}_{in_o}(s)$ takes the forms developed in sections II or III or more accurate forms that may be developed for the input impedance of the type of transmission line we are considering (with f_g factored out).

Considering the example in figure 4.2B let the outer conducting circular cylinder of radius r_o be the reference conductor since it also serves as an electromagnetic shield. Let the inner conducting cylinder of radius r_1 be conductor 1. And let the four tubes of radius a and centered on a radius r_2 be collectively conductor 2 having a common voltage and equal currents. Neglecting the effects of the small tube radius a in appropriate cases we have

$$f_{g_{1,1}} \simeq \frac{1}{2\pi}\,\ell n\left(\frac{r_o}{r_1}\right) \tag{5.3}$$

corresponding to port 1 input impedance $\tilde{Z}_{in_{1,1}}(s)$, port 1 inductance $L_{1,1}$ (self inductance) etc., obtained by leaving port 2 open circuited. Leaving port 2 again open circuited, but driving port 1 with a current and measure the voltage at port 2 gives a mutual impedance characterized by a geometrical factor

$$f_{g_{2,1}} \simeq \frac{1}{2\pi}\,\ell n\left(\frac{r_o}{r_2}\right) \tag{5.4}$$

with

$$f_{g_{1,2}} = f_{g_{2,1}} \tag{5.5}$$

by reciprocity. This gives a mutual inductance

$$L_{2,1} = L_{1,2} = \mu_o \ell f_{g_{2,1}} \tag{5.6}$$

and a corresponding mutual impedance $\tilde{Z}_{in_{2,1}}(s)$ or $\tilde{Z}_{in_{1,2}}(s)$. Between these two independent values of $f_{g_{n,m}}$ one can make a mutual-inductance type of sensor by driving one port and measuring at the other port. One may also compute $f_{g_{2,2}}$, but this is more complicated and involves a logarithmic function of the tube radius a.

VI. Measuring Inductance Change

Having a sensor with an inductance or mutual-inductance proportional to $\ell(t)$ with

$$\ell(t) - \ell_o = -(z'(t) - z) \tag{6.1}$$

where ℓ_o is the value of the length before medium flow and $z'(t)$ is the position of the short at the end of the transmission line (with z its unperturbed value), the problem is to measure an appropriate $L_{n,m}(t)$. This applies only up to such time as the medium flow reaches the input terminals of the transmission line. There are various approaches to the measurement of such inductances.

One technique would utilize the simple case of self inductance as part of an LC oscillator with a (slowly) time-varying frequency output

$$f(t) = \frac{1}{2\pi}\,\omega(t) = \frac{1}{2\pi}\,[L(t)C]^{-1} \tag{6.2}$$

This might be directly recorded after transmission via cable, or it might be first converted to an analog signal proportional to $f(t)$. Other types of reactive networks can be combined with $L(t)$ to give a different dependence of $f(t)$ on $L(t)$; having $f(t)$ then $L(t)$ is found through the designed relationship. Note that the period of $f(t)$, i.e., $f(t)^{-1}$, and the response time of the oscillator to follow changes in $L(t)$, should be small compared to times of interest in following $L(t)$.

Another technique would make $L(t)$ part of a voltage divider with a reference inductance L_r, or more general impedance $\tilde{Z}_r(s)$. Then with the loading cable from the sensor and the reference impedance (including source impedance) the signal at the recorder can be used to infer $L(t)$ from the time varying voltage-divider ratio at the operating frequency f, again with f^{-1} sufficiently small. A more sophisticated version of this would have L(t) as one arm of a bridge so that the initial RF amplitude going away to the recorder was essentially zero compared to the source signal, except for some small offset; as L(t) decreased the RF signal to the recorder would increase and could be amplitude discriminated.

One could also use a direct reflection technique with an RF signal transmitted down a cable to the sensor and reflected back from the input port of the sensor. By having the source impedance the same as the cable characteristic impedance, one can terminate the reflected wave at the source. Then by sampling the signal on the cable one can determine the standing wave properties as a function of time to obtain both phase and amplitude of the reflection as a function of time. Alternately one samples the reflected signal via a directional coupler. The magnitude as a function of time is directly obtained. By comparing the sampled reflected signal with a sample of the source signal phase can also be obtained, or if one wishes real and imaginary parts of the complex phasor reflection can also be generated.

A related technique has the directional coupler effectively as part of the sensor. This is the mutual-inductance type of sensor, such as the examples in figures 4.2B and 5.1. For this kind of sensor a signal is driven into one port and detected from the second port. Note the common ground which connects to the cable shields. The source signal may be generated at the input port or may be transported from some distance away via cable. In the latter case particularly there may be some impedances at the input port to make a constant current (CW) into the port, and/or minimize reflections back toward the source. The second port only has one signal to contend with, namely the signal leaving the port, since the other end of this cable is terminated in its characteristic impedance. At the recording instrumentation the signal from the second port can be compared to a sample of the source signal to obtain magnitude and phase (or equivalent) information concerning the mutual inductance of the sensor (and hence $\ell(t)$).

In various of these schemes it is the phase $\phi(t)$ of the reflected or transmitted signal that most directly relates to the inductance (self or mutual) or its change. A phase detector then basically gives $\ell(t)$ with various schemes having a more or less linear dependence of $\ell(t)$ on $\phi(t)$. If an analog signal proportional to $\phi(t)$ is differentiated with respect to time then $\ell(t)$ is differentiated with respect to time to some approximation. Before the medium flow reaches the sensor connectors $d\ell(t)/dt$ is the velocity of the flow at the shorted end. This is sometimes referred to as the particle velocity.

Note that if $\phi(t)$ is a linear function of t this corresponds to a frequency shift of amount $d\phi(t)/dt$ (in radian units). Thus we can interpret

$$\frac{d\phi}{dt} = -\Delta\omega(t) = -2\pi\Delta f(t) \tag{6.3}$$

giving a frequency shift. Thus for velocity purposes (assuming phase linear in $\ell(t)$) we can interpret particle velocity as a frequency shift, similar to a Doppler shift. In such a case FM discrimination of the reflected or transmitted signal can be used. Note for this purpose phase is a positive increase along the direction of propagation in the cables and a decreasing $\ell(t)$ decreases phase in the reflection type of sensor and increases the frequency of the reflected signal.

The experimenter is not constrained to operate the sensor at a single frequency. By operating at several frequencies one may have crosschecks on the results. As noted in sections II and III, if the operating frequency is too high the conductivity and permittivity of the medium affect the sensor impedances, making them other than pure inductances. By having several frequencies one can observe from the experimental records which frequencies give valid results at various times. In addition information is gained about the transient medium conductivity and permittivity under shock conditions. Even DC (zero frequency) excitation might be used to give a transient signal proportional to $d\ell(t)/dt$ as another check. This multifrequency operation can be accomplished by the use of several sensors (side by side) operated at different frequencies, one sensor operated at several frequencies, or some combination of these.

VII. Summary

Having worked out some of the basic equations and design concepts for measuring Eulerian position (one dimensional) in a shocked flowing medium via transmission-line structures operated in inductive configuration, there is still much detailed design work to be done.

Concerning some of the sensor basics some conclusions from differential geometry, group theory (symmetry), and logarithmic dependence on dimensions have been used to optimize the kind of transmission-line cross section one would best use. Perhaps these approaches can be carried further to obtain more design constraints. In addition to these, one can investigate the effects of small changes in the cross-section boundaries by a perturbation theory approach.

The conductivity and permittivity of the ambient medium are of potential importance to this type of measurement. The use of inductive techniques minimizes the dependence on the conductivity and permittivity, but for high frequencies, long sensors, and high conductivity and/or permittivity these parameters become important. The conductivity and permittivity effectively limit the parameter space in which this type of sensor may be used. It is therefore important to have quantitative information on the conductivity and permittivity of the ambient medium, including under high-pressure shock conditions, and including as a function of frequency. Fortunately the sensor can be made somewhat self checking in that part of the sensor output can be used to record the presence (or absence) of significant conductivity/permittivity effects, and this can be done simultaneously at several frequencies.

The properties of the conductors used to make the sensor need to be understood under the same conditions. These properties are mechanical in that they must not perturb the flow of the ambient medium. They are also electrical in that sufficiently high conductivity must be maintained. If any insulators are also used, their properties should be similarly understood.

Associated with such sensors one needs appropriate precision RF sources, networks, cables, discriminators, and recorders. Some of these items have been briefly touched on in this note. Optimum designs are needed.

Note that some of the ways these inductive sensors can be used involve passing a signal by or through the sensor. If the source and signal are available to the experimenter just before shot time, then one can determine at least some information about the operating condition of the sensor and cabling at shot time, when the sensor may be literally "cast in concrete."

While this note has not explicitly considered the EMP coupling to the sensor, including Compton currents present in nuclear source regions, there is much information on such phenomena. Techniques similar to those used in EMP sensors involving low atomic number and matched atomic number can be used [3]. Of course, high-quality shielding of the sensors, cabling, and instrumentation enclosures can be done to at least traditional EMP standards. Note that in a nuclear-source-region environment blast and shock sensors such as these and EMP sensors will have to be integrated in a common experimental layout if they are near each other. This will involve integrated cable and shield topologies.

VIII. References

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