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SIGNAL ANALYSIS IN INSTRUMENTATION

Josef F. Schneider

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Civil Engineering Research Division
Air Force Weapons Laboratory
Kirtland AFB, New Mexico

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FOREWORD

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ABSTRACT

Signal analysis is the answer to many problems that plague instrumentation engineers. It is a function that has to be organizationally recognized. The tools it provides are Fourier transform and its associated functions: frequency spectrum, transfer function, and correlation functions. Applications are: quick-look analysis, determination and development of data reduction parameters and procedures, analysis and development of components and systems, signal simulation, and quality control automation. The requirements for a basic system are discussed.

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INTRODUCTION

"The test went off beautifully; all sensors worked; we got good data" is very often the on-the-spot assessment before the smoke has cleared. The head-scratching begins later when Data Reduction does not seem to be able to come up with the right answers. What went wrong?

Signal analysis can solve this problem. It provides the tools to honestly evaluate the quality of the data signals after the damage has been done, or even more important, allows to check the instrumentation prior to a test, before the damage can be done. It strengthens the confidence of the instrumentation people in their equipment and the confidence of the analysts in the instrumentation.

CONCEPT

Every instrumentation system has problems that signal analysis can be called upon to solve. The larger the system, the more automation is needed for economical analysis. One channel of data in the laboratory can still be handled with a sine wave generator, a meter, and a writing pad; whereas, several hundred channels installed at a remote location require the sophisticated tools that have become available with the advent of the mini-computer.

If there are no problems, that is, if the instrumentation is perfect, signal analysis will not be needed. Many systems, unfortunately, are operated under this assumption and the data is fed into the computer for immediate reduction. Signal analysis is a function in between data acquisition and reduction. It can be performed by special equipment or it can be incorporated in the data reduction software. The latter concept is apt to be less flexible in application when data reduction is done on a big, central computer system with no, or very limited, interaction and turn-around problems, or even worse, when data reduction is contracted out.

The concept of having signal analysis as a planned operation in the chain of data handling procedures makes official a practice that had been followed one way or another anyway, by necessity, but most often with inadequate means. What emerges, so far, is that an analysis of the signals of a test is necessary before normal, repetitive data reduction can be done. This is the familiar quick-look concept. During this phase, not only a quality assessment of the data is made, but also data reduction parameters are determined. Given equipment that can produce more than a scope picture of the signals, that can produce frequency spectra, transfer function, histogram, and correlations on an easily available quick interaction basis, the quick-look assessment can become quick-look analysis; then data reduction cannot only be given parameters like sample rate and filter settings but also data reduction procedures can be developed and checked out before going on the big computer or specifying them for contracted service.

The analysis here is not concerned with the physical quantities of the measurement and their subsequent analysis which is the objective of the test. We are concerned with the signals that are generated in the sensors and passed through the instrumentation system to data reduction where the analysis of the data is performed. We instrumentation people have to be able to certify the quality of the signals which carry the data.

OTHER USES

The following discussion deals with the use of the analysis equipment for activities that should make the instrumentation system as perfect as can be achieved considering the randomness of nature and the unpredictability of the human element. The first such application arises in the development of a system. Sensors have to be tested and calibrated; amplifiers, etc., have to be developed; recording means have to be selected; and, eventually, data reduction will have to be considered. All this is in the category of black-box analysis. A black-box can be a simple circuit or the whole system.

If the signal a sensor produces for a certain input is reasonably well known, signal simulation can be employed to analyze the response of an instrumentation system or components thereof to this signal. If a certain performance is required of a system for known inputs, the analysis can be employed to register the effect of parameter changes in the system under test and so provides quick responses to system modeling.

After all the above efforts to guarantee a well-working instrumentation system, the gremlins still have chances to wreak damage on the system that would not show up until after the test has been run. Malfunctions of this sort are difficult and most often impossible to detect because they can occur (and most likely will) after the system had been certified to be in working condition. These defects can be minimized by applying automatic analysis to all instrumentation channels as close in time to the test as possible. The results will show where corrections have to be made or which channels will not be available for the test, if corrections cannot be made in time.

In applying signal analysis to instrumentation, one has to recognize the extremes of the size, the span of variety, and the degree of repetitiveness of an instrumentation system. On the minimum side of size is the single sensor in a controlled laboratory environment, hooked directly into a mini-computer that immediately produces the reduced answer for the analyst who is his own instrumentation engineer, data reduction programmer, and signal analyst. On the maximum side are hundreds of data channels in remote locations under severe environmental conditions whose outputs are recorded on site and carried to the home plant for data reduction. As for variety, we limit the consideration to electrical signals and their transmission rather than include, e.g., optical signals that may need only a short run as electrical signals on the way to the computer.

The degree of repetitiveness is important in the sense that the signal analysis effort can be scaled down or used selectively, as much as the probability of failure is reduced. This is the case in setups that are used over and over again in exactly the same configuration for exactly the same tests.

Where, then, signal analysis comes in as a preventive as well as an economic factor is toward the larger system that will be employed in different enough situations as to worry about the probabilities of failure or the confidence in its performance.

To sum it up, signal analysis can be applied to such a system in the following ways:

Quick-look analysis of signals
Determination of data reduction parameters
Development of data reduction procedures
Analysis of components and systems
Development of instrumentation systems
Component or system modeling
Signal simulation
Quality control automation
Data analysis

The last application does not really belong in this discussion, but because a computer of some sort normally is the central controller of the analysis system, data analysis is entirely possible depending on the capability of the computer. This seems to lead directly to the inclusion of the performance of data reduction in the analysis effort. Although this is being done occasionally, the following considerations should make it clear that these are singular cases in the context of this discussion. Reduction of data from a larger instrumentation system needs a number cruncher of a computer and is a highly repetitive operation. The mini- or micro-computer usually associated with signal analysis equipment is normally not a number cruncher. Operation of a repetitive sort on any equipment is a bore to a creative professional and is, in most cases, turned over to technicians. Signal analysis on the other hand should be handled by a professional because only he knows what he is looking for, and only he can make the necessary changes in the equipment configuration and in the computer software in the search for a solution of his problem.

Therefore, any attempt to use (or abuse) signal analysis equipment for data reduction needs careful consideration of the organizational and economical effect on the operation as a whole.

SIGNAL ANALYSIS EQUIPMENT

It seems a widespread misconception that in order to perform signal analysis, all that is needed is a computer and a fast Fourier transform. Although this is at the heart of any system, a large number of various instruments is needed to round out the basic system for general application, in most cases easily doubling or tripling the original expenses for the computer. Figure 1 shows the general requirements, and Figure 2 is a photograph of an actual implementation at the Air Force Weapons Laboratory.

The input to the analyzer may come from three sources. Data from remote operations will most likely be on magnetic tape; whereas, the results of bench tests can be routed to the analyzer directly. The third input is provided by signal generators when used for signal simulation. Data from magnetic tape, usually in FM-modulated form, will have to be demodulated either in the tape unit itself, or for typical FM telemetry systems, in separate discriminators. Timing is important for two reasons: first, to locate the data on the tape, and second, to provide a trigger for the processing action. The latter is also sometimes provided by a fiducial mark. However, these singular pulses have often proven to be unsuitable for triggering because of noise interferences.

Another function at the input side is monitoring, measuring, and where indicated, recording on strip charts of the input signals or data prior to digitization and analysis. Measurement of the input is indispensable for calibrated analysis except where a system calibration is provided with the data, as is normally done with larger instrumentation systems. Strip chart recording of inputs is often the only way of determining the proper triggering point for the analysis.

If the analysis system has a fairly high utilization, or if a large assortment of input equipment is necessary to handle the various tasks, it is almost a necessity to rack-mount all this equipment and install a patch panel through which all connections can be made. Nothing is more frustrating and distracting from a test or analysis than cables which are strung across the room, hanging from tables, covering meters, set-up and adjustment devices, and having to be removed everytime somebody else wants to use the equipment. The expense of a patch panel is small compared to the ease with which connections are made or changed in the course of the experiment. Figure 3 is an example of such a panel for a fairly large system. For smaller systems, too, a patch panel is of advantage if it has been decided to rack-mount the equipment for the analysis tasks. Only the very small operator is better off without it because he can afford to lug instruments around, piling them on top of each other (if he can find them; somebody else might have left them in another setup when he went on that long vacation), and connecting them with cables of usually the wrong length. Any such operation is poor utilization of the equipment and of human resources.

The analyzer proper consists of the analog-to-digital converter at its input, the computer with memory and I/O devices, the software package and, sometimes, a panel for quick setup of parameters and control of the software by means of knobs and switches.

These provide faster answers although they are less flexible in certain parameter requests; whereas, keyboard control is somewhat time-consuming when calling frequent parameter changes, especially in trying to find the optimum setup. However, keyboard control opens the way to programming the analysis function which offers a variety of arithmetic operations on data block, manipulation of display, and access to the computer peripherals. As the requirements for analysis evolve and the project engineer gets used to the advantage this analysis system offers him over the rather tedious way of batch operation in the central computer, the procedures for certain types of data or systems can be canned for a more routine operation by skilled technicians.

Analog-to-digital conversion should be available for two channels, simultaneously, for transfer functions and cross-correlation analysis. A 10-bit conversion usually is entirely adequate, but many people like a 12-bit conversion. It really depends on the system noise floor and the dynamic range requirements.

The computer can be any one of the many mini's available these days. If a micro is used, it winds up a mini by the time everything is added to make it into a more or less general-purpose computer. A micro will be of advantage in a dedicated system that can do the Fourier transform and a few associated mathematical operations, but the application of such a system is limited and expansion by the user is more difficult to accomplish.

Memory size can vary widely. In general, smaller systems get by with 8 K of core; whereas, larger ones need at least 16 K (this includes minimum data space of 4 K size). How much data can be accommodated in the 4 K? Let's assume a two-channel buffered input and a transfer function calculation. The input takes 4 blocks of data. Then, the average of the auto spectra of the two inputs has to be deposited; this requires 2 additional blocks. Next, in order to arrive at the transfer function, the cross-spectrum has to be computed and deposited, one more block. Finally, the transfer function and the coherence function which are normally computed along with it need 2 more blocks. All in all, therefore, 9 data blocks are needed for the transfer function calculation. If the block size is 1 K (2^{10} = 1024 = 1 K is the usual convention), a total of 9 K of core is needed. With a software package that requires 12 K for itself, memory size has to be at least 21 K for this application. Disks can be used to extend required memory size if a large amount of data has to be handled or if data have to be stored before processing.

Necessary I/O devices are a keyboard for communication and a scope for the display of results. Printers, plotters, interactive terminals, magnetic tape are quite useful, if economically justified. Hard copy from a scope display is a fast way for obtaining the results in a reportable format. Sometimes a scope camera will do as well. Nothing, however, can beat a digital plotter for obtaining camera-ready copy.

CAPABILITY OF THE ANALYZER

The technical specifications of the system, like frequency range, analysis bandwidth, input sensitivity, etc., differ with the system and the requirements.

Important, however, are the analysis capabilities and the results that can be obtained to solve the problems at hand.

The Fourier transform is the basic function for most operations of the analyzer. It has a one-to-one relationship between the two domains. The transform pair $x(t) \leftrightarrow X(\omega)$ has the formal definition

$$X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t} dt$$

and for the inverse transform

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega)\, e^{j\omega t} d\omega$$

The following functions of the analyzer are based on the Fourier transform.

The autospectrum, or the power spectrum density function of a signal $x(t)$, is the square of the transform $X(\omega)$:

$$G_{xx}(\omega) = X \cdot X^*$$

This is a real function, i.e., it has no imaginary components, or in other words, the phases of the Fourier transform have been lost in the process. The autospectrum, therefore, has not a one-to-one relationship between the time and frequency domain. Although the forward transform produces the power spectral density of the signal, there is no inverse transform that would lead back to the signal in the time domain, because there are many different time domain signals that have the same power spectrum. White noise and the impulse function are the two extreme examples.

An analyzer with a two-channel capability can also produce the cross-spectrum

$$G_{yx}(\omega) = Y \cdot X^*$$

This is an expression of the power vs. frequency that the two channels have in common. The cross-spectrum is a complex function and, therefore, does preserve phase information. It is useful in connection with the transfer function where it enables transfer function measurements of systems with several inputs and/or outputs.

The transfer function is commonly defined as

$$H(\omega) = \frac{Y(\omega)}{X(\omega)}$$

that is, the ratio of the output transform vs. the input transform. Aside from the nuisance of complex division that would be required in the evaluation of the function, it is clear from whatever was said about the importance of the cross-spectrum that this definition only makes sense for a laboratory-clean environment which ensures that there are no unwanted inputs significantly contributing to the output. This matter can be taken care of by the following expansion

$$H(j\omega) = \frac{Y \cdot X^*}{Y \cdot X^*} = \frac{G_{yx}}{G_{xx}}$$

which provides that only correlated signals are measured.

The transfer function is complex

$$H(j\omega) = Re(\omega) + j\ m(\omega)$$

or

$$H(j\omega) = |H|\ e^{-j\phi}$$

where $|H(\omega)|$ is the familiar frequency response and $\phi(\omega)$ is the corresponding phase response of the Bode diagram.

Circumventing all the complex mathematics, the transfer function has been produced for decades by measuring the amplitude and phase at the output of a device at single frequencies (sine waves), keeping the input amplitude carefully at a constant level. This method is still very economical for one-of-a-time requirements. But when volume and time of testing become economical factors, an analyzer of the kind described herein is a necessity. The single sine waves have to be replaced by a signal that contains all these frequencies. Such a signal is white noise. It transforms into the frequency domain (i.e., its frequency spectrum) as a continuum with constant amplitude. Figure 4a is the frequency spectrum of a white noise signal. The reason why it is not a straight line is twofold: The signal generator is not perfect, and the measurement of a statistical signal represents its true power value only when it can be observed for an infinitely long time. We have to be content that we can describe the signal only with certain limits of confidence; e.g., we can be confident that the true value lies within 50% of the measurement for 80% of the measurements taken.

White noise characteristic is not always obtainable in practical situations. Its alternate, the impulse function, is never obtainable in pure form, but a reasonable facsimile can be provided. The optimum turns out to be not too short a pulse, whose frequency spectrum is shown in Figure 4b. The downward slope toward higher frequencies is not detrimental for the analysis because of the normalizing in the transfer function calculation and as long as there is enough energy left in the higher frequencies within the desired analysis range. The continuity of the spectrum is far from perfect and depends on pulse width and sampling rate. Figure 4c is the frequency spectrum of a unit step that fulfills the requirements of continuity much better.

An example of a filter transfer function obtained with continuous white noise is provided in Figure 5a. Figure 5b has the same transfer function produced with a single pulse, and Figure 5c has been derived from a unit step input. There is, of course, no significant difference between these three curves. The remaining deviations have their origin in the differences in the power spectra of the input signals.

It should be mentioned here that all these functions are valid for linear and time-invariant systems only. Another function that can be derived from the transfer function is the not so well-known coherence function. It is expressed as a square ratio vs. frequency

$$\gamma^2(\omega) = \frac{|G_{yx}|^2}{G_{xx} \cdot G_{yy}} \qquad 0 \leq \gamma \leq 1$$

and is a dimensionless number between 0 and 1. It expresses the degree of coherence between the input and the output. If the output autospectrum G_{yy} has other components not coming from the input x(t) with autospectrum G_{xx}, then clearly this ratio has to be less than one. This function is helpful in checking the presence of, e.g., noise that contaminates a measurement channel. Figure 6a is an example of a coherence function without extraneous signals, and Figure 6b is another one with uncorrelated noise originating inside the system under test.

The autocorrelation function brings us back into the time domain. This function is defined as

$$R_{xx}(\tau) = \int_{-\infty}^{\infty} x(t)x(t-\tau)dt$$

but its computation in the time domain is cumbersome. The analyzer makes use of the Wiener-Khintchin theorem which states that power spectrum density and correlation functions are Fourier transform pairs.

$$R(\tau) \leftrightarrow G(\omega)$$

$$G(\omega) = \int_{-\infty}^{\infty} R(\tau)e^{-j\omega\tau}d\tau, \text{ and}$$

$$R(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} G(\omega)e^{j\omega\tau}d\omega$$

This theorem means that the autocorrelation can be obtained by the inverse transform (from the frequency to the time domain) of the autospectrum. (As a note of interest, before the fast Fourier transform was known, the power spectrum density was obtained by computing the autocorrelation function and transforming it into the frequency domain to obtain the spectrum.)

The autocorrelation yields one important information: Its value at $\tau = 0$ is the total power of the signal measured. This is very helpful in the case of statistical signals whose power is often difficult to obtain from the signal directly. It could also be obtained by direct integration of the autospectrum over the entire frequency range. But this is a more cumbersome process. The respective relations are:

$$E = \frac{1}{\Delta T} \int_{t}^{t+\Delta T} x^2 (t)dt \qquad \text{for } f=0 \text{ to } F \text{ within } \Delta T \qquad (1)$$

$$E = \int_{0}^{F} p(f)df \qquad \text{for a data sample of } \Delta T \text{ length} \qquad (2)$$

$$E = R(0) \qquad \text{for } \Delta T \text{ and } f = 0 \text{ to } F \qquad (3)$$

where

ΔT is the data window, i.e., the time during which the data is observed.

F is the upper frequency of the spectrum

p is the power spectral density in V^2/Hz

E is the total energy in the signal during the time ΔT and for all frequencies 0 to F, in V^2.

Form (1), if measurable, can be used for checking (2) or (3) if it has to be proven that the spectrum processing was done correctly. The operations to obtain the results from form (1) are available with the peripheral equipment that was described earlier.

Further use of the autocorrelation can be made for judging the general nature of the signal. Samples of important groups of signals are presented in Figure 7.

The same method as had been used to produce the autocorrelation function is employed to calculate the crosscorrelation function which, therefore, is the inverse Fourier transform of the cross-spectrum.

The crosscorrelation function yields the signal delay (group delay) through a system under test. Figure 8 gives an example with white noise as measurement signal. Another useful relation is the following

$$h(t) = R_{yx}(\tau)$$

which says that the crosscorrelation yields the impulse response (in the time domain) of a system.

Formally, the impulse response and the transfer function are a Fourier transform pair

$$h(t) \leftrightarrow H(j\omega)$$

The significance of the latter relation is that the impulse response can always be obtained from the transfer function, no matter what signals were used to calculate. The crosscorrelation on the other hand can produce the impulse function only when an impulse is applied to the input of the system under test.

The next function provided by the analyzer is the histogram or probability density function.

Its usefulness does not seem obvious at the outset, but consider the following. Suppose a triangular wave is input to a system. The probability density function of this signal has a uniform distribution (see Figure 9), except maybe for the edges because of the low-pass nature of a reasonable system. If the system has any amplitude distortion, it will show nicely in the histogram because the straight line will also be distorted as in Figure 10. Similar reasoning applies to other test signals.

The last of the functions available from the analyzer is waveform averaging. This operation allows recovery of repetitive signals from noise by virtue of the linear and quadratic-adding properties of synchronized signals and random noise, respectively. Figure 11 shows the input to the analyzer which is essentially noise burying the signal completely. The result of 128 iterations is seen in Figure 12.

Crowded portions of a display can be expanded horizontally to any reasonable length, with the scale factor automatically being readjusted.

At the input side, attenuators provide amplitude range flexibility, and a trigger circuit allows the exact repetition of transient events.

Very useful are antialiasing filters that are available for most of the ranges of the analyzer.

APPLICATIONS

Two major applications are immediately obvious; that is, the analysis of signals by frequency spectrum and autocorrelation, and the analysis of systems by transfer function and crosscorrelation.

Typical for the first application is the investigation of unwanted frequencies that appear with the data from a particular transducer or data transmission system, like 60 Hz and its harmonics or other power supply interference; or of resonant frequencies of underdamped systems; or of frequencies that are generated by intermodulation distortion of components which themselves lie beyond the usable range of the system. This happens in the audio business due to non-linearities in the power output stages; or of frequencies that are generated as undesirable side effects by the environment of a sensor, or induced in cables, or picked up due to crosstalk between channels, etc.

If the signal appears to be all noise, a determination still can be made as to what kind of noise it is, and whether it is all noise or there is a signal in it which resembles the expected one. If the expected signal is known to that extent that it can be simulated, a crosscorrelation between the simulated signal and the one buried in noise can be done in order to see how much of an agreement exists.

The transfer function application is equally useful for small components like an amplifier or a filter as it is for whole instrumentation systems including transducers and magnetic tape. A frequency response can be produced from signals that defy the time-honored method of simple sine wave measurements. Consider a velocity transducer that, for reasons not important for this discussion, cannot be tested on a shake table to conventionally obtain its frequency response. The only other method is impulse or unit step testing. The analyzer can calculate the transfer function with these inputs and can do it even faster.

Or consider high frequency pressure transducers, accelerometers, transducers with a large sensitized area like polymer stress gages. In each case it is easier to obtain a frequency response by a Fourier transform analyzer than by any other method. The phase response is provided in the same operation, free of charge, as is the coherence function.

The requirement on a data transmission system is that it should have a frequency response which does not affect the transducer output. Knowing the bandwidth of the transducer, the transmission system can then be defined. The transfer function helps both in the design and the maintenance of such a system. Perhaps the latter is the more important one because it is desirable to ascertain the integrity of a transmission system as shortly before a test as possible.

Consider an instrumentation van with several hundred data channels that are used for tests in remote areas. The vans have to be in place a long time before the test. The channels are set up and tested weeks before the actual test is run. Dry runs are conducted, more often to ensure smooth operation than to have a close look at the status of the data channels. They are looked at, of course, but how much time and personnel are available to look at hundreds of channels with the built-in calibration step as input. This usually involves the wading through yards of strip charts of recorded outputs.

The Fourier transform analyzer provides the means for the automation of this process. Consider a suitable input signal that can be applied to all data channels at preferably the same time, record the outputs on magnetic tape, and ship the tapes back home for analysis. This requires little effort and should hardly distract from the other operations going on during the time shortly before a test. The data from these tapes are analyzed in the laboratory in a highly automatic procedure for frequency response, system noise, distortion, and other unsanitary ingredients. Automation can be provided by comparing the results of each channel with a standard function in the computer, and flagging those that are outside of established tolerances, for closer investigation. It is mandatory, for this purpose, that the software package of the Fourier transform and its associate functions of a particular analyzer can be modified or added to in order to make this automation possible. A suitable input signal is the calibration step because it is already available in most systems. If properly timed in the analyzer, the step produces a good-looking transfer function.

An even greater amount of automation can be achieved by applying white noise as the measuring signal and recording continuously through the whole length of tape so that, on playback, the tape does not have to be shuttled back to the same point, but can be left running while the computer can switch from channel to channel; e.g., when a tunable discriminator is used for FM multiplexed data. The only manual operation left would be the switch from one track of the tape to the next. This way, complete analyses can be obtained in a few seconds per channel.

The analysis system is heavily used for quick-look analysis of multichannel tests, before the data reduction requests are written out.

THE ANALYSIS FUNCTION

It occurs very often that the data from a test is turned over to data reduction without giving it a glance. The data reduction people are thus left with the head-scratching when the results are contrary to expectations. Considering data reduction a perfect operation for the purpose of this discussion, the reason why results don't look like they should lies either in wrong predictions or in bad instrumentation.

It should be obvious that an assessment of these problems can best be made by the people responsible for their appearance and not by data reduction people. The latter people don't know about predictions, and if they point out problems in instrumentation, they only antagonize the instrumentation people who themselves believe they are perfect. I hope it has become abvious, during this whole discussion, that the signal analysis function is a necessary one. Although its organizational implementation depends largely on the size of the organization and its management policy, the best place for it in a medium-to-large-size organization with centralized data reduction is in the instrumentation group. These people should not let the data pass out of their hands before they have not ascertained its signal quality in a quick-look analysis operation. The signal analysis system described here gives them the tool, and management has to give them the time to perform this function.

Only after certification of the instrumentation quality can data reduction be attempted and the analyst can work his manipulations on the data without having to account for or remove instrumentation artifacts. Although a person with engineering background will be necessary to adequately conduct the quick-look analysis, this expense is compensated for by less data reduction runs, faster turn-around times and fewer heated discussions of where to put the blame. The intangible benefits are increased confidence in the compentence of the instrumentation group and the integrity of their systems, as well as improved relations with data reduction on one side and the analyst on the other.

ACCURACY

The accuracy of analyzer results is defined by three terms, each with a different meaning. One is the usual definition for amplitude errors of the equipment due to hardware tolerances or least significant bit software limits.

The second term affects the amplitudes of periodic components in the spectrum. By the mechanics of the analysis procedure, the amplitude of periodic, or sine wave, components are no better reproduced than within 3 db (unless the sine wave is synchronized with the analyzer operation so that its peak falls in the center of a filter slot).

The third term has to do with the fact that accuracy in its usual definition does not apply to random signals. It has to be replaced with the concept of confidence. It works like this: Based on data window width and analysis bandwidth which determine the number of degrees of freedom of the signal, one can have confidence that, e.g., the true value of 80% of the results lies within, let's say, a total range of 50% of the measurement. The mathematics for this concept makes use of the chi-square distribution. Table 1 is a sample of selected confidence values taken from this distribution. The degrees of freedom are defined on the basis of information and have nothing to do with those that are defined for a mechanical system.

TABLE 1. Confidence Limits

For No. of Degrees of Freedom	PROBABILITY that the true value lies within the stated limits of the measured value is 60%		80%		90%	
	Lower	Upper	Lower	Upper	Lower	Upper
2	.62	4.48	.43	9.49	.33	19.50
5	.69	2.13	.54	3.11	.45	4.37
10	.74	1.62	.63	2.06	.55	2.54
20	.80	1.37	.70	1.61	.64	1.84
50	.86	1.21	.79	1.33	.74	1.44
100	.90	1.14	.84	1.21	.80	1.28
200	.92	1.09	.88	1.14	.85	1.19
500	.95	1.06	.92	1.09	.90	1.11
1000	.96	1.04	.95	1.06	.93	1.09

The basic value of the number of degrees of freedom is two for the normal analyzer operation. The confidence in the measurement of random data can be improved by taking more data samples and averaging them. For example, the average over four samples produces 8 degrees of freedom; that is, each additional sample in the average increases this number by 2. The reason that the uncertainties have to be expressed in this way is the finite time of observation of the data.

The next two figures illustrate the ranges of uncertainty for a low and a high number of degrees of freedom. Figure 13 is an example of a white noise after 64 averages, on the same scale. Note that these uncertainties do not result from the analyzer mechanics but are a consequence of the actual amplitudes of the random signal that were available for observation. Thus, exactly the same sample of data will produce exactly the same spectrum.

This is an important point for the transfer function. Because the estimates at the output of a system are normalized by the estimates of the input to the system and because the output amplitudes occur at the same times (if slightly delayed) as the input amplitudes, this normalization cancels the effects of the uncertainties due to randomness. Except for a small amount due to minor inaccuracies in the system, e.g., phase differences in the two channels, the transfer function cannot much be improved in its appearance by averaging.

The correlation functions of sinusoids are not affected by the analysis mechanism. The autocorrelation shows the true power no matter what the synchronization with the internal clock. The averaging of random signals does improve the power measurement of the signal because a longer observation time is obtained. Each individual sample, however, is measured accurately to as much power as it actually has. Although this measurement is equally subject to the statistical laws of the chi-square distribution, the number of degrees of freedom is in the area of several hundred because the measurement extends over all frequencies (0 to F) which results in uncertainties of less than 10% (approximately 1 db).

BIBLIOGRAPHY

Rather than cluttering the text with numerous references, a few worthwhile books on the subjects are cited here for what they can contribute to the understanding of the analysis operation.

Reference 1 can be recommended for the study of transforms, in general. It is well structured, concise, and extremely logical in its progression. Reference 2 is a very detailed development of the principles of analysis of signals and systems; it is slanted toward the vibration specialist. For those who want to go back in history to the time when these concepts were introduced to engineering by the venerable Norbert Wiener, Reference 3 is recommended. In Reference 4 and 5, the first computer implementations of the two Fourier transform approaches are discussed: the original computerized Fourier transform and the fast Fourier transform, respectively. Reference 6 is a thorough run through the fast Fourier transform for the really interested. Finally, in Reference 7, the table of confidence limit versus the number of degrees of freedom can be found on pages 332 and 333.

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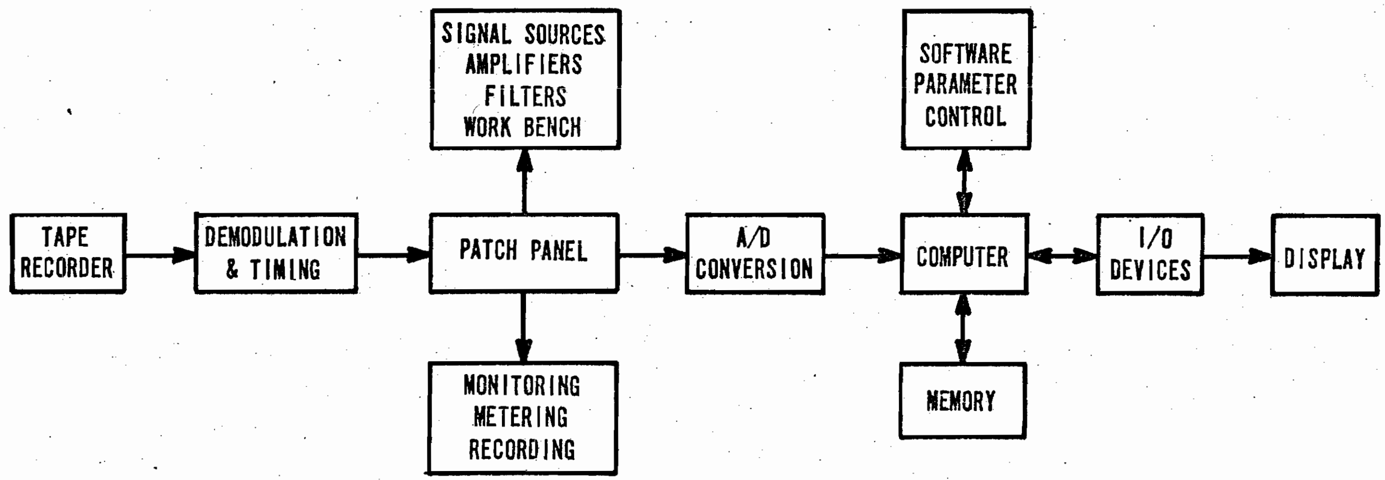

FIG. 1 SIGNAL ANALYSIS EQUIPMENT CONFIGURATION

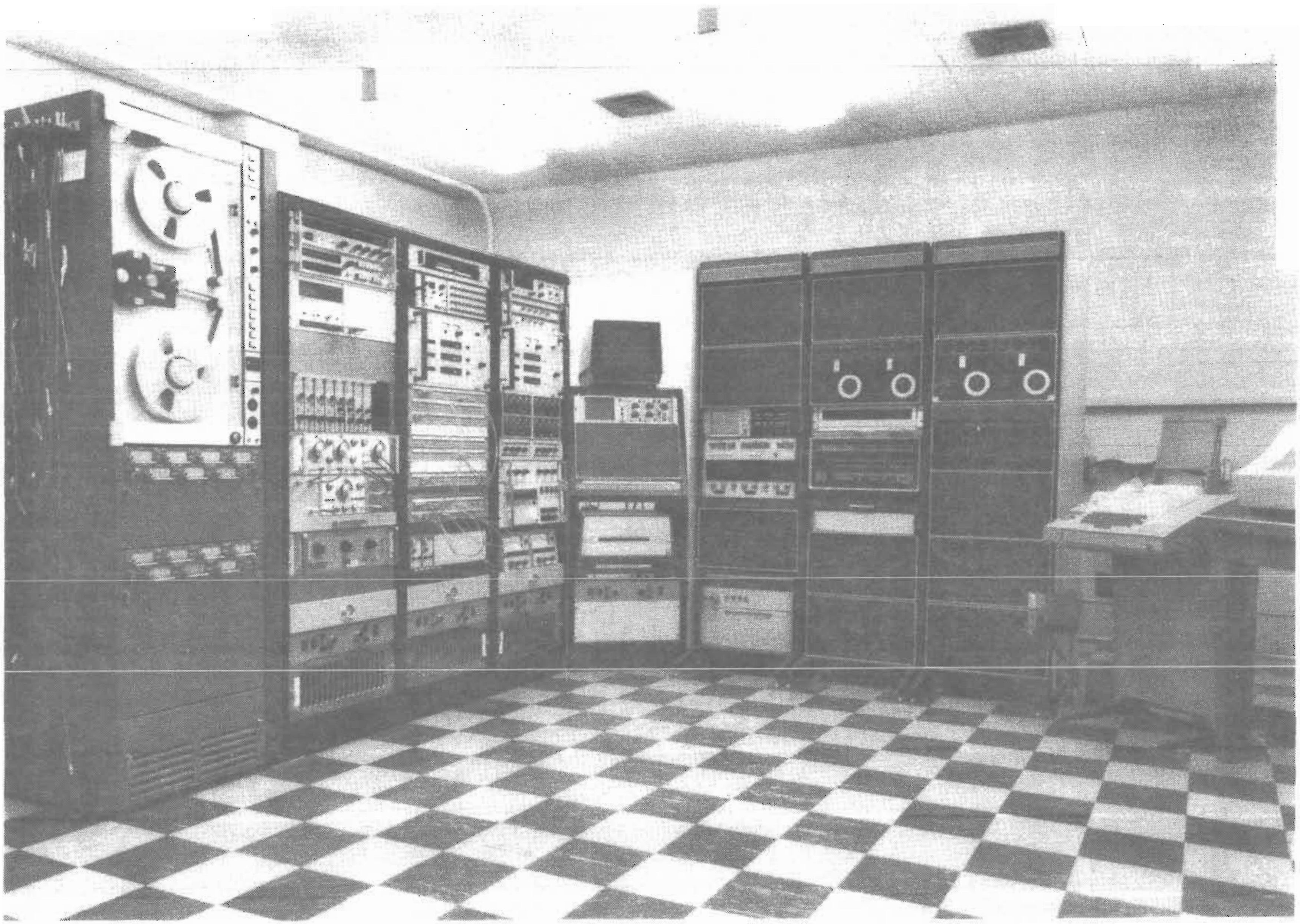

FIGURE 2. SIGNAL ANALYSIS SYSTEM

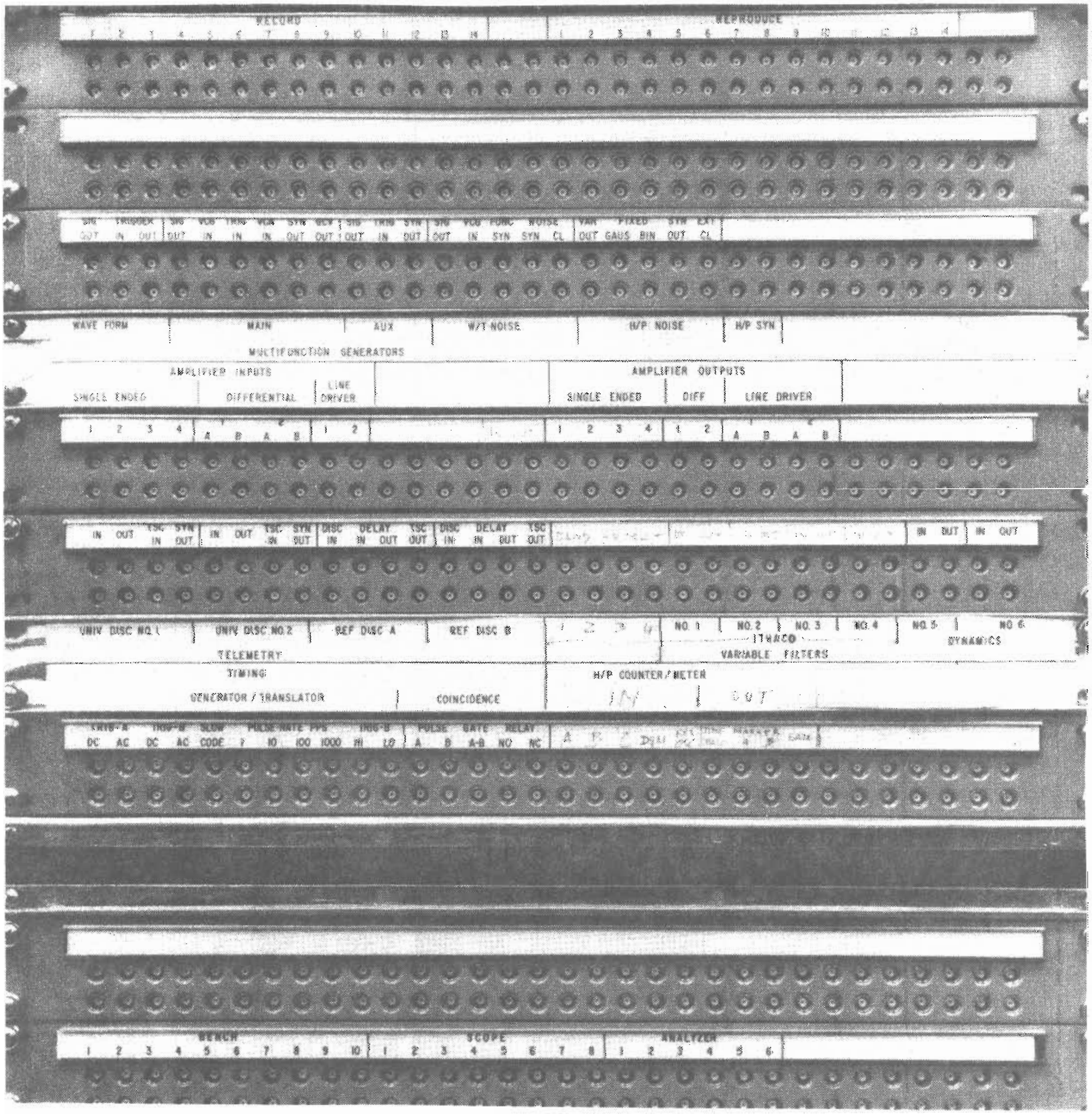

FIGURE 3. PATCH PANEL

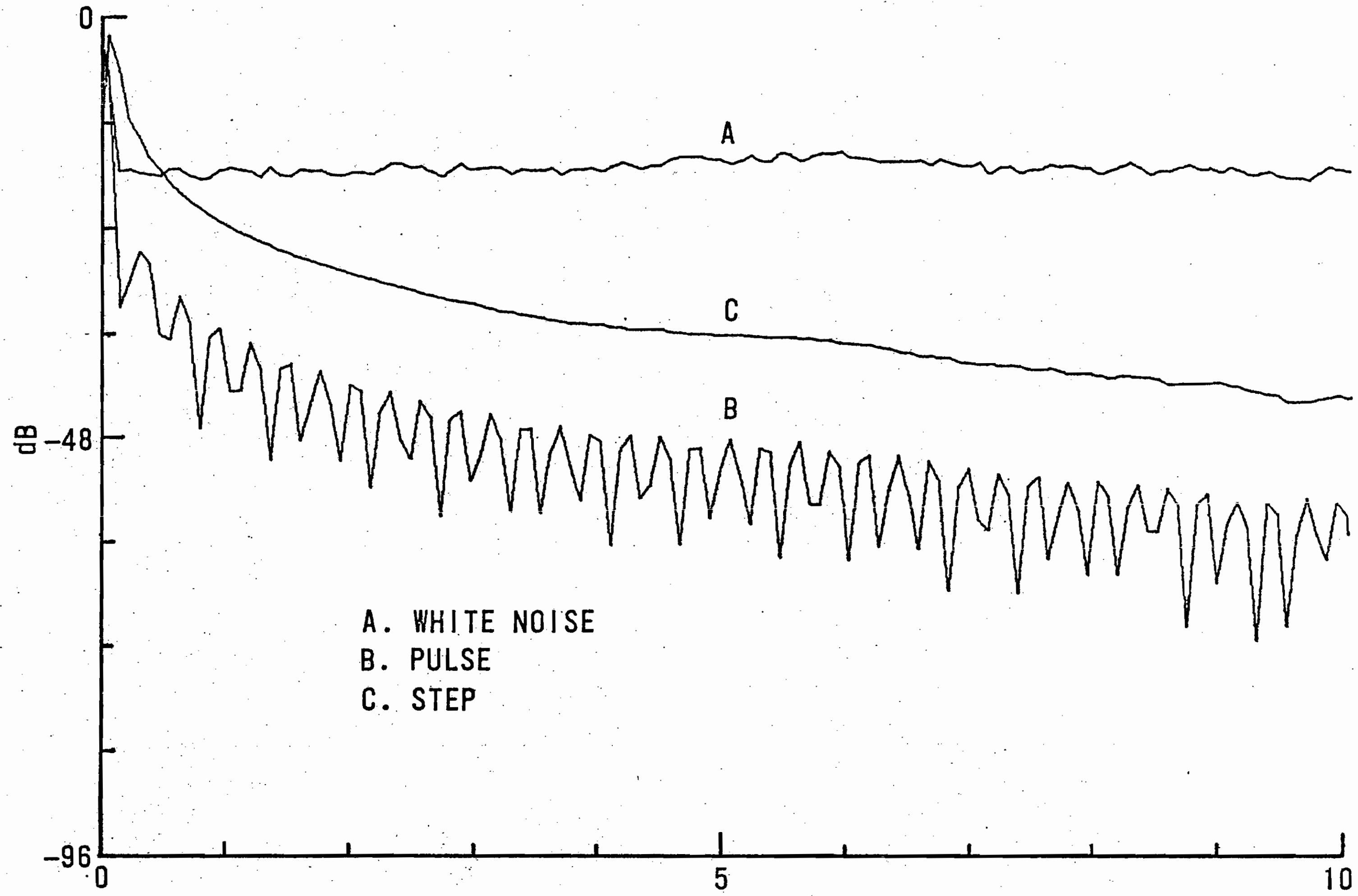

FIG. 4 AUTOSPECTRA

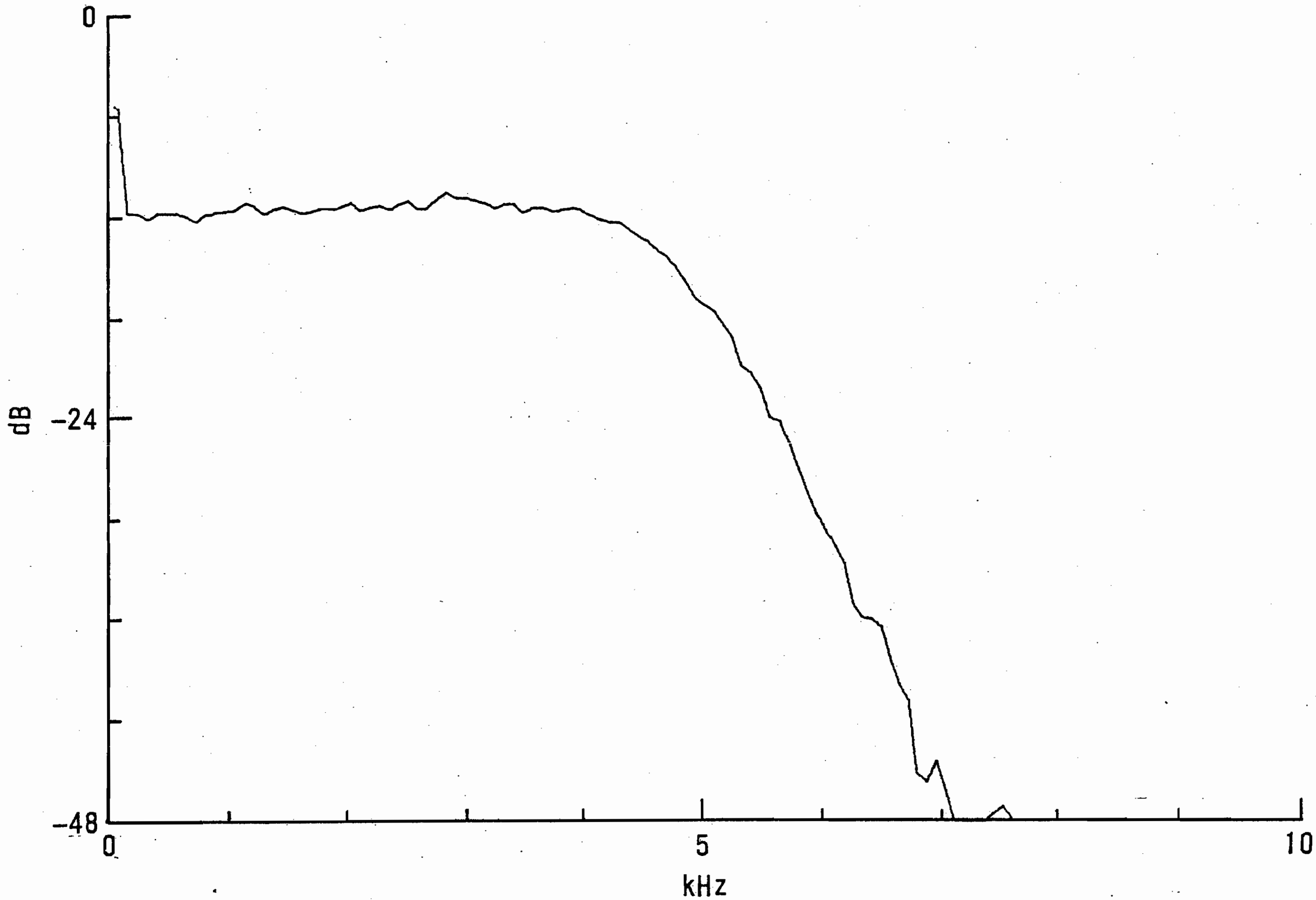

FIG. 5a TRANSFER FUNCTION WITH WHITE NOISE

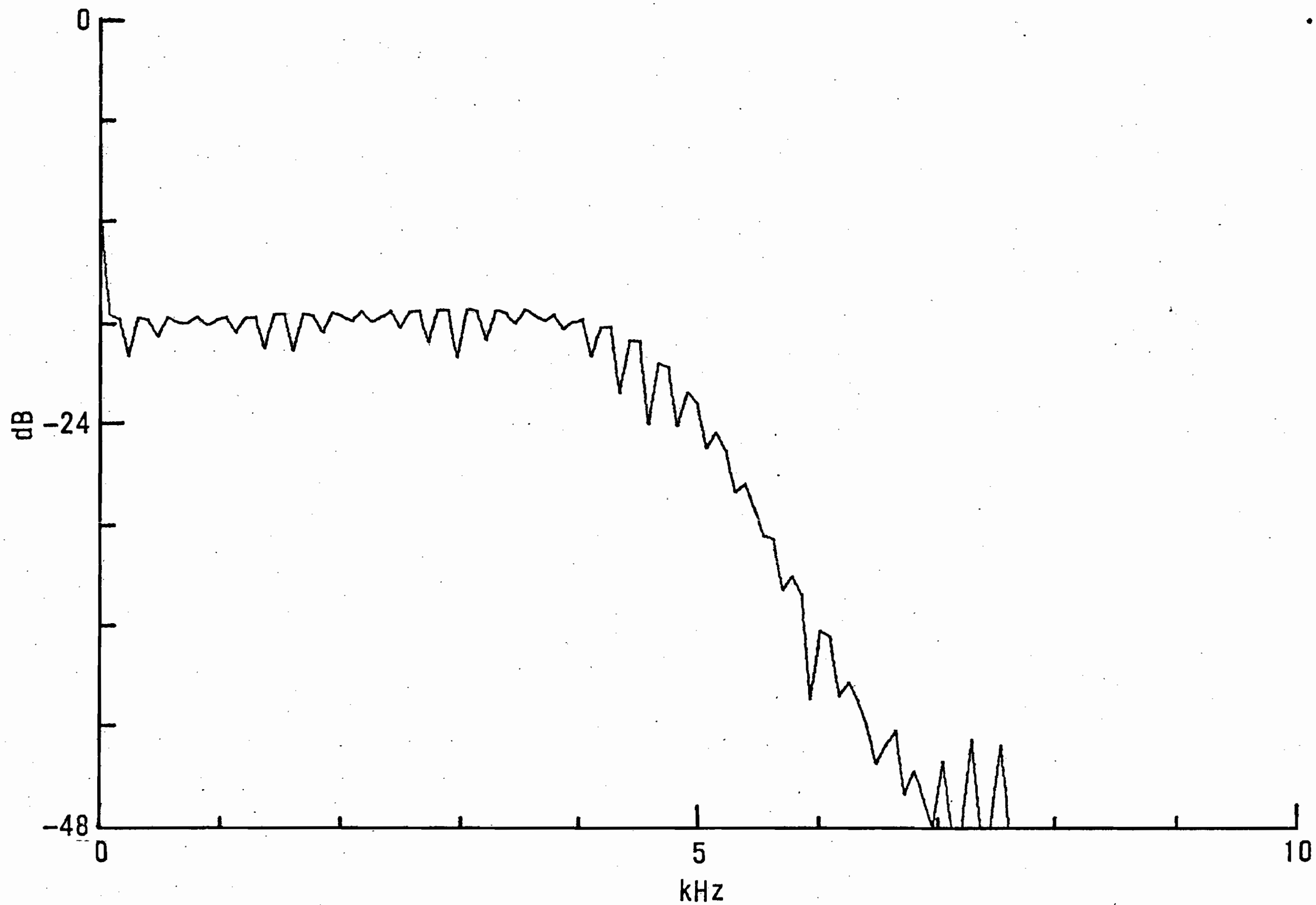

FIG. 5b TRANSFER FUNCTION WITH PULSE

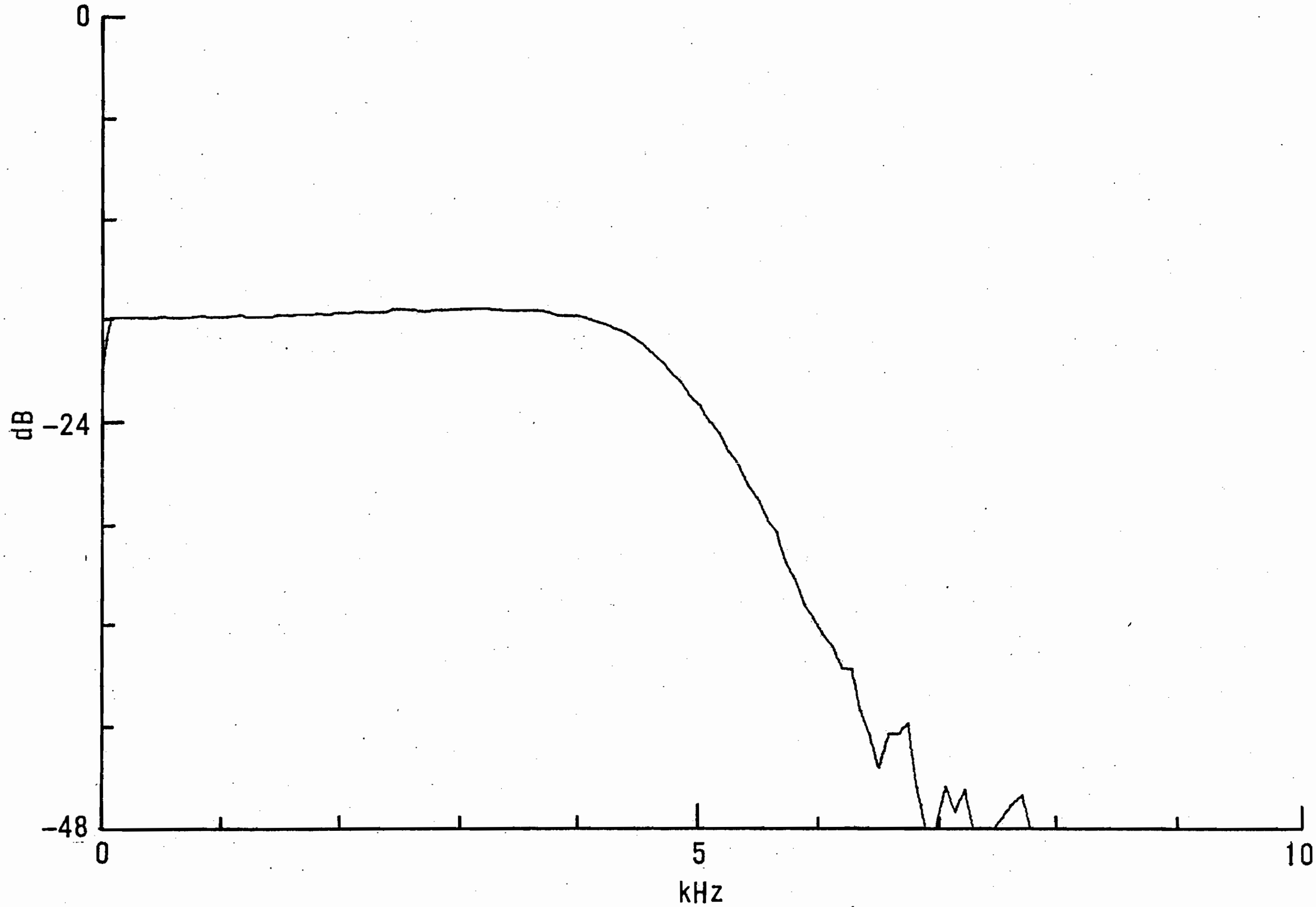

FIG. 5c TRANSFER FUNCTION WITH STEP

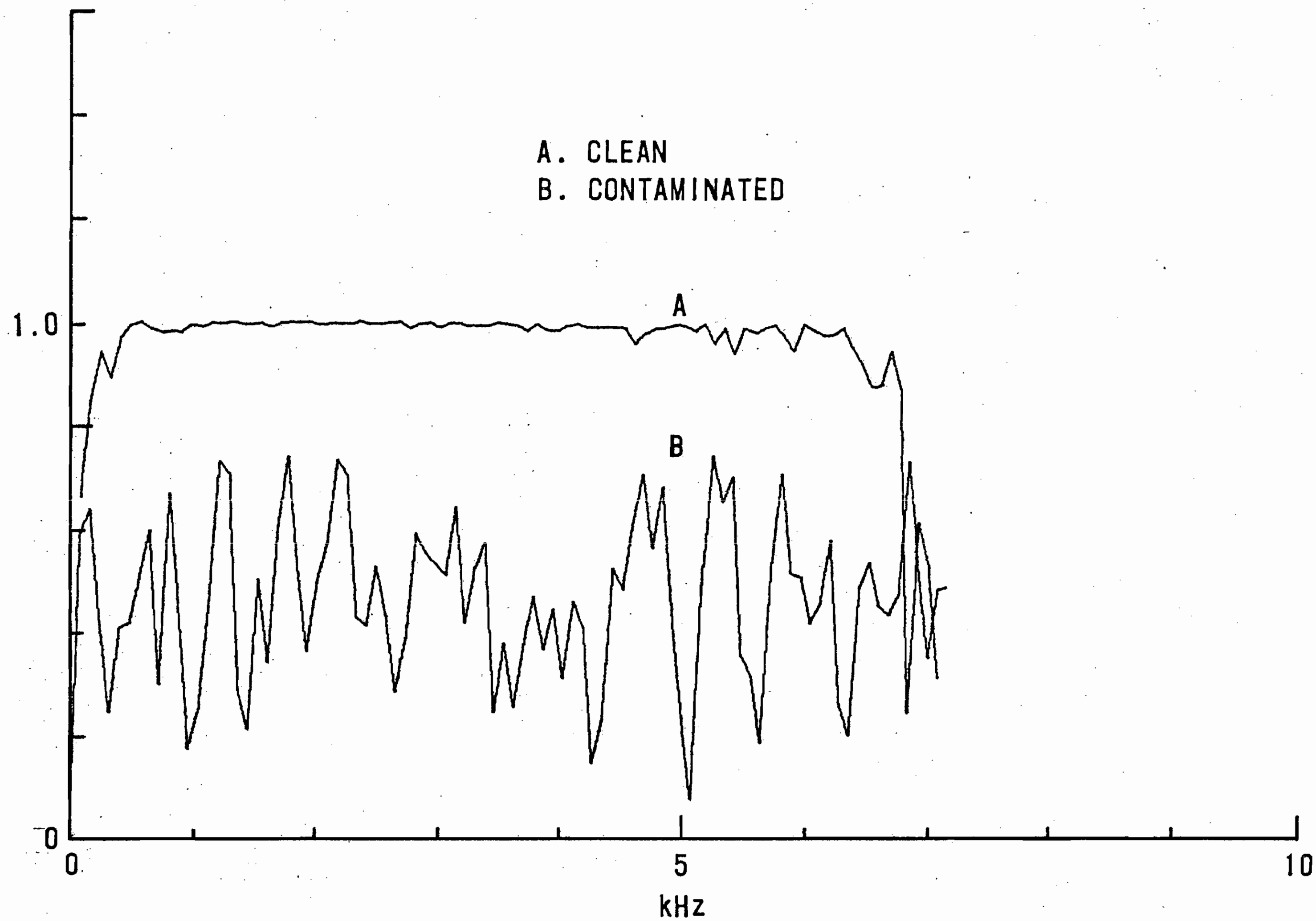

FIG. 6 COHERENCE FUNCTION

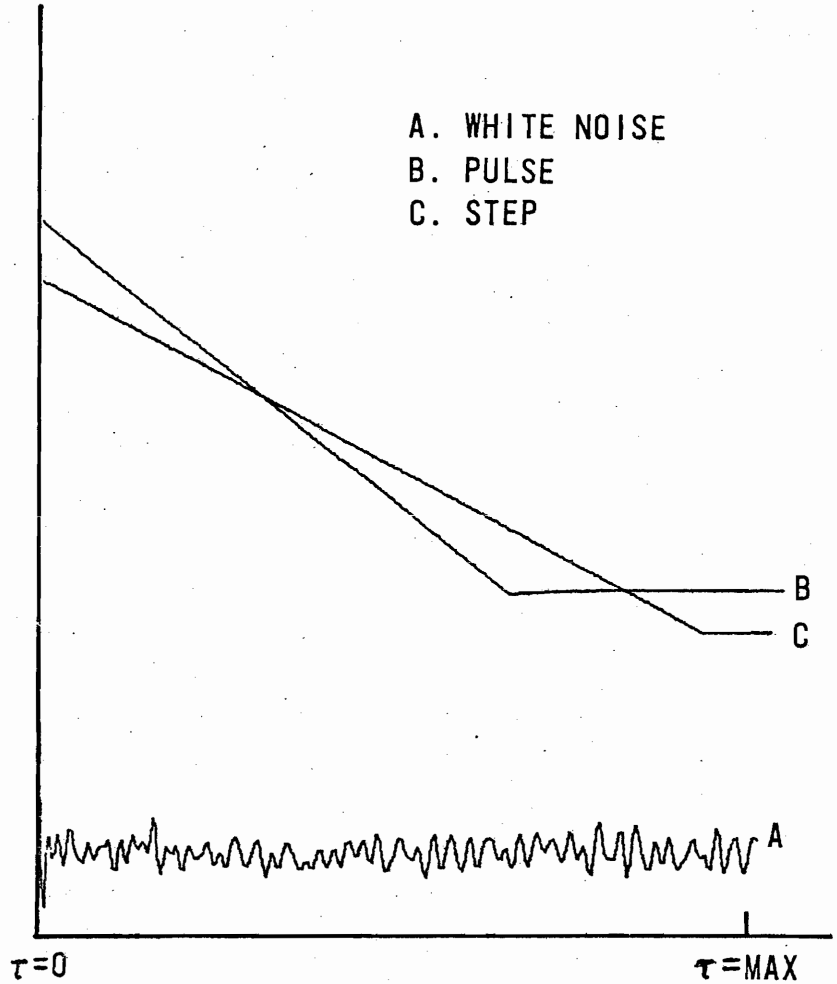

FIG. 7 AUTOCORRELATIONS

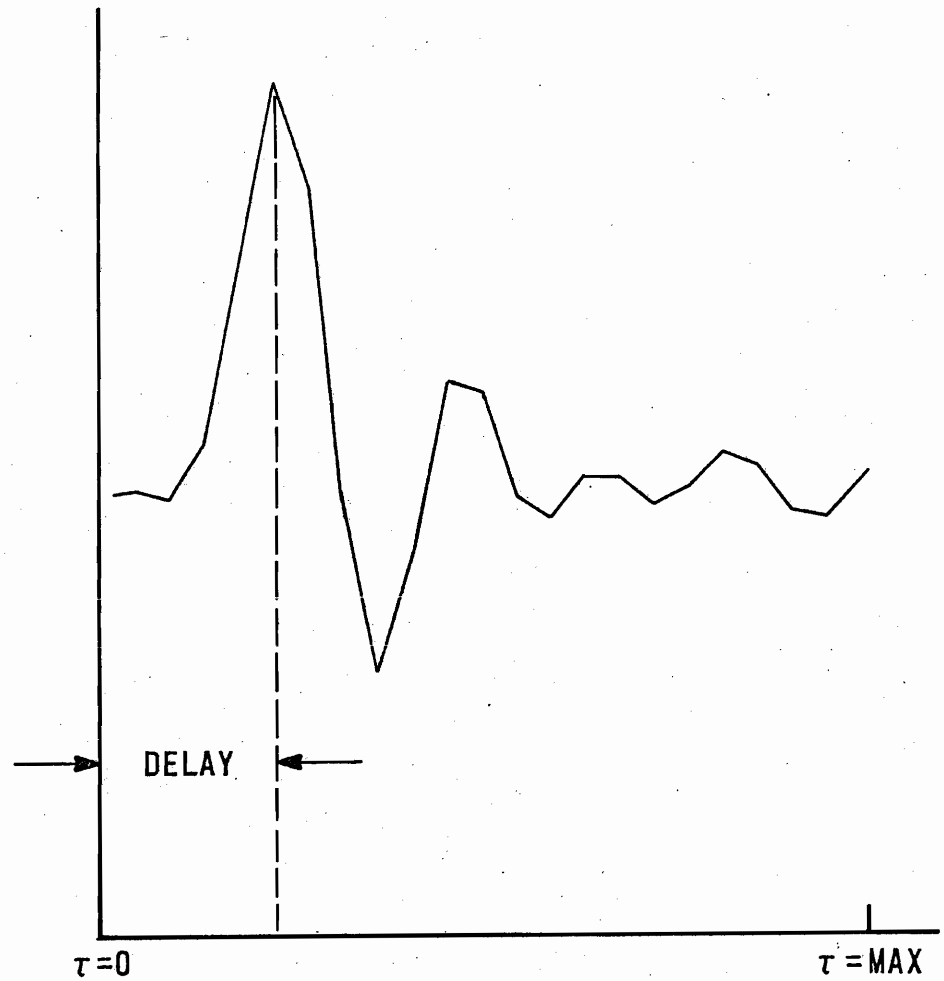

FIG. 8 CROSSCORRELATION WITH WHITE NOISE

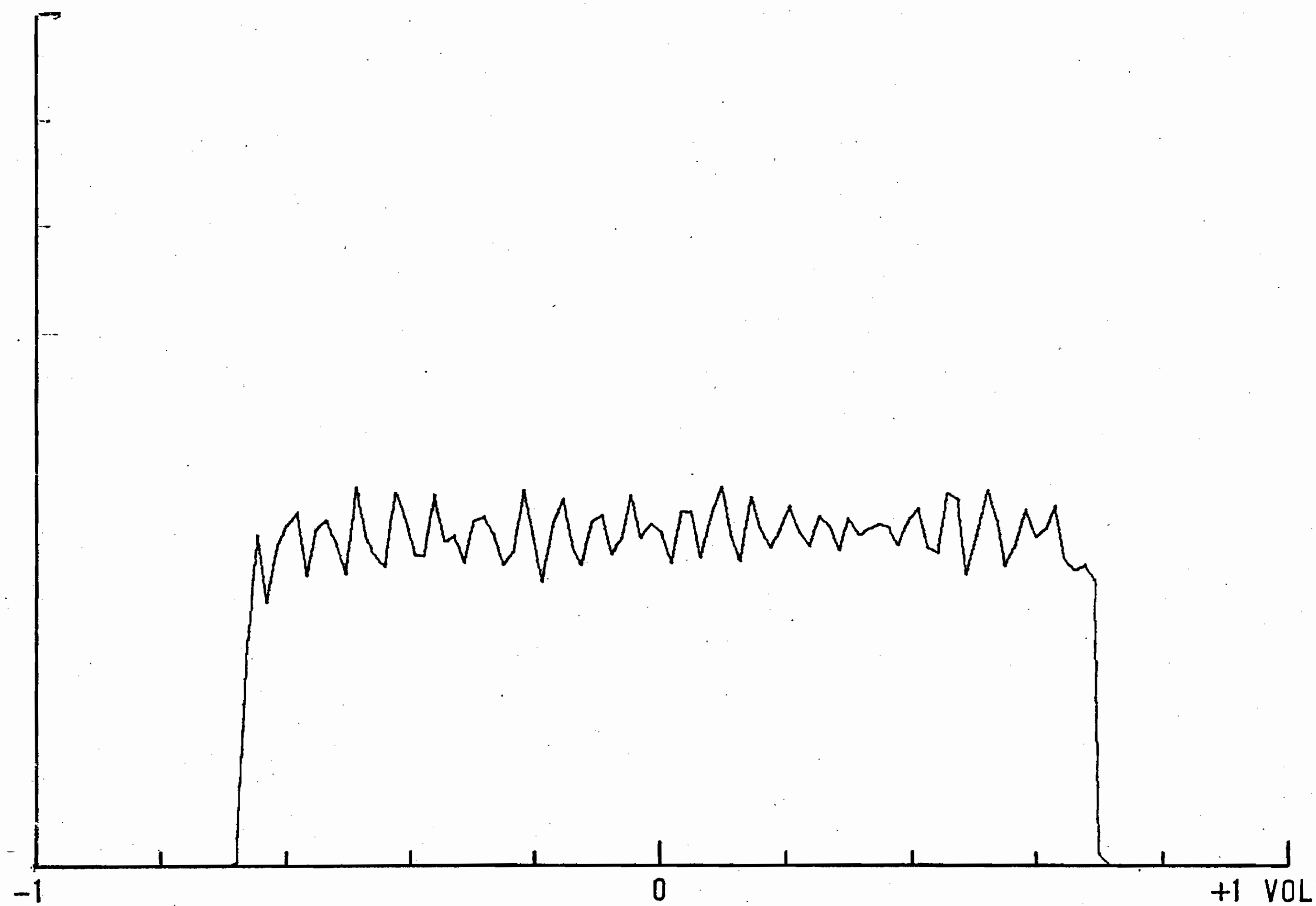

FIG. 9 HISTOGRAM OF TRIANGULAR WAVE

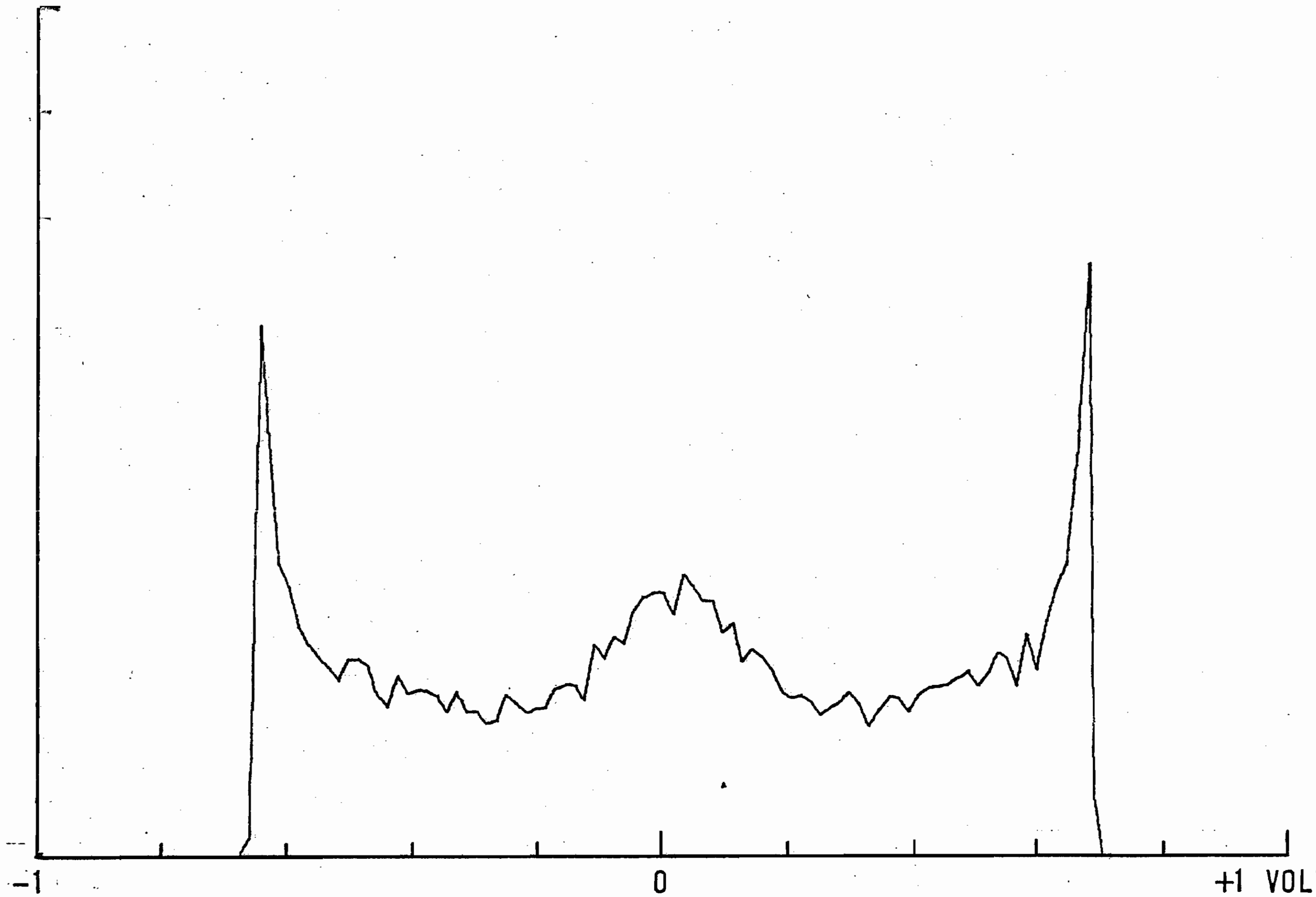

FIG. 10 HISTOGRAM OF DISTORTED TRIANGULAR WAVE

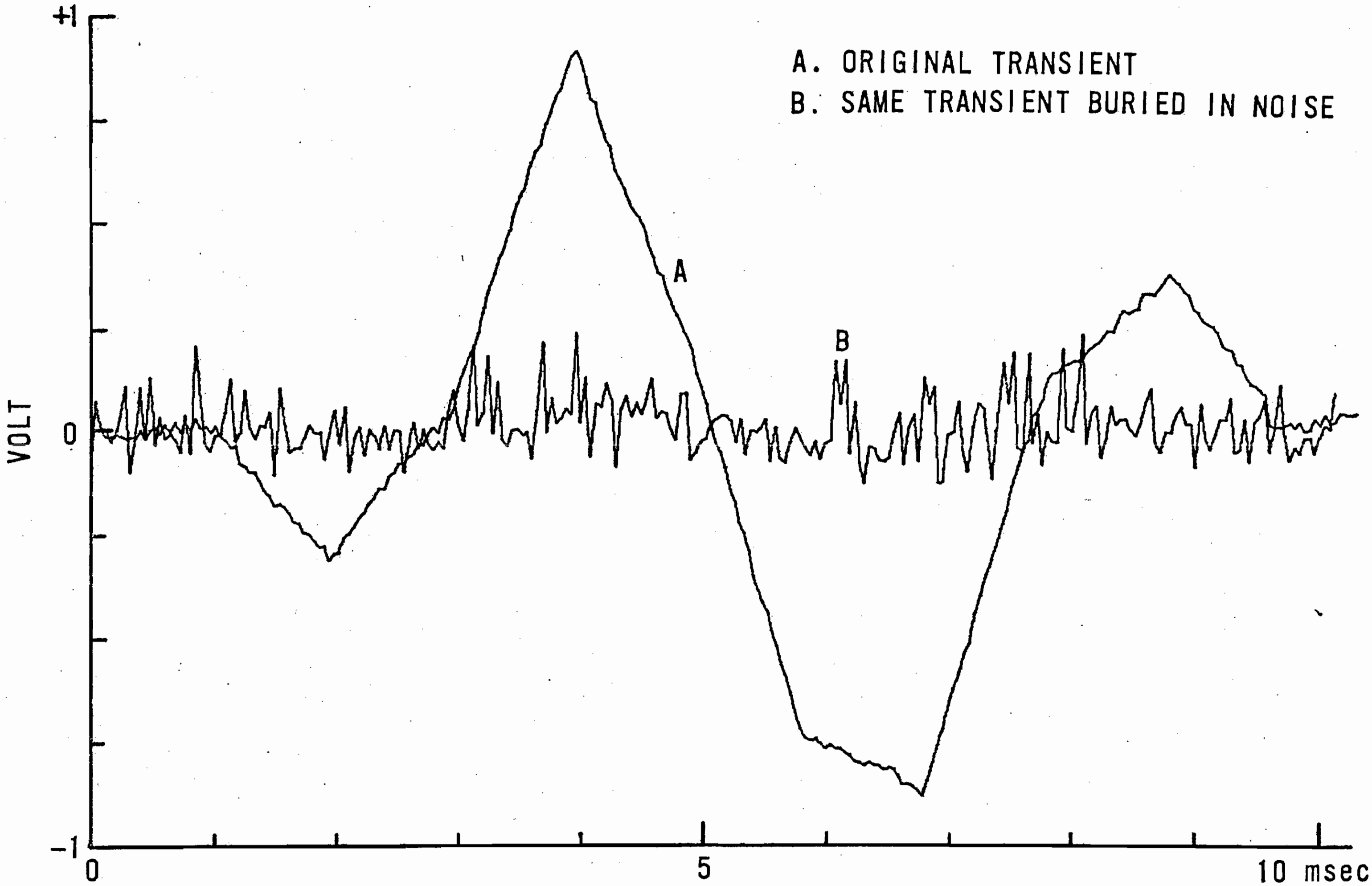

FIG. 11 WAVEFORM AVERAGING

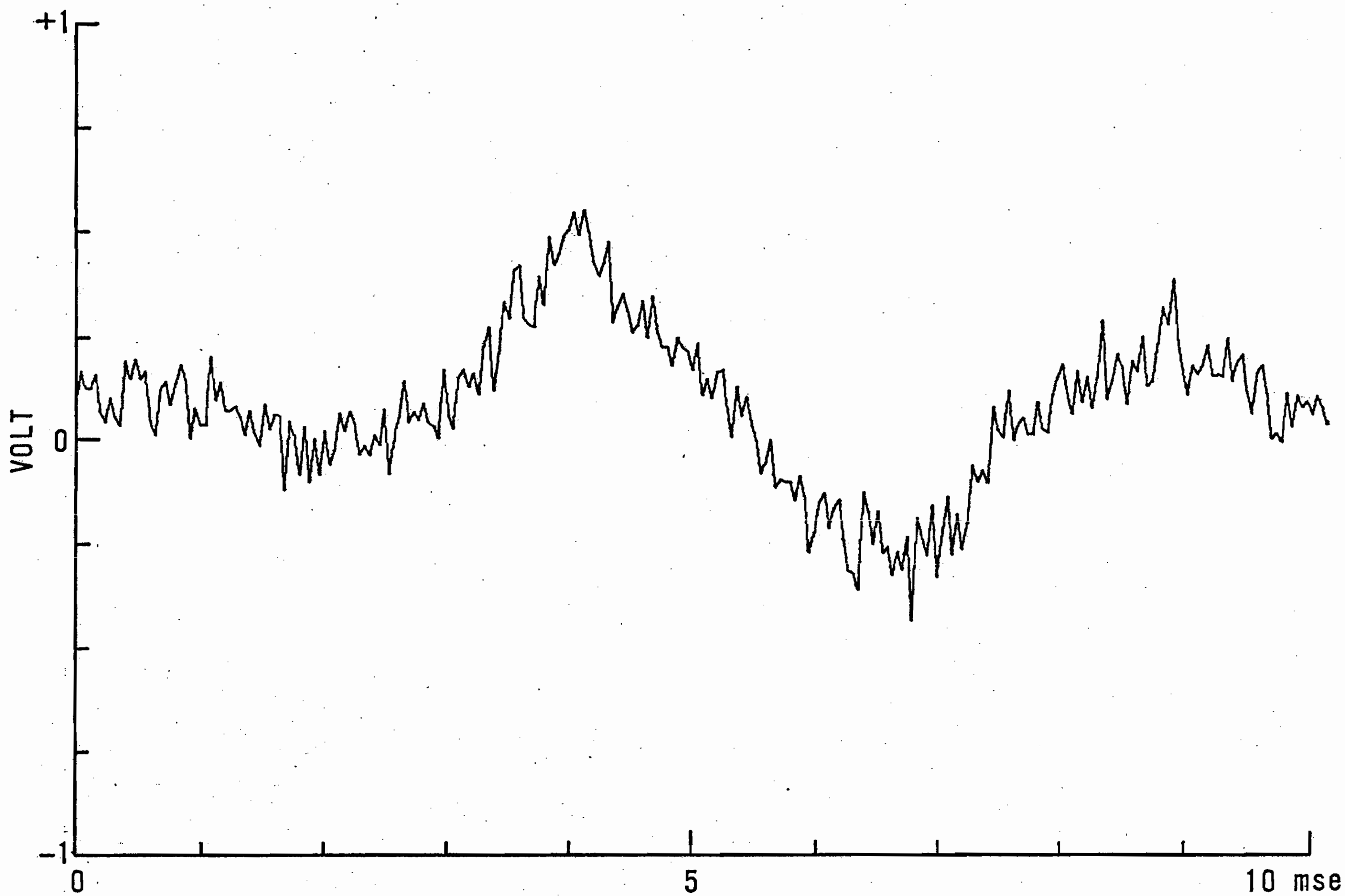

FIG. 12 TRANSIENT AFTER 128 AVERAGES

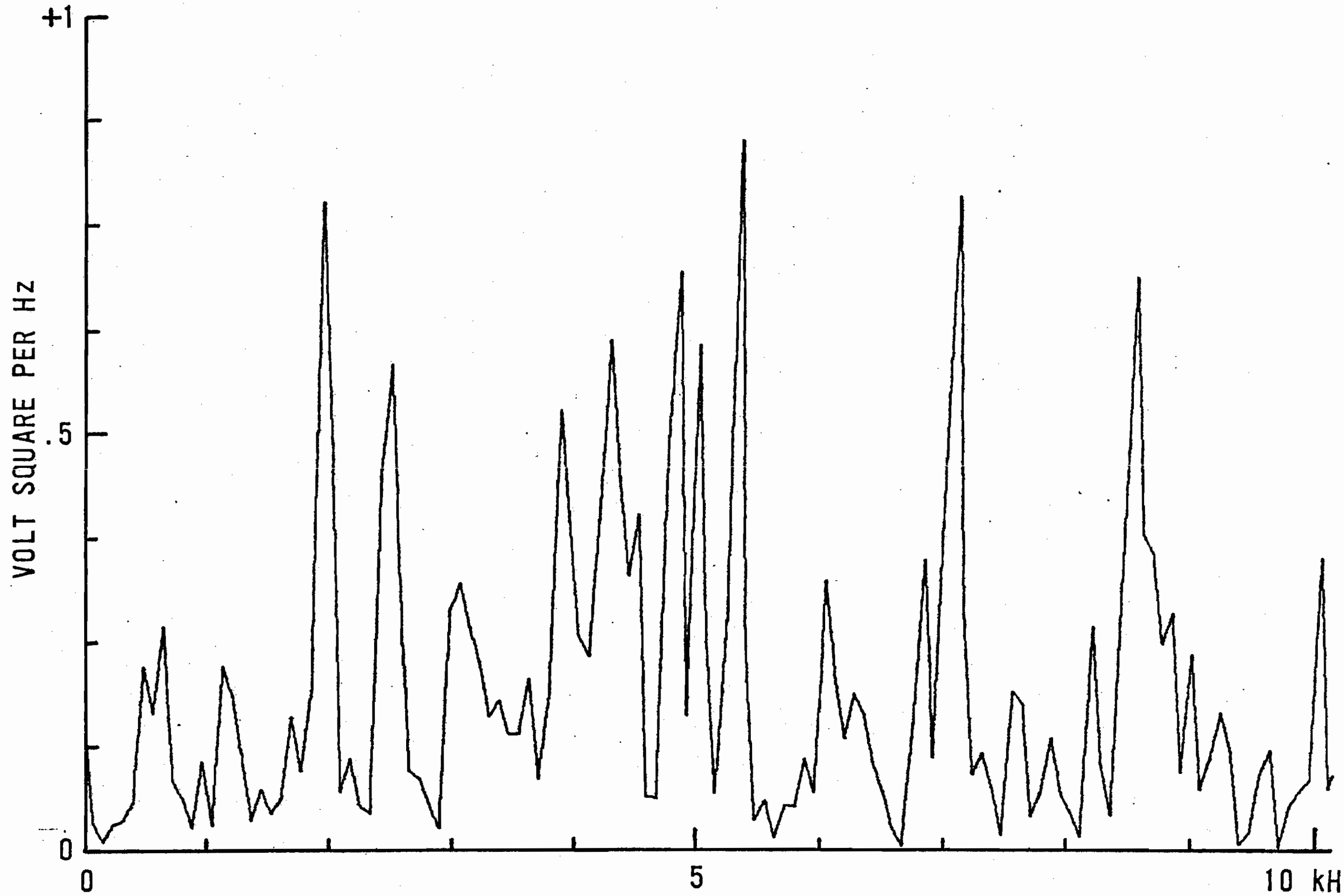

FIG. 13 AUTOSPECTRUM OF WHITE NOISE WITH NO. OF DEGREES OF FREEDOM = 2

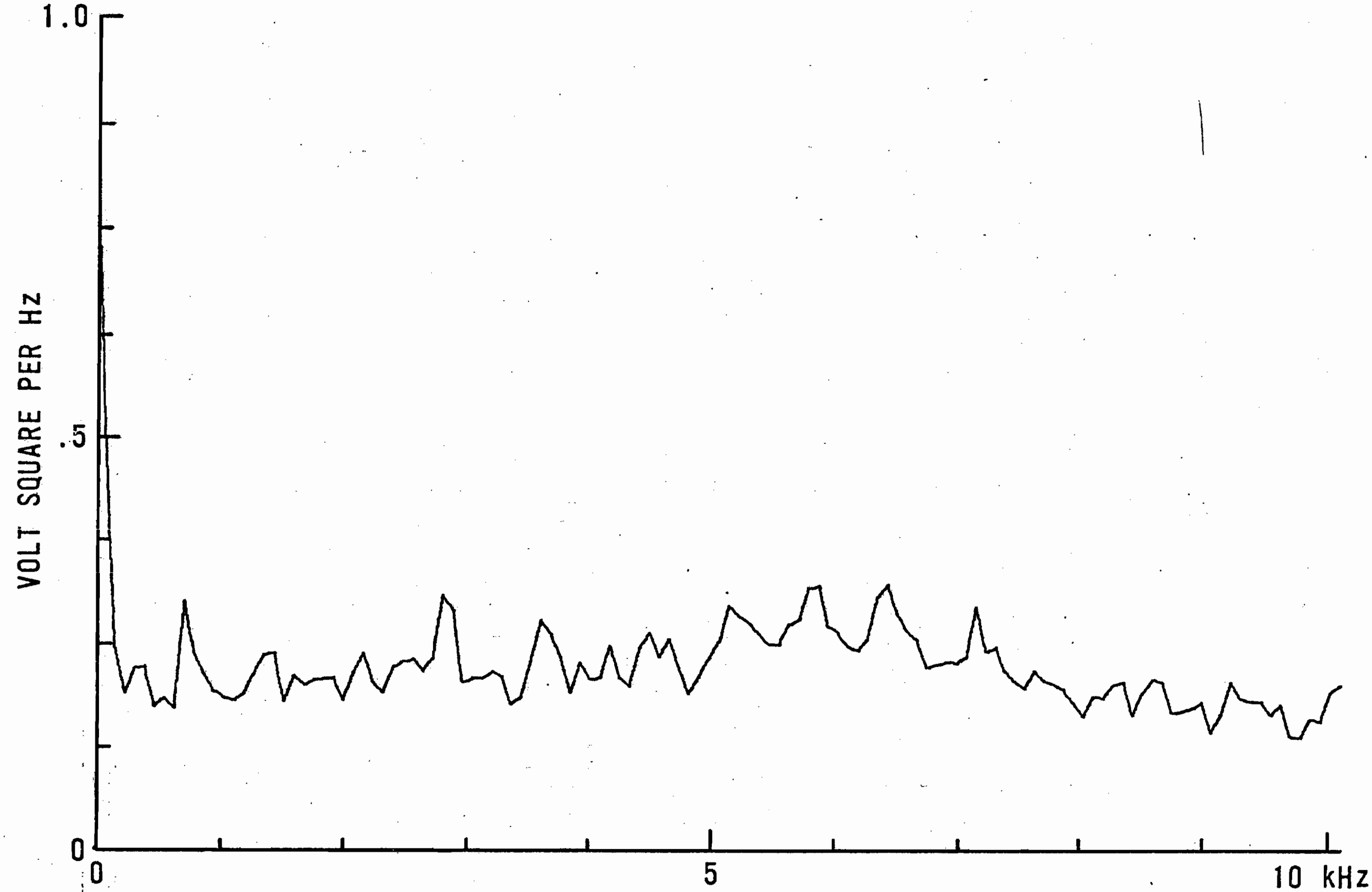

FIG. 14 AUTOSPECTRUM OF WHITE NOISE WITH NO. OF DEGREES OF FREEDOM=128